S. Chand's Smart Maths

NCF Mapping for Classes 6 to 8

MIDDLE STAGE		
Curricular Goals	**Competencies**	**Covered in Classes 6 to 8**
CG-1 Understands numbers and sets of numbers (whole numbers, fractions, integers, rational numbers, and real numbers), looks for patterns, and appreciates relationships between numbers	C-1.1 Develops a sense for and an ability to manipulate (e.g., read, write, form, compare, estimate, and apply operations) and name (in words) large whole numbers of up to 20 digits, and expresses them in scientific notation using exponents and powers	Class 6 – Ch 1 – Knowing Our Numbers Class 7 – Ch 12 – Exponents and Powers Class 8 – Ch 12 – Exponents and Powers
	C-1.2 Discovers, identifies, and explores patterns in numbers and describes rules for their formation (e.g., multiples of 7, powers of 3, prime numbers), and explains relations between different patterns	Class 6 – Ch 2 – Whole Numbers Class 7 – Ch 13 – Algebraic Expressions Class 8 – Ch 9 – Algebraic Expressions and Identities
	C-1.3 Learns about the inclusion of zero and negative quantities as numbers, and the arithmetic operations on them, as given by Brahmagupta	Class 6 – Ch 7 – Integers Class 7 – Ch 1 – Integers
	C-1.4 Explores and understands sets of numbers, such as whole numbers, fractions, integers, rational numbers, and real numbers, and their properties, and visualises them on the number line	Class 6 – Ch 2 – Whole Numbers Class 7 – Ch 1 – Integers Class 8 – Ch 1 – Rational Numbers
	C-1.5 Explores the idea of percentage and applies it to solve problems	Class 7 – Ch 8 – Comparing Quantities Class 8 – Ch 8 – Comparing Quantities
	C-1.6 Explores and applies fractions (both as ratios and in decimal form) in daily-life situations	Class 6 – Ch 8 – Fractions Class 7 – Ch 2 – Fractions and Decimals
CG-2 Understands the concepts of variable, constant, coefficient, expression, and (one-variable) equation, and uses these concepts to solve meaningful daily-life problems with procedural fluency	C-2.1 Understands equality between numerical expressions and learns to check arithmetical equations	Class 6 – Ch 12 – Algebra Class 7 – Ch 4 – Simple Equations Class 8 – Ch 2 – Linear Equations in One Variable

	C-2.2 Extends the representation of a number in the form of a variable or an algebraic expression using a variable	Class 6 – Ch 12 – Algebra Class 7 – Ch 13 – Algebraic Expressions Class 8 – Ch 9 – Algebraic Expressions and Identities
	C-2.3 Forms algebraic expressions using variables, coefficients, and constants and manipulates them through basic operations	Class 6 – Ch 12 – Algebra Class 7 – Ch 13 – Algebraic Expressions Class 8 – Ch 9 – Algebraic Expressions and Identities
	C-2.4 Poses and solves linear equations to find the value of an unknown, including to solve puzzles and word problems	Class 6 – Ch 1 – 4-Digit Numbers Class 7 – Ch 4 – Simple Equations Class 8 – Ch 16 – Playing with Numbers
	C-2.5 Develops own methods to solve puzzles and problems using algebraic thinking	Class 6 – Ch 12 – Algebra Class 7 – Ch 13 – Algebraic Expressions Class 8 – Ch 16 – Playing with Numbers
CG-3 Understands, formulates, and applies properties and theorems regarding simple geometric shapes (2D and 3D)	C-3.1 Describes, classifies, and understands relationships among different types of two- and three-dimensional shapes using their defining properties/attributes	Class 6 – Ch 5 – Understanding Elementary Shapes Class 7 – Ch 15 – Visualising Solid Shapes Class 8 – Ch 10 – Visualising Solid Shapes
	C-3.2 Outlines the properties of lines, angles, triangles, quadrilaterals, and polygons and applies them to solve related problems	Class 6 – Ch 4 – Basic Geometrical Ideas Class 7 – Ch 6 – The Triangle and its Properties Class 8 – Ch 3 – Understanding Quadrilaterals
	C-3.3 Identifies attributes of three-dimensional shapes (cubes, parallelepipeds, cylinders, cones), works hands-on with aterial to construct these shapes, and also uses two-dimensional representations of three-dimensional objects to visualise and solve problems	Class 6 – Ch 5 – Understanding Elementary Shapes Class 7 – Ch 15 – Visualising Solid Shapes Class 8 – Ch 10 – Visualising Solid Shapes
	C-3.4 Draws and constructs geometric shapes, such as lines, parallel lines, perpendicular lines, angles, and simple triangles, with specified properties using a compass and straightedge	Class 6 – Ch 15 – Practical Geometry Class 7 – Ch 5 – Lines and Angles Class 8 – Ch 4 – Practical Geometry

	C-3.5 Understands congruence and similarity as it applies to geometric shapes and identifies similar and congruent triangles	Class 7 – Ch 7 – Congruence of Triangles
CG-4 Develops understanding of perimeter and area for 2D shapes and uses them to solve day-to-day life problems	C-4.1 Discovers, understands, and uses formulae to determine the area of a square, triangle, parallelogram, and trapezium and develops strategies to find the areas of composite 2D shapes	Class 6 – Ch 11 – Mensuration Class 7 – Ch 11 – Perimeter and Area Class 8 – Ch 11 – Mensuration
	C-4.2 Learns the Baudhayana-Pythagoras theorem on the lengths of the sides of a right-angled triangle, and discovers a geometric proof using areas of squares erected on the sides of the triangle, and other related geometric constructions from the Sulba-Sutras	Class 6 – Understanding Elementary Shapes Class 7 – Ch 6 – The Triangle and its Properties Class 8 – Ch 6 – Squares and Square Roots
	C-4.3 Constructs various designs (using tiling) on a plane surface using different 2D shapes and appreciates their appearances in art in India and around the world	Class 6 – Ch 14 – Symmetry Class 7 – Ch 14 – Symmetry
	C-4.4 Develops familiarity with the notion of fractal and identifies and appreciates the appearances of fractals in nature and art in India and around the world	Class 6 – Ch 14 – Symmetry Class 7 – Ch 14 – Symmetry
CG-5 Collects, organises, represents (graphically and in tables), and interprets data/information from daily-life experiences	C-5.1 Collects, organises, and interprets the data using measures of central tendencies such as average/mean, mode, and median	Class 6 – Ch 10 – Data Handling Class 7 – Ch 3 – Data Handling Class 8 – Ch 5 – Data Handling
	C-5.2 Selects, creates, and uses appropriate graphical representations (e.g., pictographs, bar graphs, histograms, line graphs, and pie charts) of data to make interpretations	Class 6 – Ch 10 – Data Handling Class 7 – Ch 3 – Data Handling Class 8 – Ch 5 – Data Handling
CG-6 Develops mathematical thinking and the ability to communicate mathematical ideas logically and precisely	C-6.1 Applies both inductive and deductive logic to formulate definitions and onjectures, evaluate and produce convincing arguments/ proofs to turn these definitions and conjectures into theorems or correct statements, particularly in the areas of algebra, elementary number theory, and geometry	Class 6 – Ch 12 – Algebra

CG-7 Engages with puzzles and mathematical problems and develops own creative methods and strategies to solve them	C-7.1 Demonstrates creativity in discovering one's own solutions to puzzles and other problems, and appreciates the work of others in finding their own, possibly different, solutions C-7.2 Engages in and appreciates the artistry and aesthetics of puzzle-making and puzzle-solving	Class 6 – Ch 3 – Playing with Numbers Class 7 – Ch 4 – Simple Equations Class 8 – Ch 16 – Playing with Numbers
CG-8 Develops basic skills and capacities of computational thinking, namely, decomposition, pattern recognition, data	C-8.1 Approaches problems using programmatic thinking techniques such as iteration, symbolic representation, and logical operations and reformulates problems into series of ordered steps (i.e., algorithmic thinking)	Class 7 – Ch 4 – Simple Equations Class 8 – Ch 9 – Algebraic Expressions and Identities
	C-8.2 Learns systematic counting and listing, systematic reasoning about counts and iterative patterns, and multiple data representations; learns to devise and follow algorithms, with an eye towards understanding correctness, effectiveness, and efficiency of algorithms	Class 7 – Ch 4 – Simple Equations Class 8 – Ch 9 – Algebraic Expressions and Identities
CG-9 Knows and appreciates the development of mathematical ideas over a period of time and the contributions of past and modern mathematicians from India and across the world	C-9.1 Recognises how concepts (like counting numbers, whole numbers, negative numbers, rational numbers, zero, concepts of algebra, geometry) evolved over a period of time in different civilisations.	Class 6 – Ch 2 – Whole Numbers Class 7 – Ch 9 – Rational Numbers Class 8 – Ch 1 – Rational Numbers
	C-9.2 Knows and appreciates the contributions of specific Indian mathematicians (such as Baudhayana, Pingala, Aryabhata, Brahmagupta, Virahanka, Bhaskara, and Ramanujan)	Class 6 – Special Pages (at the end of textbook) Class 7 – Special Pages (at the end of textbook) Class 8 – Special Pages (at the end of textbook)
CG-10 Knows about and appreciates the interaction of Mathematics with each of their other school subjects	10.1 Recognises interaction of Mathematics with multiple subjects across Science, Social Science, Visual Arts, Music, Vocational Education, and Sports	Class 6 – Tagging of CC, EL, AIL and 21st CS icons done Class 7 – Tagging of CC, EL, AIL and 21st CS icons done Class 8 – Tagging of CC, EL, AIL and 21st CS icons done

Revised Edition

As per NCF 2023

S. Chand's

Smart MATHS

A Complete Course in Mathematics

6

Anita Sharma
Dr K P Chinda

This Book Belongs to:

Name ....

Roll No. ....

Class and Section ....

School ....

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(An imprint of S Chand Publishing)
A Division of S Chand And Company Limited
(An ISO 9001 Certified Company)
Head Office : D-92, Sector 2, Noida, Uttar Pradesh-201 301
Phone : 0120-468 2700, **e-mail :** info@schandpublishing.com
Registered Office : A-27, 2nd Floor, Mohan Co-operative Industrial Estate, New Delhi–110 044
www.schandpublishing.com; **e-mail :** helpdesk@schandpublishing.com

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Mumbai	:	Ph: 022-2500 0297, mumbai@schandpublishing.com
Patna	:	Ph: 0612-226 0011, patna@schandpublishing.com

First Published in 2020
This Revised Edition 2024
Third Impression 2024

ISBN : 978-93-55019-23-3 **Product Code :** SCS2SMA060MATAB24CBY

PRINTED IN INDIA
By Vikas Publishing House Private Limited, Plot 20/4, Site-IV, Industrial Area Sahibabad, Ghaziabad–201 010
and Published by S Chand And Company Limited, A-27, 2nd Floor, Mohan Co-operative Industrial Estate, New Delhi–110 044.

PREFACE

S. Chand's **Smart Maths** is a carefully graded Mathematics series of 9 books for KG to Class 8. The series has been designed in accordance with the latest guidelines laid down by the National Curriculum Framework for School Education, 2023. This series is aimed at developing the power of logical and mathematical reasoning.

National Education Policy, 2020 intends to revamp the education system of India in order to meet the requirements of fast-changing world and knowledge-based society to firm-up India's leadership at the global stage. Attaining the goal of a strong foundation of mathematical competencies amongst all children up to the age of 11 years is a national mission. To reach this goal, great emphasis is placed on **interactive** and **fun classrooms, experiential learning, exploration, interdisciplinary approach, collaborative projects, stimulating** and **creative exercise** and **grade-appropriate practice opportunities**.

The text of this book is written to be user-friendly and easy to read and understand. There is not a lot of text, and then a lot of exercises, as in traditional textbooks. It is all thoughtfully mixed together with explanation, exploration, examples, exercises, etc. It is written for students to learn in a collaborative group setting, where students read and work together, helping each other to learn.

THE SALIENT FEATURES OF THE SERIES ARE:

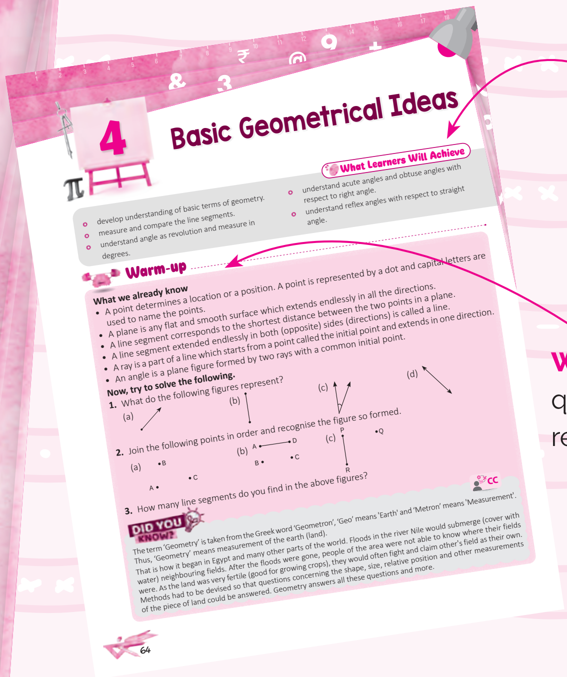
In addition to features such as **Remember**, **HOTS**/**Challenge**, **Did You Know**, and **Competency Based Exercise** S. Chand's **Smart Maths** has the following unique features:

- **What Learners Will Achieve/Learning Outcomes** at the beginning of each chapter clearly define the goals of study.
- **Warm-up** for quick and easy recapitulation.
- **Watch Your Step!** section warns against likely mistakes and misconceptions.
- **Let Us Do** within the text are exploratory in nature to help the learners to draw conclusions on their own or sometimes to verify the concepts with hands-on practice.
- **Skill Check/Practice** section interspersed within the text for self-testing.
- **Assertion-Reasoning Questions** are to ensure enough quick practice opportunities for the learner as per the CBSE guidelines.
- **Smart Time** is to engage children in mathematical discussions and activities/projects to establish a learning environment of fun and enjoyment.
- **Let's Work in Mind** to develop skills in rapid calculations.
- **Case Study** helps to reinforce concepts in an interesting way by providing questions on real-life scenarios.

We would greatly welcome suggestions and feedback for further improvement.

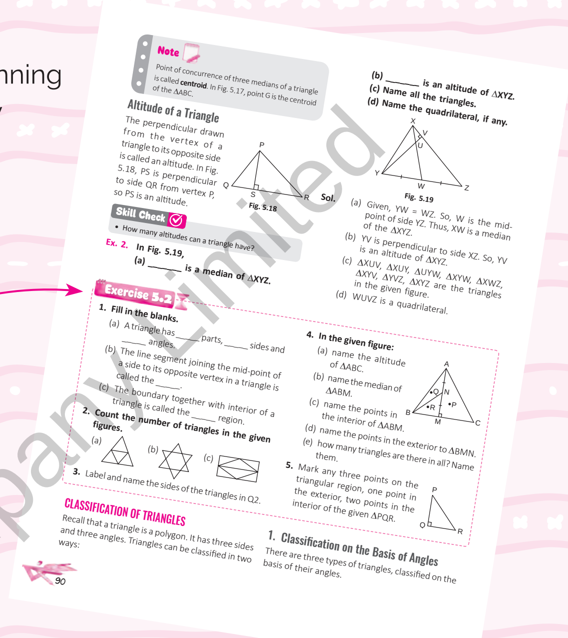
Authors

Features of the Series



What Learners Will Achieve/Learning Outcomes at the beginning of each chapter clearly define the goals of study.

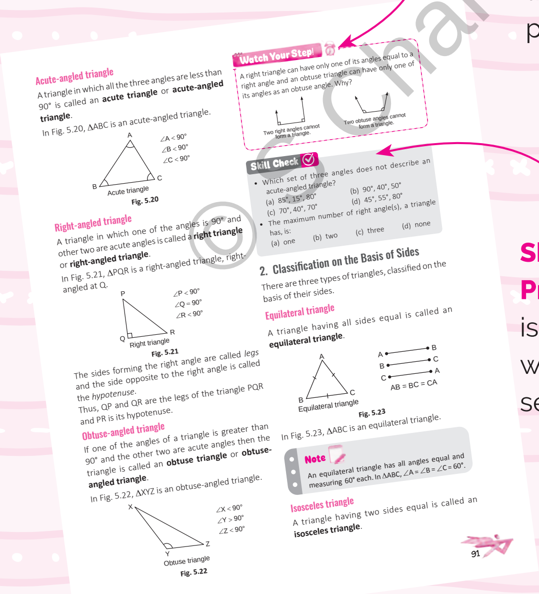
Warm-up for quick and easy recapitulation.

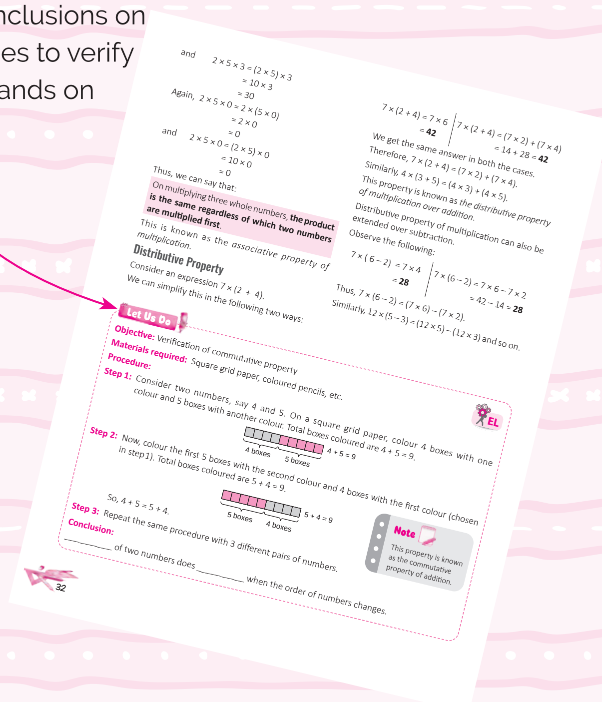


Exercises are extensive exercises for ample practise for learners.

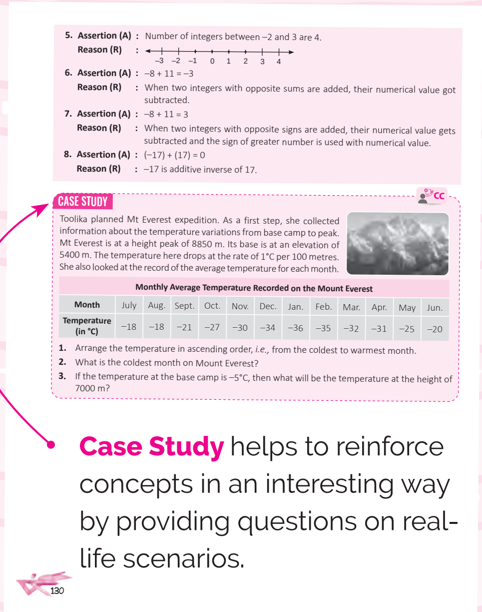
Watch Your Step! section warns against likely mistakes and misconceptions.

Let Us Do within the text are exploratory in nature to help the learners to draw conclusions on her own or sometimes to verify the concepts with hands on practice.





Skill Check/Practice section is interspersed within the text for self-testing.



5. **Assertion (A) :** Number of integers between –2 and 3 are 4.
Reason (R) : –3 –2 –1 0 1 2 3 4

6. **Assertion (A) :** –8 + 11 = –3
Reason (R) : When two integers with opposite sums are added, their numerical value got subtracted.

7. **Assertion (A) :** –8 + 11 = 3
Reason (R) : When two integers with opposite signs are added, their numerical value gets subtracted and the sign of greater number is used with numerical value.

8. **Assertion (A) :** (–17) + (17) = 0
Reason (R) : –17 is additive inverse of 17.

CASE STUDY

Toolika planned Mt Everest expedition. As a first step, she collected information about the temperature variations from base camp to peak. Mt Everest is at a height peak of 8850 m. Its base is at an elevation of 5400 m. The temperature here drops at the rate of 1°C per 100 metres. She also looked at the record of the average temperature for each month.

Monthly Average Temperature Recorded on the Mount Everest

Month	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.
Temperature (in °C)	–18	–18	–21	–27	–30	–34	–36	–35	–32	–31	–25	–20

1. Arrange the temperature in ascending order, *i.e.*, from the coldest to warmest month.
2. What is the coldest month on Mount Everest?
3. If the temperature at the base camp is –5°C, then what will be the temperature at the height of 7000 m?

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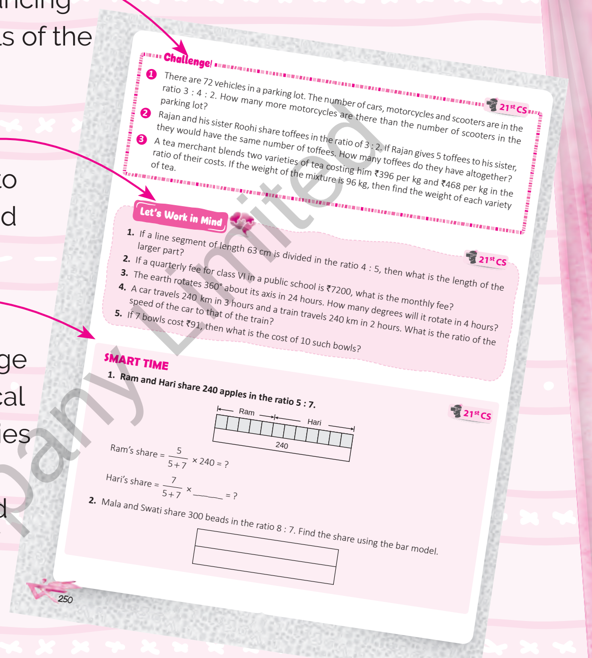
Challenge for enhancing critical thinking skills of the learners.



Challenge

1. There are 72 vehicles in a parking lot. The number of cars, motorcycles and scooters are in the ratio 3 : 4 : 2. How many more motorcycles are there than the number of scooters in the parking lot?
2. Rajan and his sister Roohi share toffees in the ratio of 3 : 2. If Rajan gives 5 toffees to his sister, they would have the same number of toffees. How many toffees do they have altogether?
3. A tea merchant blends two varieties of tea costing him ₹396 per kg and ₹468 per kg in the ratio of their costs. If the weight of the mixture is 96 kg, then find the weight of each variety of tea.

Let's Work in Mind

1. If a line segment of length 63 cm is divided in the ratio 4 : 5, then what is the length of the larger part?
2. If a quarterly fee for class VI in a public school is ₹7200, what is the monthly fee?
3. The earth rotates 360° about its axis in 24 hours. How many degrees will it rotate in 4 hours?
4. A car travels 240 km in 3 hours and a train travels 240 km in 2 hours. What is the ratio of the speed of the car to that of the train?
5. If 7 bowls cost ₹91, then what is the cost of 10 such bowls?

SMART TIME

1. Ram and Hari share 240 apples in the ratio 5 : 7.

Ram Hari 240

Ram's share = $\frac{5}{5+7} \times 240$ = ?

Hari's share = $\frac{7}{5+7} \times$ ______ = ?

2. Mala and Swati share 300 beads in the ratio 8 : 7. Find the share using the bar model.

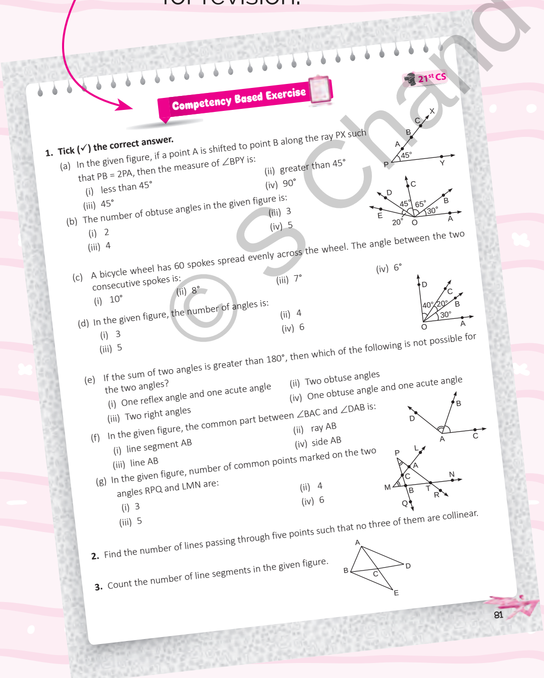
250

Let's Work in Mind to develop skills in rapid calculations.

Case Study helps to reinforce concepts in an interesting way by providing questions on real-life scenarios.

Smart Time is to engage children in mathematical discussions and activities to establish a learning environment of fun and enjoyment.

Competency Based Exercise consisting of a mix of objective- and subjective-type questions for revision.



Competency Based Exercise

1. Tick (✓) the correct answer.
(a) In the given figure, if a point A is shifted to point B along the ray PX such that PB = 2PA, then the measure of ∠BPY is:
(i) less than 45° (ii) greater than 45° (iii) 45° (iv) 90°
(b) The number of obtuse angles in the given figure is:
(i) 2 (ii) 3 (iii) 4 (iv) 5
(c) A bicycle wheel has 60 spokes spread evenly across the wheel. The angle between the two consecutive spokes is:
(i) 10° (ii) 8° (iii) 7° (iv) 6°
(d) In the given figure, the number of angles is:
(i) 3 (ii) 4 (iii) 5 (iv) 6
(e) If the sum of two angles is greater than 180°, then which of the following is not possible for the two angles?
(i) One reflex angle and one acute angle (ii) Two obtuse angles (iii) Two right angles (iv) One obtuse angle and one acute angle
(f) In the given figure, the common part between ∠BAC and ∠DAB is:
(i) line segment AB (ii) ray AB (iii) line AB (iv) side AB
(g) In the given figure, number of common points marked on the two angles RPQ and LMN are:
(i) 3 (ii) 4 (iii) 5 (iv) 6
2. Find the number of lines passing through five points such that no three of them are collinear.
3. Count the number of line segments in the given figure.

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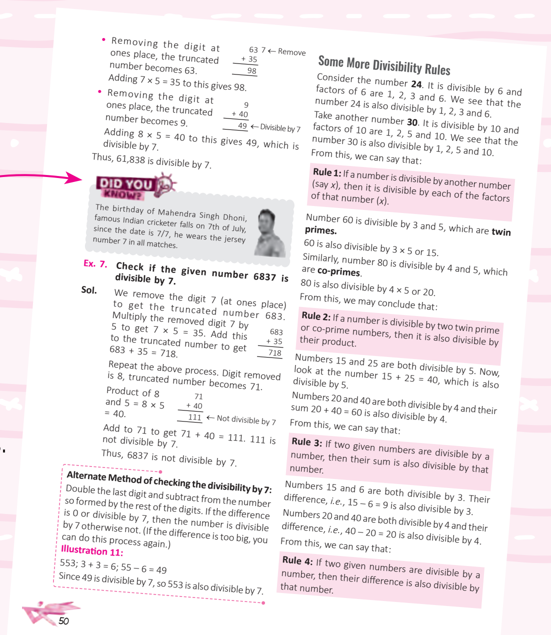

- Removing the digit at ones place, the truncated number becomes 63. Adding 7 × 5 = 35 to this gives 98.
- Removing the digit at ones place, the truncated number becomes 9. Adding 8 × 5 = 40 to this gives 49, which is divisible by 7.

Thus, 61,838 is divisible by 7.

DID YOU KNOW?

The birthday of Mahendra Singh Dhoni, famous Indian cricketer falls on 7th of July, since the date is 7/7, he wears the jersey number 7 in all matches.

Ex. 7. Check if the given number 6837 is divisible by 7.

Sol. We remove the digit 7 (at ones place) to get the truncated number 683. Multiply the removed digit 7 by 5 to get 7 × 5 = 35. Add this to the truncated number to get 683 + 35 = 718.

Repeat the above process. Digit removed is 8, truncated number becomes 71. Product of 8 and 5 = 8 × 5 = 40.

Add to 71 to get 71 + 40 = 111. 111 is not divisible by 7.

Thus, 6837 is not divisible by 7.

Alternate Method of checking the divisibility by 7: Double the last digit and subtract from the number so formed by the rest of the digits. If the difference is 0 or divisible by 7, then the number is divisible by 7 otherwise not. (If the difference is too big, you can do this process again.)

Illustration 11: 553; 3 + 3 = 6; 55 – 6 = 49
Since 49 is divisible by 7, so 553 is also divisible by 7.

Some More Divisibility Rules

Consider the number **24**. It is divisible by 6 and factors of 6 are 1, 2, 3 and 6. We see that the number 24 is also divisible by 1, 2, 3 and 6.

Take another number **30**. It is divisible by 10 and factors of 10 are 1, 2, 5 and 10. We see that the number 30 is also divisible by 1, 2, 5 and 10.

From this, we can say that:

Rule 1: If a number is divisible by another number (say *x*), then it is divisible by each of the factors of that number (*x*).

Number 60 is divisible by 3 and 5, which are **twin primes**. 60 is also divisible by 3 × 5 or 15.

Similarly, number 80 is divisible by 4 and 5, which are **co-primes**. 80 is also divisible by 4 × 5 or 20.

From this, we may conclude that:

Rule 2: If a number is divisible by two twin prime or co-prime numbers, then it is also divisible by their product.

Numbers 15 and 25 are both divisible by 5. Now, look at the number 15 + 25 = 40, which is also divisible by 5.

Numbers 20 and 40 are both divisible by 4 and their sum 20 + 40 = 60 is also divisible by 4.

From this, we can say that:

Rule 3: If two given numbers are divisible by a number, then their sum is also divisible by that number.

Numbers 15 and 6 are both divisible by 3. Their difference, *i.e.*, 15 – 6 = 9 is also divisible by 3. Numbers 20 and 40 are both divisible by 4 and their difference, *i.e.*, 40 – 20 = 20 is also divisible by 4.

From this, we can say that:

Rule 4: If two given numbers are divisible by a number, then their difference is also divisible by that number.

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Did You Know? to provide interesting mathematical facts.

Contents

1 Knowing Our Numbers

What Learners Will Achieve

- recapitulate the concepts of reading and writing large numbers.
- solve problems involving large numbers by applying appropriate operations (+, –, × and ÷).
- estimate the sum, difference, product and quotient of large numbers by rounding off.
- know the use of brackets.
- write Hindu-Arabic numerals as Roman numerals and vice versa.

Warm-up

What we already know

- The numbers 1, 2, 3, 4, 5, ... are called *counting numbers* or *natural numbers*.
- There are infinite *counting numbers* as every number is followed by the next number which is greater than the previous number by 1.
- There are two place value systems—Indian and International, used to read and write numbers.
- The place value of a digit depends upon its position in the number whereas the face value of a digit is the value of the digit itself.
- There are seven symbols I, V, X, L, C, D and M which are used to write all the numbers in Roman numerals. This does not have a place value system.

Now, try to solve the following.

Successor = Number + 1
Predecessor = Number – 1

1. Fill in the blanks.

(a) There are __________ 3-digit numbers from __________ to __________.

(b) There are __________ 5-digit numbers from __________ to __________ .

(c) The place value and face value of a digit at the __________ place is always the same.

(d) The place value and face value of 5 in 13,546 are __________ and __________, respectively.

(e) The greatest 4-digit number is __________ and the smallest 5-digit number is __________.

(f) The predecessor and successor of 76,901 are __________ and __________, respectively.

2. Write 38,298 in expanded form.

3. Is 90,000 the greatest 5-digit number?

4. What is the product of the place values of digit 3 in 63,532?

Roman numeral system does not have any symbol to represent 0.

5. Write:

(a) 67 in Roman numeral. (b) LXXV in Hindu-Arabic numeral.

WHAT DO WE KNOW ABOUT NUMBERS?

Numbers play an important role in our day-to-day life. For example, to count the number of students going to picnic, to know the amount contributed at CRPF welfare association, to record the performance of participants, to tell the time, etc.

In previous classes, we have learnt to read and write numbers up to lakhs place. Before proceeding further, let us revise the concept learnt.

Place Value and Face Value

Place value: The place value of a digit in a number depends upon its position in the number.

Face value: The face value of a digit is the value of the digit itself.

For example, in

3 7 5 4 6	Place value	Face value
6 →	6 ones = 6 × 1 = 6	6
4 →	4 tens = 4 × 10 = 40	4
5 →	5 hundreds = 5 × 100 = 500	5
7 →	7 thousands = 7 × 1000 = 7000	7
3 →	3 ten thousands = 3 × 10000 = 30000	3

Remember

The place value and face value of 0 is always 0, irrespective of its place in the number.

Expanded Form and Short Form

The **expanded form** of a number shows the place value of each of its digits.

For example, the number 42,369 can be written in expanded form as:

42,369 = 40,000 + 2000 + 300 + 60 + 9

(42,369: Short form; 40,000 + 2000 + 300 + 60 + 9: Expanded form)

A number written as numeral using commas is called **short form** of the number.

Place Value Chart

The number we use is based on the decimal system. In this system, the value of a digit increases 10 times when it moves to the left place.

For example, observe the place values of 6 in the numbers 39,652 and 86,417.

TTh	Th	H	T	O
3	9	⑥	5	2
8	⑥	4	1	7

The values are 600 and 6000 respectively.

We see 600 × 10 = 6000.

[**Note:** The place value chart is very helpful in comparing and building numbers.]

DID YOU KNOW?

Indian Mathematicians discovered the decimal system, *i.e.,* use of ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to represent any number. They were aware of the significance of zero and place value.

Though these were invented by Hindus some time before 200 BCE, but were popularised throughout the world by Arabic Mathematicians, so these numerals are popularly known as **Hindu-Arabic numerals**.

Successor and Predecessor

The **successor** of a number is one more than the given number.

For example, 8206 + 1 = 8207.

The **predecessor** of a number is one less than the given number.

For example, 206 – 1 = 205.

Comparing Numbers

If two numbers are not equal, then one is either **greater than** or **less than** the other. To find which of the two numbers is **greater**, we follow a simple procedure to compare the numbers:

Case 1. The number having more digits is greater than the other.

Illustration 1: Let us compare 21,752 and 9580. Since 21,752 has more digits than 9580; so 21,752 **is greater than** 9580.

Case 2. If the number of digits in both the numbers is the same, then follow the given steps:

Step 1: Compare the digits in the corresponding places of the two numbers one by one, starting from the highest place.

Step 2: If the digits at the highest place are equal, move one place to the right and compare the digits at this place in both the numbers.

Step 3: Continue this process till you get two different digits at the same place in the two numbers.

Step 4: Stop when you get different digits for the first time. The number having greater digit is greater.

Illustration 2: Let us compare 24,566 and 24,578. Both have 5 digits. We observe that the first 3 digits from the left in both the numbers are the same. So, we now compare the digits at the 4th place from the left of the numbers. At this place, the digits are 6 and 7 respectively.

TTh	Th	H	T	O
2	4	5	6	6
2	4	5	7	8

Observe: → Same Different (6 < 7)

So, 24,5**6**6 < 24,5**7**8.

Ordering Numbers

Ascending order: Numbers can be arranged in order from the smallest to the largest. This order is called an **ascending order.**

For example, 4, 15, 102 and 3425 are in ascending order.

Descending order: Numbers can be arranged from the largest to the smallest. This order is called a **descending order**.

For example, 3245, 1038, 435, 65 and 9 are in descending order.

Skill Check

- In the number 18,439, which digit has the greatest place value?
- Are 2345, 3456, 4567 and 5678 in descending order?
- Which among the numbers: 9850, 5098, 8905 and 5089 is the smallest?
- Write 5382 in expanded form.

BUILDING NUMBERS

Using the given digits, we can make different numbers by placing these digits (without repetition) in different positions.

Building the Largest Number

To build the largest number using the given digits, follow these steps:

Step 1: Place the largest digit in the highest place value.

Step 2: Now, place the next largest digit at the next highest place value and so on till all the digits are placed.

Illustration 3: To build the greatest 4-digit number using the digits 3, 5, 1 and 7 (without repetition), we observe that 7 > 5 > 3 > 1.

So, to form the greatest 4-digit number, place the digits in the place value chart in descending order as shown.

Th	H	T	O
7	5	3	1

Think! 21st CS

Why does the extreme left position hold the highest place value?

Building the Smallest Number

To build the smallest number using the given digits, place the digits in ascending order starting from the highest place value in the chart.

Illustration 4: To build the smallest 4-digit number using the digits 3, 1, 5 and 7 (without repetition), we observe that 1 < 3 < 5 < 7.

So, to form the smallest 4-digit number, arrange the digits in ascending order in the place value chart as shown on the next page.

Th	H	T	O
1	3	5	7

Building the smallest number when 0 is one of the digits

Illustration 5: To form the smallest 4-digit number, using the digits 5, 7, 1 and 0 (without repetition).

- First observe the smallest non-zero digit, *i.e.,* 1 here.
- Now, place the smallest non-zero digit at the highest place as shown.

Th	H	T	O
1			

- Place the smallest digit (0) next to the highest place.
- Place the rest of the digits, *i.e.*, 5 and 7 in ascending order in the place value chart.

Thus, the number so formed is:

Th	H	T	O
1	0	5	7

Building numbers using repetition of digits

Illustration 6: To form the greatest and smallest 4-digit numbers using any one digit twice: 8, 3, 4.

The greatest 4-digit number can be formed by placing the greatest digit twice at the highest place and the place next to it in the place value chart.

So, the greatest 4-digit number = 8843.

The smallest 4-digit number can be formed by placing the smallest (non-zero) digit twice at the highest place and the place next to it in the place value chart.

So, the smallest 4-digit number = 3348.

Skill Check

- What is the largest 4-digit number that can be formed using the digits 2, 4, 1 and 6?
- What is the smallest 4-digit number that can be formed using the digits 0, 3, 5 and 4?

Building new number by interchanging digits

By changing the place value of the digits of a number, new numbers are formed.

Illustration 7: Take a 3-digit number, say 285 and interchange the digit at the ones place with the digit at the hundreds place.

Before shifting: 285 After shifting: 582

H	T	O
2	8	5

H	T	O
5	8	2

Interchange 2 and 5

Note here that the number 582 is greater than 285.

Now, if we interchange the digits at the tens place with that at the ones place (in the same number, 285), we get

Before shifting: 285 After shifting: 2**58**.

Note here that the number 258 is smaller than 285.

Illustration 8: Let us consider a 4-digit number, say 4372.

Now, find a new number by interchanging its ones digit with the tens digit.

4372 (Original number) 43**27** (New number)

(Interchanging the places of the digits 7 and 2)

Further, find another number by interchanging the hundreds digit with the thousands digit, in the new number.

4327 **34**27

(Interchanging the places of the digits 4 and 3)

In this way, we can make many numbers by interchanging the digits of a number.

Let us study some more examples.

Ex. 1. Arrange the following numbers in ascending and descending orders:

847, 9754, 8320, 571

Sol. We know that ascending order means an arrangement from the smallest to the greatest.

	847	9754	8320	571
	↓	↓	↓	↓
Number of digits:	3	4	4	3

	Th	H	T	O	
5 < 8		5	7	1	3-digit numbers
		8	4	7	
9 > 8	8	3	2	0	4-digit numbers
	9	7	5	4	

Therefore, arrangement in ascending order is 571, 847, 8320, 9754.
And, arrangement in descending order is 9754, 8320, 847, 571.

Ex. 2. Make the greatest and smallest 4-digit numbers by using any one digit twice: 8, 0, 4

Sol. The greatest 4-digit number can be formed by using the greatest digit twice at the highest place and the place next to it.

So, the greatest 4-digit number is 8840.

The smallest 4-digit number can be formed by using the smallest non-zero digit at the highest place value and the digit 0 twice at the places next to it.

So, the smallest 4-digit number is 4008.

Watch Your Step!

The smallest 4-digit number using the digits 4, 0, 8 is neither 0408 nor 0048. 0 can be repeated but not at the highest place.

Ex. 3. Make the greatest and smallest 4-digit numbers using the digits 6, 7, 8 and 9, and satisfying the condition that the digit 9 is always at the tens place.

Sol. It is given that the digit 9 is always at the tens place.

Th	H	T	O
		9	

To write the greatest 4-digit number using 6, 7, 8 and 9 under the given condition, write the remaining digits 6, 7, 8 in descending order, *i.e.*, 8, 7, 6.

Th	H	T	O
8	7	9	6

Thus, the required greatest 4-digit number is 8796.

To write the smallest 4-digit number using 6, 7, 8 and 9 under the given condition, write the remaining digits 6, 7, 8 in ascending order, *i.e.*, 6, 7, 8.

Th	H	T	O
6	7	9	8

Thus, the required smallest 4-digit number is 6798.

Exercise 1.1

1. Identify the statement as true or false.

(a) 42,276 < 85,710 (b) 86,982 < 27,875 (c) 34,370 > 30,439 (d) 10,000 < 9999

2. Compare the following numbers using >, = or <.

(a) 4259 ☐ 2594 (b) 39,751 ☐ 39,715 (c) 24,680 ☐ 24,680 (d) 93,571 ☐ 95,731

3. Form a new number by interchanging the digits for the following places.

(a) 2378; hundreds place and ones place (b) 4079; hundreds place and tens place

(c) 3999; thousands place and tens place (d) 6725; thousands place and hundreds place

4. Write the smallest and greatest 4-digit numbers using the following digits.

(a) 3, 7, 1, 5 (b) 7, 2, 3, 9

5. Arrange the following numbers in ascending order.

(a) 48,807; 84,087; 87,084; 47,808 (b) 92,360; 98,632; 32,960; 32,999

6. Arrange the following numbers in descending order.

(a) 235; 628; 98,570; 1257; 7541 (b) 25,837; 73,258; 85,237; 75,328

7. Form the greatest 5-digit number using the digits 3, 6, 7, 8 and 0.
8. Form the greatest 5-digit number using the digits 6, 2, 3 and when repetition of digits is allowed.
9. **Write the greatest and smallest 4-digit numbers, satisfying the conditions given.**
   (a) 5, 0, 2, 7 (Condition: Digit 0 is always at the tens place.)
   (b) 7, 6, 3, 5 (Condition: Digit 7 is always at the ones place.)
   (c) 2, 0, 6, 4 (Condition: Digit 6 is always at the hundreds place.)
   (d) 5, 6, 1, 9 (Condition: Digit 1 is always at the ones place.)
   (e) 5, 3, 7, 2, 6 (Using any one digit twice)
10. Form the least 5-digit number using the digits 1, 4 and 7 with the condition that the digit at the ones place is 4 times the digit at the thousands place. (Repetition of digits is allowed.)

INTRODUCING LARGE NUMBERS

We often come across large numbers in our real life while reading an important information.

Read the following statements:

- 27,000 trees are cut every day to make tissue paper.
- We consume 1,60,000 plastic bags every second and hence dump 5000 billion plastic bags every year in the rivers, seas and oceans.
- In India, through television and other online sources, over 320 lakh people watched Final match of FIFA 2022 football world cup held in Qatar.
- The number of mobile phone users in the world is expected to cross the 7.49 billion mark by 2025.
- Indian railways is one of the largest railway networks in the world. It operates a total of about 12,617 passenger trains transporting 230 lakhs passengers daily across the country.

To understand the given information, we shall learn to read and write large numbers containing 6 or more digits.

Large numbers are numbers that are significantly larger than those normally used in day-to-day life. Numbers like **one lakh, ten lakh, one crore, ten crore, etc.**, are examples of large numbers.

We know that:

- The greatest 4-digit number = 9999.
  On adding 1 to it, we get 9999 + 1 = 10,000 or **ten thousand**, the smallest 5-digit number.
- The greatest 5-digit number = 99,999.
  On adding 1 to it, we get 99,999 + 1 = 1,00,000 or **one lakh**, the smallest 6-digit number.

We can extend the number system further in the same way.

- The greatest 6-digit number = 9,99,999.
  On adding 1 to it, we get 9,99,999 + 1 = 10,00,000 or **ten lakh**, the smallest 7-digit number.
- The greatest 7-digit number = 99,99,999.
  On adding 1 to it, we get 99,99,999 + 1 = 1,00,00,000 or **one crore**, the smallest 8-digit number and so on.

Reading and Writing Large Numbers

There are two systems of reading and writing large numbers—**the Indian system of numeration** and the **International system of numeration**.

Indian system of numeration

In order to read and write large numbers in the Indian system of numeration, we group place values into periods or groups like **Ones**, **Thousands**, **Lakhs**, **Crores**, **Arabs**, etc.

The Indian place value chart is as follows (see Table 1.1):

Indian Place Value Chart

PERIODS →	CRORES		LAKHS		THOUSANDS		ONES		
PLACES →	TC	C	TL	L	TTh	Th	H	T	O
	10,00,00,000	1,00,00,000	10,00,000	1,00,000	10,000	1000	100	10	1

Table 1.1

In this system, first comma comes after the hundreds place and other commas come after every two digits to the left of the previous comma, starting after the first comma.

Illustration 1: Let us try to read 73250422.

Use of commas makes number reading easy. We can now read the number **7,32,50,422** easily as **seven crore thirty-two lakh fifty thousand four hundred twenty-two**.

Remember

- We do not use the periods in plural while reading or writing number names.
  For example, we read ten crore not ten crores.
- We may leave space between two periods instead of using commas.
  For example, 10,15,75,009 is the same as 10 15 75 009.

International system of numeration

In order to read numbers in the International system of numeration, we use **Ones**, **Thousands**, **Millions**, **Billions** and **Trillions** as periods.

The International place value chart is as follows (see Table 1.2):

International Place Value Chart

PERIODS →	MILLIONS			THOUSANDS			ONES		
PLACES →	HM	TM	M	HTh	TTh	Th	H	T	O
	100,000,000	10,000,000	1,000,000	100,000	10,000	1000	100	10	1

Table 1.2

In the International system, comma is placed after every three places, starting from the rightmost digit.

Illustration 2: Let us try to read 7636902.

In the International system of numeration, we write the number as **7,636,902** and read as **seven million six hundred thirty-six thousand nine hundred two**.

Let Us Do EL

Objective: Reading and writing large numbers

Materials required: Paper slips, a bowl, a pencil

Procedure:

Step 1: Write the large numbers on paper slips and put them in a bowl.

Step 2: Divide the class in two equal groups and name them as **Indian delegation** and **American delegation**. Tell the class that both delegations are meeting to make a business deal. During their talk, they will be using their systems of numeration to read large numbers.

Step 3: Call one student from each group. Ask them to pick a slip.
For example, they pick a slip with number 234698678.

Step 4: Now, instruct both of them to read the number in their respective systems of numeration.

Indian delegate will speak: Mr John, I will invest *twenty-three crore forty-six lakh ninety-eight thousand six hundred seventy-eight* rupees in your project but I expect the same amount of investment from your side.

Delegate from American group will reply: Ok, you mean that I must invest *two hundred thirty-four million six hundred ninety-eight thousand six hundred seventy-eight* rupees.

Step 5: If both of them read and speak the number correctly, the teacher will say, 'the deal is final'.

Step 6: Repeat the activity with two new students (one from each group).

Skill Check

- What is the difference between the values of two 9s in the number 196097?
- What is the place value of 5 in 3057421?
- What is the place value of the digit underlined in $\underline{6}50236$?
- Thirty-five thousand two hundred nine is written in numerals as ____________.
- Write the numeral which represents 'five hundred five million one thousand five'.

Expanded Form of Numbers

To write a number in expanded form, follow these steps:

Step 1: Obtain the place value of each digit in the number in order from left to right.

Step 2: Write the sum of their place values.

Illustration 3: Let us write 6,54,934 in its expanded form.

In the number 6,54,934,

The place value of 6 = 6 × 1,00,000 = 6,00,000

The place value of 5 = 5 × 10,000 = 50,000

The place value of 4 = 4 × 1000 = 4000

The place value of 9 = 9 × 100 = 900

The place value of 3 = 3 × 10 = 30

The place value of 4 = 4 × 1 = 4

So, the expanded form of 6,54,934 = 6,00,000 + 50,000 + 4000 + 900 + 30 + 4.

Note

Place value can be written in short as PV.

Remember

The number 6,54,934 is read as 'six lakh fifty-four thousand nine hundred thirty-four'. It is called as number name.

Let us study some more examples.

Ex. 4. Fill in the blank: 1 million = _____ lakhs.

Sol. 1 million = 1,000,000; 1 lakh = 1,00,000

We can write 1 million = 1,000,000 = 10 × 1,00,000 = 10 lakhs

Thus, 1 million = 10 lakhs.

Ex. 5. Write the number name for 73458695 according to the International system of numeration.

Sol. Place the digits of 73458695 in the International place value chart as follows (see Table 1.3):

PERIODS →	MILLIONS			THOUSANDS			ONES		
PLACES →	HM	TM	M	HTh	TTh	Th	H	T	O
		7	3	4	5	8	6	9	5

Table 1.3

Now, mark the commas as 73,458,695. Thus, the number name for 73,458,695 is seventy-three million four hundred fifty-eight thousand six hundred ninety-five.

Ex. 6. Write the numeral for the number name of 'seven hundred fifty-six million three hundred forty-two'.

Sol. Writing the given number in the place value chart (see Table 1.4), we have

PERIODS →	MILLIONS			THOUSANDS			ONES		
PLACES →	HM	TM	M	HTh	TTh	Th	H	T	O
	7	5	6				3	4	2

Table 1.4

Since thousands are not used in the number name, place the digit zero at the Th, TTh and HTh places. Thus, the numeral is 756,000,342.

Ex. 7. Write the numeral for 'nine crore four lakh forty-two'.

Sol. In the Indian system of numeration, commas are used to mark thousands, lakhs and crores period. In numerals, we write the given number as 9,04,00,042.

Exercise 1.2

1. **Fill in the blanks.**
   (a) 1 crore = ________ thousands
   (b) 1 million = ________ thousands
   (c) 10 millions = ________ lakhs
   (d) 100 thousands = ________ lakh(s)
2. Write the smallest 7-digit number and the greatest 5-digit number; and find their difference.
3. **Place commas according to the Indian system of numeration in the following numbers. Also, write their number names.**
   (a) 22578093 (b) 79985786 (c) 300508599
4. **Place commas according to the International system of numeration in the following numbers. Also, write their number names.**
   (a) 56842372 (b) 666666666 (c) 38607500
5. **Write the numeral for the given number names in the place value chart.**
   (a) Eight million twenty-three thousand eight hundred
   (b) Sixty-two million eighty-three thousand five hundred sixty-one
   (c) Two hundred million three hundred fifty-four
6. **Write the expanded form for the given number names.**
   (a) Six crore fifteen lakh twenty-two (b) Seventy crore forty-eight lakh eighty-three
   (c) Ninety-four lakh fifty-two thousand
7. There are about 1,55,015 post offices in India. Write the number name for the number of post offices in India.
8. **The following table shows the average number of spectators per FIFA WORLD CUP from 2002 to 2022.**

CC

Year	FIFA World Cup	Total Number of Spectators
2022	FIFA WORLD CUP QATAR	3,400,000
2018	FIFA WORLD CUP RUSSIA	3,031,768
2014	FIFA WORLD CUP BRAZIL	3,441,450
2010	FIFA WORLD CUP SOUTH AFRICA	3,167,984
2006	FIFA WORLD CUP GERMANY	3,367,000
2002	FIFA WORLD CUP SOUTH KOREA/JAPAN	2,724,604

   (a) Arrange the number of spectators for the years 2002 to 2022 in ascending order.
   (b) Also, find in which years of the FIFA WORLD CUP, there were the greatest number of spectators and the smallest number of spectators.
9. **Compare the following:**
   (a) 9,40,625 ☐ 9,41,625
   (b) 1,00,000 ☐ 9,99,999
   (c) 1,11,654 ☐ 1,111,654
   (d) 10,00,000 ☐ 1 million

10. Build the greatest and the smallest numbers, as directed.

(a) 7-digit number, using the digits 1, 5, 7, 8, 0, 9 and 6

(b) 8-digit number, using any digits

(c) 6-digit number, using the digits 7, 0, 9 and 5 (with repetition)

(d) 7-digit number, using the digits 0, 3 and 2 (with repetition)

11. If the digits of the numbers are interchanged (as marked), find the new number. Also, write the expanded form of the new number, thus formed.

(a) 1,36,582 (b) 77,78,688 (c) 90,00,583 (d) 68,88,888

LARGE NUMBERS IN PRACTICE

Conversion of Length, Mass and Capacity

In our daily life, we often measure length, mass and capacity. For measuring length, we use millimetre (mm), centimetre (cm), metre (m) and kilometre (km).

The relationship among these units are as follows:

1 cm = 10 mm
1 m = 100 cm = 100 × 10 mm = 1,000 mm
1 km = 1000 m = 1000 × 100 cm = 1,00,000 cm
= 1,00,000 × 10 mm = 10,00,000 mm

For measuring mass, we generally use milligram (mg), gram (g) and kilogram (kg).

The relationship among these units are as follows:

1 g = 1000 mg
1 kg = 1000 g = 1000 × 1000 mg = 10,00,000 mg

Similarly, for measuring capacity, we generally use millilitre (mL), litre (L) and kilolitre (kL).

The relationship among these units are as follows:

1 L = 1000 mL
1 kL = 1000 L = 1000 × 1000 mL = 10,00,000 mL

Illustration 1: Let us convert 12 kg into g and mg.

12 kg = 12 × 1000 g (∵ 1 kg = 1000 g)
= 12,000 g
= 12,000 × 1000 mg (∵ 1 g = 1000 mg)
= 1,20,00,000 mg

Illustration 2: Let us convert 2 kL 50 L into mL.

We have, 2 kL 50 L
= 2 × 1000 L + 50 L (∵ 1 kL = 1000 L)
= (2000 + 50) L = 2050 L
= 2050 × 1000 mL (∵ 1 L = 1000 mL)
= 20,50,000 mL

Illustration 3: Let us compare 45 mm, 6 km and 15 m.

We have, 45 mm, 6 km and 15 m.

So, 6 km = 6 × 10,00,000 mm = 60,00,000 mm

15 m = 15 × 1000 mm = 15,000 mm

Clearly, 45 mm < 15,000 mm < 60,00,000 mm,

i.e., 45 mm < 15 m < 6 km.

Skill Check

Compare using <, = or >.

(a) 2 m 5 cm ________ 500 mm

(b) 6 kg ________ 6000 mg

(c) 30 L 30 mL ________ 3,00,000 mL

(d) 5 km 5 m ________ 5,00,500 cm

Applications

In our day-to-day life, we come across several situations where we need to solve problems involving addition, subtraction, multiplication and division dealing with large numbers.

Let us study some more examples.

Ex. 8. In 2023, there were 53,801 males and 45,115 females in a city. What was the population of the city in the year 2023?

Sol. Number of males = 53,801
Number of females = 45,115
Thus, the population in the year 2023
= 53,801 + 45,115 = 98,916.

Ex. 9. The sum of two numbers is 79,615. One of the numbers is 48,530. Find the other number.

Sol. Sum of two numbers = 79,615

One of the two numbers = 48,530

Thus, the other number

= 79,615 – 48,530 = 31,085

Ex. 10. Find the difference between the largest 4-digit number and the smallest 7-digit number.

Sol. Largest 4-digit number = 9999

Smallest 7-digit number = 10,00,000

Difference = 10,00,000 – 9999 = 9,90,001

Ex. 11. Find the difference between the greatest and the smallest numbers that can be formed, using each of the digits 6, 2, 7, 4 and 3 only once.

Sol. By using the digits 6, 2, 7, 4 and 3,

the greatest number formed = 76,432

the smallest number formed = 23,467

Difference = 76,432 – 23,467 = 52,965

Ex. 12. The number of sheets of paper available for making notebooks is 75,000. Each sheet makes 8 pages of a notebook. Each notebook contains 200 pages. How many notebooks can be made from the paper available?

Sol. One sheet makes 8 pages.

75,000 sheets make 75,000 × 8 pages = 6,00,000 pages

Each notebook contains 200 pages, *i.e.*, 200 pages make 1 notebook.

Therefore, 6,00,000 pages make (6,00,000 ÷ 200) notebooks = 3000 notebooks

Ex. 13. The cost of 15 flats constructed by DDA is ₹4,87,50,000. What is the cost of each flat?

Sol. Total cost of 15 flats = ₹4,87,50,000

Thus, cost of each flat = ₹4,87,50,000 ÷ 15 = ₹32,50,000

Ex. 14. Biscuits are packed in cartons, each packed carton weighs 4 kg 500 g. How many such cartons can be loaded in a van which cannot carry beyond 900 kg?

Maximum load carried by the van

= 900 kg

= 900 × 1000 g (∵ 1 kg = 1000 g)

= 9,00,000 g

Weight of one carton = 4 kg 500 g

= 4000 g + 500 g (∵ 1 kg = 1000 g)

= 4500 g

Number of cartons that can be loaded in the van = 9,00,000 ÷ 4500 = 200

Therefore, 200 cartons can be loaded in the van.

Exercise 1.3

1. **Fill in the blanks.**

   (a) 100 m = ________ mm
   (b) 5 km = ________ m
   (c) 4 m = ________ mm
   (d) 2 m 5 cm = ________ cm = ________ mm
   (e) 20 kg = ________ g
   (f) 8 kg 10 g = ________ g
   (g) 5 L 100 mL = ________ mL
   (h) 4 kL 1 L = ________ L = ________ mL

2. **Following is the runs made by some world famous cricketers in Test Matches.**

   Sachin Tendulkar : 15,921
   Ricky Ponting : 13,378
   Jacques Kallis : 13,289
   Rahul Dravid : 13,288

   Write the number name for the sum of runs made by these cricketers in Test Matches according to the International system of numeration.

3. The sum of two numbers is 9,23,568. If one of the numbers is 4,23,456, find the other number.

4. Find the difference between the smallest 6-digit number and the greatest 4-digit number.

5. Number of students enrolled in schools across the country in a certain year were 26,70,396 in primary classes and 25,59,769 in upper primary classes in India. Find the total number of students enrolled in schools in that year.
6. **Following is the list of cab services in India and the daily bookings of passengers travelling through them.**
   Maxi : 35,000; Mega : 15,000; Easy : 15,000; Ola : 2,40,000; Uber : 44,000
   How many passengers travel daily using these cabs?
7. According to the 'Central Pollution Control Board of India', 15,432 tonnes of plastic waste is generated in a year, of which 9,205 tonnes are recycled. How much plastic waste remains uncollected and littered? CC
8. There are 40 students in a class. Each student is given 250 mL of glucose solution. A total of 600 L of glucose solution is prepared. In how many classes can it be served?
9. Nobel Laureate Kailash Satyarthi started a campaign in 1980 'Bachpan Bachao Andolan' to eradicate child labour from India. India had 1,26,66,377 child labour in 2016 out of which he succeeded in rescuing 83,512 children. How many more children are to be rescued?
10. If we consume 1,60,000 plastic bags every second, how many plastic bags are consumed by us in one hour?
11. If 27,000 trees are cut every day to make tissue paper, how many trees are cut in a leap year to make tissue paper?
12. The fastest train on the Indian Railway Network "The King" Rajdhani Express covers 13,932 km in 162 hours. How much distance is covered by the train in one hour?
13. Find the sum of the greatest and the smallest numbers that can be formed using the digits 6, 2, 7, 4 and 3 only once.

ESTIMATING AND ROUNDING OFF NUMBERS

There are many situations where we do not need exact quantity but need only a reasonable guess or an estimate. For example, while stating how many people watched a cricket match in a stadium, we say approximately 40,000 or 42,000, etc. We do not need to state the exact number of people. Similarly, when we say that ₹8000 crore is required for a particular project, it means approximately ₹8000 crore. This gives us an idea of estimation or approximation.

In this section, we shall discuss estimating and rounding off numbers.

DID YOU KNOW?

It is said that the total distance covered by all trains (passenger + freight) on any single day in India is about 30 lakhs kilometres, which is about 10 times the distance between the earth and the moon.

Meaning of Estimation and Rounding Off

In mathematics, estimation often refers to predicting an answer before the exact calculation is made. In other words:

To round off a given number means to find another number close to the given number.

A number can be rounded off to the nearest tens, hundreds or any other desired by using the following rules:

Rule 1: If the digit to the right of the rounding place is 5 or greater, round up by adding 1 to the digit at the rounding place and changing the remaining digits to the right of it to zeros.

Rule 2: If the digit to the right of the rounding place is less than 5, round down by changing the digits to the right of the rounding place to zeros.

Rounding Off to the Nearest Tens

Illustration 1: 468 —(Rounded off to the tens place)→ 470

468 ≈ 470

↑

Symbol for "approximately equal to" or "nearly equal to".

(Digit at ones place is 8 > 5. Next digit is 6 (at tens place). We write 7 in place of 6 and 0 in place of 8 in ones place.)

Illustration 2: 964 —(Rounded off to the tens place)→ 960

964 ≈ 960

(Digit at ones place is 4 < 5. Next digit is 6 (at tens place). We write 0 in place of 4 at ones place and do not disturb the digit at tens place.)

Rounding Off to the Nearest Hundreds

Illustration 3: Let us round off 5,78,679 to the nearest hundreds.

We place an arrow under 6, the digit in the hundreds place.

5 7 8 6 ⑦ 9

Hundreds place ↑ ↑ (greater than 5) use Rule 1

So, 5,78,679 rounded off to hundreds place = 5,78,700.

Illustration 4: Let us round off 5,78,623 to the nearest hundreds.

We place an arrow under 6, the digit in the hundreds place.

5 7 8 6 ② 3

Hundreds place ↑ ↑ (less than 5) use Rule 2

So, 5,78,623 ≈ 5,78,600. (6 remains the same)

Rounding Off to the Nearest Thousands

Illustration 5: Let us round off 5,78,623 to the nearest thousands.

Place an arrow under the digit in the thousands place.

5 7 8 ⑥ 2 3

Hundreds ↑ ↑ (greater than 5)

5,78,623 ≈ 5,79,000

(rounded off to the thousands place)

Illustration 6: Let us round off 6,68,325 to the nearest ten thousands.

6 6 ⑧ 3 2 5

Ten thousands ↑ ↑ (greater than 5)

So, 6,68,325 rounded off to the nearest ten thousands is 6,70,000.

Rounding Off to the Nearest Lakhs, Ten Lakhs, Crores and Ten Crores

Illustration 7: Let us round off the numbers 27,53,26,498 and 91,00,56,824 to the nearest lakhs, ten lakhs, crores and ten crores as given in the Table 1.5.

Number	Rounding off to the nearest			
	Lakh	Ten Lakh	Crore	Ten Crore
275326498	2 < 5, 275300000	3 < 5, 275000000	5 = 5, 280000000	7 > 5 300000000
910056824	910100000	910000000	910000000	900000000

Table 1.5

Skill Check

- When 365 is rounded off to 400, it is rounded off to the nearest ___________.
- If 37,521 is rounded off to the nearest thousands, the number becomes ___________.
- 5780 rounded off to the nearest hundreds is ___________.
- To round off 13,564 to the nearest thousands, which digit would determine whether the numeral is to be rounded up or down?

Estimating Sum or Difference

To estimate a sum or a difference of numbers, we first round off the numbers so that addition and subtraction can be done mentally.

Estimating sum

To estimate a sum of two or more numbers, follow the given steps:

Step 1: Round off each number to the place mentioned. If it is not mentioned, round off each number to its largest place value.

Step 2: Add the rounded off numbers.

Ex. 15. Estimate the sum: 973 + 32 + 156

Sol. We have,

973	≈	1000
32	≈	30
+ 156	≈	+ 200
1161		1230

The actual sum 1161 is close to 1230.

Estimating difference

To estimate the difference of two numbers, follow the given steps:

Step 1: Round off each number to the place mentioned. If it is not mentioned, round off each number to its largest place value.

Step 2: Subtract the rounded off numbers.

Ex. 16. Estimate the difference 48,696 – 31,976 to the nearest ten thousands.

Sol. We have,

48,696	≈	50,000
– 31,976	≈	–30,000
16,720		20,000

The actual difference is 16,720 and the estimated difference is 20,000.

Estimating Product or Quotient

Estimating product

To estimate the product of two numbers, follow the given steps:

Step 1: Round off each number to its largest place value (unless mentioned otherwise).

Step 2: Multiply the rounded off numbers.

Ex. 17. Estimate the product of 8593 and 472.

Sol. We have,

8593 ≈ 9000 and 472 ≈ 500

∴ 9000 × 500 = 45,00,000

The estimated product is 45,00,000.

The actual product = 8593 × 472
= 40,55,896.

Note

The estimate is higher because both the factors were rounded up.

Estimating quotient

To estimate the quotient of two numbers, follow the given steps:

Step 1: Round off each number to its largest place value.

Step 2: Divide the rounded off numbers.

Ex. 18. Estimate the quotient of 9258 and 45.

Sol. We have,

9258 ≈ 9000 and 45 ≈ 50

So, 9258 ÷ 45 ≈ 9000 ÷ 50.

```
      180
50 ) 9000
    –50
     400
    –400
       0
      –0
       0
```

Thus, the estimated quotient is 180.

By actual division, we get 9258 ÷ 45 = 205 R 33.

Therefore, actual quotient is 205, which is close to the estimated quotient.

Exercise 1.4

1. Round off the following numbers to the nearest tens.

(a) 7834 (b) 69,866 (c) 5,37,051

2. Round off the following numbers to the nearest hundreds.

(a) 27,956 (b) 34,786 (c) 43,054

3. **Round off the following numbers to the nearest thousands.**
   (a) 66,786 (b) 93,235 (c) 1,50,125
4. **Round off the following numbers to the nearest lakhs, ten lakhs, crores and ten crores.**
   (a) 1,00,57,236 (b) 78,65,457 (c) 5,12,35,680 (d) 45,60,08,543
5. **Estimate the sum.**
   (a) 647 + 736 + 1548 (b) 157 + 3534 + 2564
6. **Estimate the difference to the nearest hundreds.**
   (a) 2908 – 437 (b) 5606 – 325
7. **Estimate the product.**
   (a) 285 × 5463 (b) 786 × 1111
8. **Estimate the quotient.**
   (a) 7928 ÷ 24 (b) 5436 ÷ 17 (c) 3648 ÷ 929
9. **Add the following numbers and round off their actual sum to the nearest hundreds.**
   1394, 7364, 5855
   Also, estimate the sum to the nearest hundreds.
   What do you observe?
   Is it correct to say that estimating the sum is the same as rounding off the actual sum?
10. In 2011, the number of children deprived from school in Uttar Pradesh were 8,96,301 and in Madhya Pradesh they were 2,86,310. Estimate the total number of children deprived from school education in these two states.
11. In 2001, Delhi had 41,899 child labour between the age group of 5–14 years. In 2011, it was reduced to 26,473. Estimate the difference in number of child labour in the given two years.
12. One tree makes 8333 sheets of paper. One ream of paper consists of 500 sheets. Estimate the number of reams that can be produced from one tree. Estimate the number of reams that can be produced using 6 trees.
13. India's population is 1,350,000,000 and the population of trees is 35,000,000,000. Estimate the number of trees per person in India.
14. Number of private two wheelers and four wheelers on Delhi roads are respectively 53,85,153 and 26,67,354. Estimate the total number of private vehicles.
15. Sachin Tendulkar has the record of the highest number of runs 15,921 in 329 innings (in Test matches). Estimate the number of runs scored per inning by rounding off to the nearest tens.

USE OF BRACKETS

Suppose, you are asked to write an expression for "Five multiplied by the sum of six and seven".

Can we write it as 5 × 6 + 7?

Here, there is a need to use brackets such as ( ). Using brackets, we write the above expression correctly as:

$$\underset{\text{Five}}{5} \; \underset{\uparrow \text{ Multiplied by}}{\times} \; \underbrace{(6 + 7)}_{\text{Sum}}$$

In this section, we shall learn the use of brackets in writing expressions and their expansions.

Illustration: Shridhi does homework for 3 hours every day and Medhanshu does homework for 2 hours every day. We can express how many hours they work for in a week in the following ways:

Method 1.

In a week, Shridhi works for 7 × 3 hours = 21 hours and Medhanshu works for 7 × 2 hours = 14 hours.

In a week, together they work for (21 + 14) hours = 35 hours.

Method 2. Using brackets.

In a day, Shridhi and Medhanshu together work for (3 + 2) hours.

So, in a week, together they work for 7 × (3 + 2) hours = 7 × 5 hours = 35 hours.

Here, we observe that 7 × (3 + 2) = 7 × 3 + 7 × 2.

Expanding Brackets

Ex. 19. Solve the following:

(a) 8 × 105 **(b) 203 × 107**

(c) 3(5 + 7) – 11

Sol. (a) 8 × 105 = 8 × (100 + 5)
= 8 × 100 + 8 × 5 = 800 + 40 = 840

(b) 203 × 107 = 203 × (100 + 7)
= 203 × 100 + 203 + 7
= (200 + 3) × 100 + (200 + 3) × 7
= 200 × 100 + 3 × 100 + 200 × 7 + 3 × 7
= 20,000 + 300 + 1400 + 21 = 21,721

(c) 3(5 + 7) – 11 = 3 × 12 – 11
= 36 – 11 = 25

Ex. 20. Write a situation for the expression 6 × (15 + 10).

Sol. Many situations can be created for an expression. Here is one situation.
Anil bought 6 notebooks and 6 pens. The cost of a notebook is ₹15 and that of a pen is ₹10. How much did he spend in all?

Exercise 1.5

1. **Write the expression using brackets.**
   (a) Divide the difference of 11 and 4 by five.
   (b) Divide the sum of 17 and 22 by two.
   (c) Forty-eight divided by the difference of 23 and 7.
   (d) Multiply the sum of 6 and 28 by seven.
   (e) Divide the difference of 19 and 4 by three.
2. **Simplify the following expressions.**
   (a) 3(3 + 2) – 11 (b) 4 + 2(3 + 5) (c) 15 × 204 (d) 106 × 109
3. Sneha plays for 2 hours and watches TV for 1 hour daily. How much time does she spend on these activities in a week?
4. **Think and write a situation for each of the following expressions.**
   (a) 5 × (7 + 12) (b) 12 × (60 – 20)

ROMAN NUMERALS

Roman Numeral System

There are several systems of reading and writing numbers, used by different civilizations in the world. One of the systems is Roman Numeral System.

In this system, numerals are written by using symbols like I, V, X, L, C, D and M.

We use Roman numerals in some special situations like symbols for hours in a clock, symbols for indicating chapter numbers in a book, etc. It is strange that Romans, when they invented Roman numeral system, did not think of a symbol to represent zero. So, Roman system does not have a symbol for zero.

Remember

Numerals made up of using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called Hindu-Arabic numerals.

Reading and Writing Roman Numerals

Letters of the English alphabet are assigned to express Roman Numerals. Let us read the value of each Roman numeral with the corresponding Hindu-Arabic numeral.

I = 1, V = 5, X = 10, L = 50, C = 100, D = 500 and M = 1000

We can write any number using the combination of these symbols.

We use the following rules to read and write Roman numerals.

Rule 1: If a symbol is repeated, its value is added as many times as it occurs.

Illustration 1: II = 2, XXX = 30, CC = 200, etc.

A symbol can be repeated only up to three times. But the symbols V, L and D are never repeated.

Illustration 2: VV = 10 and XXXX = 40 are not allowed.

↑ V is repeated. ↑ X is repeated more than 3 times.

Rule 2: If one or more symbols are placed after another symbol of greater value, add that value.

Illustration 3:

VII = 7 (5 + 1 + 1 = 7)

LXXX = 80 (50 + 10 + 10 + 10 = 80)

MC = 1100 (1000 + 100 = 1100)

Rule 3: If a symbol is placed before another symbol of greater value, subtract that value.

Illustration 4:

IV = 4 (5 – 1 = 4)

XC = 90 (100 – 10 = 90)

CM = 900 (1000 – 100 = 900)

The symbols V, L and D are never written to the left of a symbol of greater value. This means V, L and D are never subtracted.

Illustration 5: VC = 95 is not allowed.

For writing 95, we use XCV.

The symbol I can be subtracted from V and X only, the symbol X can be subtracted from L and C only and the symbol C can be subtracted from D and M only.

Illustration 6: IC = 99 and IM = 999 are not allowed. Write XCIX = 99 and CMXCIX = 999.

Skill Check

- The Hindu-Arabic numeral equivalent to CDXVI is:
  (a) 461 (b) 349 (c) 369 (d) 416
- The Hindu-Arabic numeral equivalent to CCXLIV is ________.
- For which number, the Roman system does not have any symbol?

Let us study some more examples.

Ex. 21. Write Roman numerals for:

(a) 67 (b) 95 (c) 36

Sol. (a) 67 = 60 + 7 = (50 + 10) + 7
= LX + VII = LXVII

(b) 95 = 90 + 5 = (100 – 10) + 5
= XC + V = XCV

(c) 36 = 10 + 10 + 10 + 5 + 1
= X + X + X + V + 1
= XXXVI

Remember

Breaking up helps in writing Roman numerals.

Ex. 22. Write in Roman numerals.

(a) 87 (b) 213 (c) 105

Sol. (a) 87 = 80 + 7
= (50 + 10 + 10 + 10) + 7
= (L + XXX) + VII = LXXXVII

(b) 213 = 200 + 10 + 3 = CCXIII

(c) 105 = 100 + 5 = CV

Ex. 23. Write the following in Hindu-Arabic numerals.

(a) CXXXV (b) XCVIII

Sol. (a) C + XXX + V = 100 + 30 + 5 = 135

(b) XCVIII = XC + VIII = (100 – 10) + 8
= 90 + 8 = 98

Exercise 1.6

1. Write the following numbers in Roman numerals.

(a) 84	(b) 107	(c) 69	(d) 155	(e) 351	(f) 138	(g) 56
(h) 231	(i) 220	(j) 198	(k) 178	(l) 233	(m) 950	(n) 75
(o) 413	(p) 252	(q) 321	(r) 514	(s) 1100		

2. **Write the Hindu-Arabic numerals for each of the following.**
   (a) CCLVI (b) MCVI (c) CLVI (d) MCLVI (e) CM (f) MC (g) MCX
   (h) CMLXX (i) XXXI (j) XLIV (k) CCXXV
3. Write the equivalent Hindu-Arabic numeral for XCIV + CD.
4. **Which of the following numbers in Roman numeral is incorrect?**
   (a) LXXX (b) LXX (c) LX (d) LLX

Competency Based Exercise

21st CS

1. **Tick (✓) the correct answer.**
   (a) Which of the following numbers in Roman numerals is incorrect?
   (i) LXII (ii) XCI (iii) LC (iv) XLIV
   (b) Two billion is equal to:
   (i) 200 millions (ii) 20 millions (iii) 2000 lakhs (iv) 20,000 lakhs
   (c) The digit at the ten million place of the numeral 129,345,607 is:
   (i) 1 (ii) 2 (iii) 3 (iv) 9
   (d) If a private college received four bids for the construction of Maths lab room for ₹3,95,679; ₹3,96,759; ₹3,97,695 and ₹3,99,765, then the lowest bid received is:
   (i) ₹3,95,679 (ii) ₹3,96,759 (iii) ₹3,97,695 (iv) ₹3,99,765
   (e) Which of the following symbols is never repeated while writing a Roman numeral?
   (i) C (ii) I (iii) L (iv) M
2. **Write the following numbers as per International and Indian systems of numeration.**
   (a) 65823425 (b) 1005895 (c) 70000007 (d) 99999989
3. **Arrange the following numbers in ascending and descending orders.**
   (a) 25,43,260; 5,43,326; 5,54,326; 5,55,43,326
   (b) 11,11,110; 0; 1,11,127; 10,12,708; 10,00,538; 1005
   (c) 2 lakhs; 20 millions; 20 lakhs; 2 billions
   (d) MCXI, MMCD, MCMXL, CXLI
4. Find the sum of the greatest and smallest 5-digit numbers formed by using the digits 5, 0, and 2.
5. **Simplify:**
   (a) 30,30,300 + 8521 (b) 41,73,019 – 3,68,427 (c) 5638 × 942 (d) 10,95,250 ÷ 50
6. Find the number by interchanging the digits at the tens and thousands places in the number 25,436.

7. **Use the digits 2, 8, 7 and 4 (without repetition), and make the:**

   (a) greatest 4-digit number. (b) smallest 4-digit number.

8. Estimate the sum 197 + 345 to the nearest tens place.
9. Estimate the difference 8538 – 580 to the nearest hundreds place.
10. **Which of the following are not meaningful?**

    (a) VXXIX (b) XXXI (c) XLIV (d) CXCLXV

11. Write 'Divide the difference of 91 and 7 by 6' using brackets and solve.
12. A packet contains 250 mL fruit juice. There are 24 such packets in a carton. A shopkeeper orders for 36 cartons. How many litres of fruit juice will be supplied?
13. 5,23,468 students are studying in 958 colleges. Estimate how many students are there in each college.

Challenge!

21st CS

1. If 5 frogs can catch five flies in five minutes. How many frogs are required to catch hundred flies in hundred minutes?
2. Write ten million ten thousand ten hundred ten in numerals.

Let's Work in Mind

21st CS

1. How will you read 10325436 in the International system?
2. How many 3-digit numbers can be formed using the digits 2, 0 and 7 (without repetition)?
3. How many millions make one crore?
4. How many millimetres make 4 metres?
5. What are the greatest 4-digit number and the smallest 4-digit number, formed with different digits?

CASE STUDY

CC

Estimate the total consumption of noodles per day in India as per the given information:

Number of Rural Households: 227.5 Million

Number of Urban Households: 97.5 Million

1. If one person in each family in both rural and urban area consumes one packet of noodles every week, estimate the consumption of noodles per week.
2. If two persons in urban household consumes one packet each per week, then estimate the total consumption in urban areas.
3. If two persons in urban household consumes one packet each and one person in rural household consumes one packet per week, then estimate the consumption of noodles per week.

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A)** : Ten lakhs is the same as one million.
   Reason (R) : In Indian numeral system, one lakh has five zeros and in International numeral system, one million has six zeros.
2. **Assertion (A)** : Successor of 99,525 is 99,526.
   Reason (R) : The successor of a number is obtained by subtracting 1 from the given number.
3. **Assertion (A)** : Predecessor of 7,66,666 is 7,66,665.
   Reason (R) : The predecessor of a number is obtained by subtracting 1 from the given number.
4. **Assertion (A)** : 7684 can be rounded off to 7680 to the nearest tens.
   Reason (R) : If the digit to the right of the rounding place is less than 5, rounding off is done by changing the digits to the right of the rounding place to zeros.
5. **Assertion (A)** : 4 metres = 4000 millimetres
   Reason (R) : 1 metre = 1000 millimetres
6. **Assertion (A)** : Estimated sum of 973 and 32 is 1030.
   Reason (R) : 973 can be rounded off to 1000 and 32 can be rounded off to 30.
7. **Assertion (A)** : Estimated value of product 878 × 42 is 36000.
   Reason (R) : 873 can be rounded off to 800 and 45 can be rounded off to 40.
8. **Assertion (A)** : Four multiplied by the sum of 6 and 8 can be written as 4 × 6 + 8.
   Reason (R) : BODMAS Rule.
9. **Assertion (A)** : 30(28 + 42) = 30 × 28 + 30 × 42
   Reason (R) : $a(b + c) = a \times b + a \times c$
10. **Assertion (A)** : 36 + 58 = 58 + 36
    Reason (R) : $(a + b) + c = a + (b + c)$
11. **Assertion (A)** : XCXXXV = 145
    Reason (R) : X = 10, C = 100, V = 5

2 Whole Numbers

What Learners Will Achieve

- represent whole numbers on the number line.
- compare, add, subtract, multiply and divide whole numbers.
- understand the properties of whole numbers.
- simplify the expressions using DMAS or BODMAS rule.
- formulate the rules of the given patterns and extend them.
- explore some new patterns.

Warm-up

What we already know

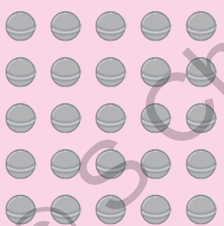
- 1, 2, 3, 4, ... are *natural numbers*. When 0 joins this family, we get a new family 0, 1, 2, 3, 4, ... called *whole numbers*.
- There are infinite whole numbers.
- When a number is subtracted from itself, the difference is 0.
- When 0 is added to a number, the result is the number itself.
- The product of a number and 0 is always 0.

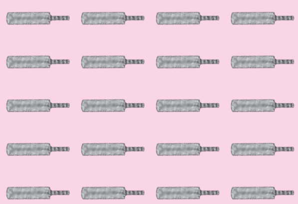
Now, try to solve the following.

1. Which natural numbers will come just before and just after 1000?

2. Write the next three numbers just after 10,099.

3. Count the number of objects given below. Write a multiplication fact for each.

(a) 

(b) 

(c) 

What do you observe? _________

4. Identify the rules and extend the patterns.

(a) 0, 1, 1, 2, 3, 5, 8, ______, ______, ______

(b)
$1 \times 1 = 1$
$101 \times 101 = 10,201$
$10,101 \times 10,101 = 10,20,30,201$
____

(c)
$1 \times 9 + 1 = 10$
$12 \times 9 + 2 = 110$
$123 \times 9 + 3 = 1110$
____

NATURAL NUMBERS AND WHOLE NUMBERS

In the previous chapter, we have learnt about *counting numbers* or *natural numbers*, *i.e.*, 1, 2, 3, … . We also know about the digit '0' (zero).

- 0 is also used as a number to count the number of objects in a collection having **'no' object**.
- The natural numbers along with 0 form a collection of numbers called *whole numbers*.

Thus, the collection of whole numbers is 0, 1, 2, 3, … and so on.

Complete the following table.

Natural Number	Whole Number
It starts from 1.	It starts from 0.
1, 2, 3, 4, …	0, 1, 2, 3, …
The smallest natural number is ________.	The smallest whole number is ________.
There is ___________ largest natural number.	There is ___________ largest whole number.

Table 2.1

DID YOU KNOW?

- In spelling, the first 100 natural numbers, 'a' does not occur even once.
- Zero is derived from the Sanskrit word 'Shunya' – meaning void or emptiness.

PREDECESSOR AND SUCCESSOR

Predecessor

The **predecessor** of a given natural number is the number obtained by subtracting 1 from the given number.

For example, predecessor of 25 is 25 – 1 = 24.
(Note that 24 comes before 25)

Successor

The **successor** of a given natural number is the number obtained by adding 1 to the given number.

For example, successor of 25 is 25 + 1 = 26.

Remember

The number 1 has no predecessor in the collection of natural numbers. However, 0 is the predecessor of 1 in the collection of whole numbers. 0 has no predecessor in the collection of whole numbers.

Skill Check

- Can all natural numbers be called whole numbers?
- Can all whole numbers be called natural numbers?
- Which whole number is not a natural number?
- What is the successor of 2,53,658?
- What is the predecessor of 89,800?

THE NUMBER LINE FOR WHOLE NUMBERS

Let us draw a line and mark a point on it. Label this point as O. Let it represent the number 0.

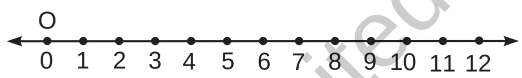


Fig. 2.1 Number line

Now, mark a second point at a suitable distance to the right of 0. Label it as 1. Distance between 0 and 1 is called a **unit distance**. Mark the other points on the number line at the same distance and label them as 2, 3, 4, … .

Comparing Numbers on the Number Line

A number on the **right** of a given number on the number line is **greater than** the given number, or equivalently, a number on the **left** of a given number on the number line is **less than** the given number.

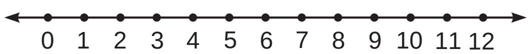


Fig. 2.2

- 11 is on the right of 6. So, 11 > 6.
- 4 is on the right of 0. So, 4 > 0.
- 5 is on the left of 8. So, 5 < 8.

Addition on the Number Line

Let us see the addition of 2 and 5 on the number line.

First jump to 2. Then count 5 units to the **right**.

We arrive at the point 7 (see Fig. 2.3).

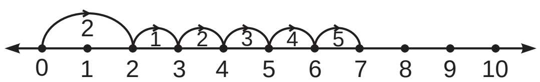


Fig. 2.3

Thus, 7 is the sum of 2 and 5, *i.e.*,2 + 5 = 7.

Subtraction on the Number Line

Let us see the subtraction of 4 from 9 on the number line.

First jump to 9. Then count (backward) 4 units to the **left** of 9.

We arrive at the point 5 (see Fig. 2.4).

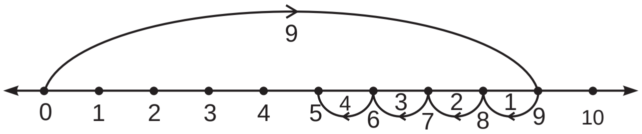


Fig. 2.4

Thus, 5 is the difference between 9 and 4, *i.e.*, 9 – 4 = 5.

Multiplication on the Number Line

Multiplication is a process of repeated addition. A method of multiplication in terms of addition of whole numbers can be demonstrated using the number line.

Let us see the multiplication of 2 and 3 on the number line.

2 × 3 = 2 + 2 + 2 (2 repeated 3 times)

Start from 0, then move 2 units to the right.

Make this move 2 more times, *i.e.*, total of 3 times.

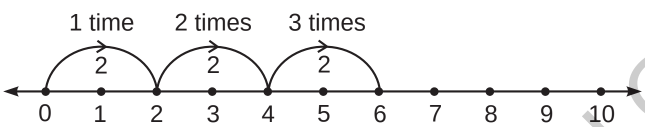


Fig. 2.5

We arrive at the point 6. Therefore, 2 × 3 = 6.

Division on the Number Line

Let us divide 12 by 3 on the number line. Start from 0, then move 12 units to the right. Then move 3 units backward at a time. Make more such moves till we reach 0. We jump a total of 4 times.

Therefore, 12 ÷ 3 = 4.

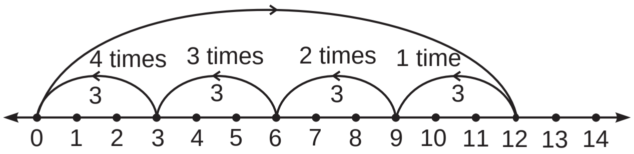


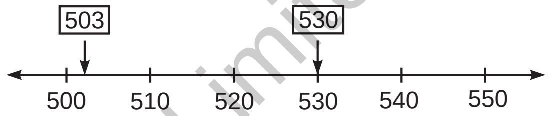
Fig. 2.6

Let us study some more examples.

Ex. 1. In each of the following pairs of numbers, state which whole number is on the left side of the other on the number line.

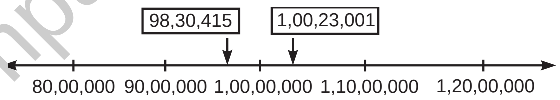
(a) 530; 503 (b) 98,30,415; 1,00,23,001

Sol. (a) Find positions of the given numbers on the number line.



Clearly, 503 is on the left side of 530. Therefore, 530 > 503.

(b) Find the positions of the given numbers on the number line.



Clearly, 98,30,415 is on the left side of 1,00,23,001.

Therefore, 98,30,415 < 1,00,23,001.

Ex. 2. How many whole numbers are there between 35 and 53?

Sol. Number of whole numbers between two whole numbers a and b, ($a < b$)

$= (b - a) - 1.$

Here, $a = 35$ and $b = 53$.

So, number of whole numbers between 35 and 53 = (53 – 35) – 1 = 18 – 1 = 17.

Exercise 2.1

1. **Write the predecessor of the following numbers.**

   (a) 5236 (b) 40,999 (c) 1,08,000 (d) 90,001

2. **Write the successor of the following numbers.**

   (a) 635 (b) 4999 (c) 25,009 (d) 9,87,450

3. **In each of the following pairs of numbers, state which whole number is on the right side of the other on the number line. Also, compare using > or <.**

   (a) 425; 452 (b) 9530; 9305 (c) 1,46,832; 1,48,326

4. Write five whole numbers just before 4,25,963.

5. How many whole numbers are there between 45 and 78?
6. How many natural numbers are there between 98 and 120?
7. **Add the numbers using the number line.**
   (a) 4 and 7 (b) 3 and 8 (c) 4 and 4
8. **Subtract the numbers using the number line.**
   (a) 3 from 6 (b) 4 from 9 (c) 0 from 8
9. **Multiply the numbers using the number line.**
   (a) 5 and 4 (b) 1 and 8 (c) 2 and 6
10. **Divide the numbers using the number line.**
   (a) 10 by 2 (b) 9 by 3 (c) 12 by 4
11. For which number, the product of its successor and predecessor is 99?

PROPERTIES OF WHOLE NUMBERS

In the previous section, we have learnt four operations of whole numbers on the number line. If you look into these operations, you will notice several properties of whole numbers. These properties help us understand the numbers in a better way. In this section, we shall discuss some properties of whole numbers.

Closure Property

Addition

Consider the addition of the following two whole numbers:

3 + 4 = 7 ← a whole number
0 + 8 = 8 ← a whole number
and 9 + 8 = 17 ← a whole number

Observe that the sum of two whole numbers is also a whole number.

Thus, we say that:

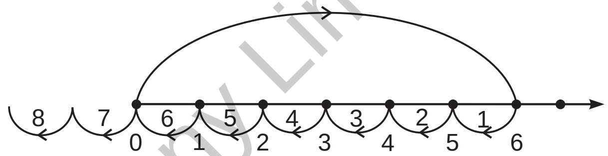
Collection of whole numbers is **closed under addition.**

This property is known as the *closure property for addition of whole numbers*.

Subtraction

Consider the subtraction of the following two whole numbers.

6 – 4 = 2 ← a whole number
6 – 6 = 0 ← a whole number
6 – 8 = ?



There is no whole number on the left of zero.

Fig. 2.7

In other words, there is no whole number which equals 6 – 8.

Observe that the subtraction of two whole numbers is **not** always a whole number.

Thus, we say that:

Collection of whole numbers is **not closed under subtraction**.

Multiplication

Consider the multiplication of the following two whole numbers:

6 × 4 = 24 ← a whole number
7 × 0 = 0 ← a whole number
and 15 × 1 = 15 ← a whole number

Observe that multiplication of two whole numbers, *i.e.*, product of two whole numbers is a whole number.

Thus, we say that:

Collection of whole numbers is **closed under multiplication**.

This property is known as the *closure property for multiplication of whole numbers*.

Division

Consider the division of two whole numbers.

$9 \div 3 = 3 \quad \leftarrow$ a whole number

$8 \div 3 = \frac{8}{3} \quad \leftarrow$ not a whole number

We see that the result of division of two whole numbers is **not** always a whole number.

Thus, we say that:

> Collection of whole numbers is **not closed under division**.

Division by Zero

Division of a whole number by zero (0) is *not defined*. We know that division by a number means subtracting that number repeatedly.

Let us apply this definition to $12 \div 3$, $15 \div 4$ and $3 \div 0$, one by one.

$$\begin{array}{rl} 12 & \\ -\ 3 & \leftarrow 1 \\ \hline 9 & \\ -\ 3 & \leftarrow 2 \\ \hline 6 & \\ -\ 3 & \leftarrow 3 \\ \hline 3 & \\ -\ 3 & \leftarrow 4 \\ \hline 0 & \end{array} \Bigg\} \text{ 4 subtractions}$$

So, $12 \div 3 = 4$.

$$\begin{array}{rl} 15 & \\ -\ 4 & \leftarrow 1 \\ \hline 11 & \\ -\ 4 & \leftarrow 2 \\ \hline 7 & \\ -\ 4 & \leftarrow 3 \\ \hline 3 & \end{array} \Bigg\} \text{ 3 subtractions and then left over 3}$$

So, $15 \div 4 =$ Q 3 R 3.

In the case of $3 \div 0$, we see that in every move, we get 3 again, so it is a never ending process. If we take any other whole number and divide by '0', the process will again be unending. So, We say that the result of the division is not defined.

$$\begin{array}{rl} 3 & \\ -\ 0 & \leftarrow 1 \\ \hline 3 & \\ -\ 0 & \leftarrow 2 \\ \hline 3 & \\ -\ 0 & \leftarrow 3 \\ \hline 3 & \end{array} \Bigg\} \text{ 3 subtractions and then left over 3 again}$$

> Division of a whole number by 0 is not defined.

Commutative Property of Addition and Multiplication

Addition

Observe that:

$4 + 8 = 12$ and $8 + 4 = 12$

$10 + 15 = 25$ and $15 + 10 = 25$

The two numbers are added in two different orders but the sum is the same. The same is true for other whole numbers too. Thus, we say that:

> Changing the order of whole numbers in addition does not change the sum. **We can add two numbers in any order**.

This is known as the *commutative property of addition*.

Multiplication

Observe that:

$4 \times 8 = 32$ and $8 \times 4 = 32$

$12 \times 9 = 108$ and $9 \times 12 = 108$

The two numbers are multiplied in two different orders but their product is the same. The same is true for other whole numbers too. Thus, we say that:

> Changing the order of whole numbers in multiplication does not change the product. **We can multiply two whole numbers in any order**.

This is known as the *commutative property of multiplication*.

Associative Property of Addition and Multiplication

Addition

Observe that: $2 + (5 + 1) = 2 + 6 = 8$

and $(2 + 5) + 1 = 7 + 1 = 8$

Again, $4 + (0 + 2) = 4 + 2 = 6$

and $(4 + 0) + 2 = 4 + 2 = 6$

Thus, we say that:

> On adding three whole numbers, the **sum is the same regardless of which two addends are added first**.

This is known as the *associative property of addition*.

Multiplication

Observe the following two multiplications:

$$\begin{aligned} 2 \times 5 \times 3 &= 2 \times (5 \times 3) \\ &= 2 \times 15 \\ &= 30 \end{aligned}$$

and $2 \times 5 \times 3 = (2 \times 5) \times 3$

$= 10 \times 3$

$= 30$

Again, $2 \times 5 \times 0 = 2 \times (5 \times 0)$

$= 2 \times 0$

$= 0$

and $2 \times 5 \times 0 = (2 \times 5) \times 0$

$= 10 \times 0$

$= 0$

Thus, we can say that:

On multiplying three whole numbers, **the product is the same regardless of which two numbers are multiplied first**.

This is known as the *associative property of multiplication*.

Distributive Property

Consider an expression $7 \times (2 + 4)$.

We can simplify this in the following two ways:

$7 \times (2 + 4) = 7 \times 6$	$7 \times (2 + 4) = (7 \times 2) + (7 \times 4)$
$= \mathbf{42}$	$= 14 + 28 = \mathbf{42}$

We get the same answer in both the cases.

Therefore, $7 \times (2 + 4) = (7 \times 2) + (7 \times 4)$.

Similarly, $4 \times (3 + 5) = (4 \times 3) + (4 \times 5)$.

This property is known as *the distributive property of multiplication over addition*.

Distributive property of multiplication can also be extended over subtraction.

Observe the following:

$7 \times (6 - 2) = 7 \times 4$	$7 \times (6 - 2) = 7 \times 6 - 7 \times 2$
$= \mathbf{28}$	$= 42 - 14 = \mathbf{28}$

Thus, $7 \times (6 - 2) = (7 \times 6) - (7 \times 2)$.

Similarly, $12 \times (5 - 3) = (12 \times 5) - (12 \times 3)$ and so on.

Let Us Do

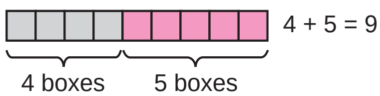
EL

Objective: Verification of commutative property

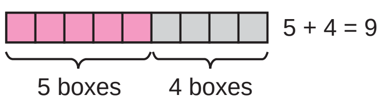
Materials required: Square grid paper, coloured pencils, etc.

Procedure:

Step 1: Consider two numbers, say 4 and 5. On a square grid paper, colour 4 boxes with one colour and 5 boxes with another colour. Total boxes coloured are 4 + 5 = 9.



Step 2: Now, colour the first 5 boxes with the second colour and 4 boxes with the first colour (chosen in step 1). Total boxes coloured are 5 + 4 = 9.



So, 4 + 5 = 5 + 4.

Note

This property is known as the commutative property of addition.

Step 3: Repeat the same procedure with 3 different pairs of numbers.

Conclusion:

___________ of two numbers does ___________ when the order of numbers changes.

Identity Property of Addition and Multiplication

Addition

We have,

$5 + 0 = 5, 0 + 5 = 5$

$12 + 0 = 12, 0 + 12 = 12$

$7 + 0 = 7, 0 + 7 = 7$

$952 + 0 = 952, 0 + 952 = 952$

Thus, 0 is called the *additive identity.*

Multiplication

We have,

$5 \times 1 = 5, 1 \times 5 = 5$

$12 \times 1 = 12, 1 \times 12 = 12$

$7 \times 1 = 7, 1 \times 7 = 7$

$952 \times 1 = 952, 1 \times 952 = 952$

Thus, 1 is called the *multiplicative identity.*

Let us study some more examples.

Ex. 3. Find the sum 837 + 208 + 363 by suitable rearrangement.

Sol. $837 + (208 + 363)$

$= 837 + (363 + 208)$ (By commutative property)

$= (837 + 363) + 208$ (By associative property)

$= 1200 + 208$

$= 1408$

Ex. 4. Find the product 2 × 1768 × 50 by suitable rearrangement.

Sol. $2 \times (1768 \times 50)$

$= 2 \times (50 \times 1768)$ (By commutative property)

$= (2 \times 50) \times 1768$ (By associative property)

$= 100 \times 1768$

$= 1,76,800$

Ex. 5. Find the value of 297 × 17 + 297 × 3 using suitable property.

Sol. $297 \times 17 + 297 \times 3$

$= 297 \times (17 + 3)$ (By distributive property)

$= 297 \times 20$

$= 5940$

Ex. 6. Find the product 738 × 103 using suitable property.

Sol. 738×103

$= 738 \times (100 + 3)$

$= 738 \times 100 + 738 \times 3$

(By distributive property)

$= 73,800 + 2214 = 76,014$

Ex. 7. A taxi-driver filled his car with 40 litres of petrol on Monday. The next day, he filled it with 50 litres of petrol. If petrol costs ₹96 per litre, how much did he spend in all on petrol?

Sol. Quantity of petrol filled in the car on Monday = 40 litres

Quantity of petrol filled in the car on next day = 50 litres

Cost of petrol per litre = ₹96

Cost of petrol filled on Monday = ₹96 × 40

Cost of petrol filled on Tuesday = ₹96 × 50

Total amount spent on petrol

= ₹96 × 40 + ₹96 × 50

= ₹96 × (40 + 50)

(By distributive property)

= ₹96 × 90 = ₹8640

Ex. 8. Find the value of the following:

(a) 887 × 10 × 461 – 361 × 8870

(b) 3845 × 5 × 782 + 769 × 25 × 218

Sol. (a) We have,

$887 \times 10 \times 461 - 361 \times 8870$

$= (887 \times 10) \times 461 - 361 \times 8870$

$= 8870 \times 461 - 8870 \times 361$

$= 8870 \times (461 - 361)$

(By distributive property)

$= 8870 \times 100 = 8,87,000$

(b) We have,

$3845 \times 5 \times 782 + 769 \times 25 \times 218$

$= (3845 \times 5) \times 782 + (769 \times 25) \times 218$

$= 19,225 \times 782 + 19,225 \times 218$

$= 19,225 \times (782 + 218)$

(By distributive property)

$= 19,225 \times 1000 = 1,92,25,000$

Exercise 2.2

1. **Complete the following table using suitable examples.**

	Properties of Whole Numbers	For Addition	For Multiplication
(a)	Closure		
(b)	Commutative		
(c)	Associative		
(d)	Identity		
(e)	Distributive property of: (i) multiplication over addition (ii) multiplication over subtraction		

2. **Find the sum of the following by suitable rearrangement.**
   (a) 1562 + 544 + 238 (b) 1117 + 506 + 283 (c) 764 + 689 + 236
3. **Find the product of the following by suitable rearrangement.**
   (a) 4 × 236 × 25 (b) 8 × 432 × 125 (c) 125 × 375 × 16
4. **Find the value of the following using suitable property.**
   (a) 37,834 × 92 + 8 × 37,834 (b) 21,775 × 189 – 89 × 21,775
   (c) 3046 × 684 – 84 × 3046 (d) 9317 × 29 + 71 × 9317
5. **Find the product of the following using suitable property.**
   (a) 825 × 101 (b) 728 × 1002 (c) 97 × 178 (d) 82 × 50
6. If the cost of petrol per litre is ₹96, then find the total amount spent by a taxi-driver in two days for purchase of 46 litres and 54 litres petrol.
7. **Identify the property demonstrated in each of the following:**
   (a) 132 × 64 = 64 × 132 (b) 368 + 0 = 368
   (c) 1 × 526 = 526 (d) (11 + 17) + 18 = 11 + (17 + 18)
   (e) (19 × 8) × 27 = 19 × (8 × 27) (f) 45 × (80 + 7) = 45 × 80 + 45 × 7
8. **Find the value of the following.**
   (a) 3125 × 211 + 89 × 5 × 625 (b) 442 × 2 × 104 + 96 × 884
   (c) 36,800 × 212 – 12 × 100 × 368 (d) 118 × 19 × 1213 – 2 × 1121 × 1213

ORDER OF OPERATIONS

We know how to add or multiply two numbers. But, if we have to simplify an expression containing both addition and multiplication, then we can get different results if we do not follow proper rules.

In the absence of such rules, we use brackets, to indicate which operation should be performed first. This is explained as follows:

Consider an expression, say 4 × 5 + 7.
One person may simplify it like:
4 × 5 + 7 = 20 + 7 = 27
Another may simplify it like: 4 × 5 + 7 = 4 × 12 = 48
We get two different results from the same numerical expression.
So, to avoid such confusions, we use brackets to indicate our intentions about the operation that we want to perform first.
If we want the addition first, then we use in the expression as 4 × (5 + 7).

Therefore, 4 × (5 + 7) = 4 × 12 = 48.

If we want multiplication first, then we use brackets in the expression as (4 × 5) + 7.

Therefore, (4 × 5) + 7 = 20 + 7 = 27.

DMAS Rule

If an expression does not have brackets, we follow DMAS rule, where:

- **D** stands for **Division**
- **M** stands for **Multiplication**
- **A** stands for **Addition**
- **S** stands for **Subtraction**

Illustration 1:

Consider 14 + 20 ÷ 5 – 3 × 4 (1st divide)

= 14 + 4 – 3 × 4 (next multiply)

= 14 + 4 – 12 (third add)

= 18 – 12 (finally subtract)

= 6

Simplifying Expressions Involving Brackets

While simplifying an expression containing brackets, first simplify the terms inside the brackets into a single term and then perform the other operations.

Illustration 2: To simplify the expression (15 – 8) of (4 + 6) ÷ 5, we first simplify the expression inside the brackets.

(15 – 8) of (4 + 6) ÷ 5

= (7) of (10) ÷ 5

= (70) ÷ 5

= 14

'of' is an operator which means 'multiplication' but it is solved before '÷'.

Remember

- If there is no sign between a number and bracket, perform multiplication.
- We can also write:
  (15 – 8) × (4 + 6) ÷ 5 as (15 – 8) (4 + 6) ÷ 5

BODMAS Rule

In simplifying an expression, we follow the following sequence of operations:

1. B → Brackets
2. O → of means '×'
3. D → Division
4. M → Multiplication
5. A → Addition
6. S → Subtraction

We now take up some examples to explain the above procedure.

Ex. 9. Simplify the following expressions:
(a) 2(5 + 3) – 10
(b) 5 + 2(4 + 3)

Sol. (a) 2(5 + 3) – 10 = 2(8) – 10
= 16 – 10 = 6

(b) 5 + 2(4 + 3) = 5 + 2(7) = 5 + 14 = 19

Ex. 10. Simplify the given expressions:
(a) (7 + 2)(10 – 3) + 7(8 – 3) ÷ 5
(b) (30 ÷ 10)(9 – 7) × 4 + 8 × (3 + 5)

Sol. (a) (7 + 2)(10 – 3) + 7(8 – 3) ÷ 5
= (9)(7) + 7(5) ÷ 5
= 63 + 35 ÷ 5 [On simplifying 63 + 35 ÷ 5, we first perform division]
= 63 + 7 = 70

(b) (30 ÷ 10)(9 – 7) × 4 + 8 × (3 + 5)
= (3)(2) × 4 + 8 × (8)
= 6 × 4 + 8 × 8
= 24 + 64 = 88 (Perform '×' before '+')

Kinds of Brackets and their Simplification

Sometimes, a mathematical expression has more than one type of brackets.

Let us consider 33 – [11 + {7 – 2(27 – 24)}].

Here, you can see three kinds of brackets [ ], { } and ( ).

Let us first know their names and then order to solve.

1. [ ] ← Square or big brackets.
2. { } ← Curly brackets or braces.
3. ( ) ← Round or simple brackets, parentheses, is also used.
4. '—' ← It is called vinculum, line or base brackets.

The brackets are removed in the following order:

'—' (Line brackets) → ( ) (Round brackets) → { } (Curly brackets) → [ ] (Square brackets)

Illustration 3: Let us solve the expression

33 – [11 + {7 – 2(27 – 24)}]

= 33 – [11 + {7 – 2 × 3}] (Removing round brackets)

= 33 – [11 + {7 – 6}]

= 33 – [11 + 1] (Removing curly brackets)

= 33 – 12 (Removing square brackets)

= 21

Ex. 11. Simplify:

(a) $15 \div 3 - 2 \text{ of } 4 + 6(2 \times \overline{3+1})$

(b) $[20 - \{3 \text{ of } (\overline{8-5} + 2) - 10\}] + 5$

Sol. (a) $15 \div 3 - 2 \text{ of } 4 + 6(2 \times \overline{3 + 1})$

$= 15 \div 3 - 2 \text{ of } 4 + 6(2 \times 4)$

$= 15 \div 3 - 2 \text{ of } 4 + 6(8)$

$= 15 \div 3 - 2 \text{ of } 4 + 48$

$= 15 \div 3 - 8 + 48$

$= 5 - 8 + 48 = 53 - 8 = 45$

(b) $[20 - \{3 \text{ of } (\overline{8 - 5} + 2) - 10\}] + 5$

$= [20 - \{3 \text{ of } (3 + 2) - 10\}] + 5$

$= [20 - \{3 \text{ of } 5 - 10\}] + 5$

$= [20 - \{15 - 10\}] + 5$

$= [20 - 5] + 5 = 15 + 5 = 20$

Exercise 2.3

Simplify the following expressions.

1. 12 – 2 × 4 + 5 **2.** 25 ÷ 5 + 3 of 7 **3.** 10 of 4 ÷ 5 – 6 **4.** 15 + 2(3 + 4 × 2)

5. 26 – {8 + (18 – 5)} **6.** [35 – 2{5 + (2 × 3 + 5)}] **7.** $11 \text{ of } 8 - 3\,(7 \times \overline{1+2})$

8. 120 ÷ (2 × 4 + 7) + {15 – 5 (2 of 3 – 6)} **9.** (102 + 30) (40 – 2) + 15(95 – 5) ÷ 3

10. 115 – 15(28 – 21) ÷ (85 – 70)

PATTERNS IN WHOLE NUMBERS

In this section, we will find some interesting number patterns–each pattern having its own particular motive and theme.

Observing Patterns

Observing patterns can help us in simplifying calculations. Observe the following patterns:

Pattern 1: Let us add 157 and 99, *i.e.,*

157 + 99 = 256.

Now, add 157 and 99 again by using a different approach.

We can write 99 as 100 – 1.

So, 157 + 99 = 157 + 100 – 1 = 257 – 1 = 256.

Observe that how easily we obtained the sum of 157 and 99 in this case.

Study the following:

157 + 9 = 157 + 10 – 1 = 167 – 1 = 166

157 – 9 = 157 – 10 + 1 = 147 + 1 = 148

157 + 99 = 157 + 100 – 1 = 257 – 1 = 256

157 – 99 = 157 – 100 + 1 = 57 + 1 = 58

This pattern helps us to add or subtract numbers like 9, 99, 999, ... to form other numbers.

Pattern 2: Let us multiply 25 and 99.

25 × 99 = 2475

Now, multiply 25 and 99 again by using a different approach.

We can write 99 as 100 – 1.

So, 25 × 99 = 25 × (100 – 1) = 2500 – 25 = 2475.

Observe that how easily we obtained the product of 25 and 99 in this case.

Study the following:

25 × 9 = 25 × (10 – 1) = 250 – 25 = 225

25 × 99 = 25 × (100 – 1) = 2500 – 25 = 2475

25 × 999 = 25 × (1000 – 1) = 25000 – 25 = 24975

This pattern helps us to multiply numbers like 9, 99, 999, ... with other numbers.

Pattern 3:

$$\mathbf{1} = 1 = 1 \times 1$$
$$1 + \mathbf{2} + 1 = 4 = 2 \times 2$$
$$1 + 2 + \mathbf{3} + 2 + 1 = 9 = 3 \times 3$$
$$1 + 2 + 3 + \mathbf{4} + 3 + 2 + 1 = 16 = 4 \times 4$$
$$1 + 2 + 3 + 4 + \mathbf{5} + 4 + 3 + 2 + 1 = 25 = 5 \times 5$$
$$\vdots \qquad \vdots \qquad \vdots$$

Write the next four lines of this pattern yourself.

Pattern 4:

$$999 \times 1 = 0999$$
$$999 \times 2 = 1998$$
$$999 \times 3 = 2997$$
$$999 \times 4 = 3996$$
$$999 \times 5 = 4995$$
$$\vdots \quad \vdots \quad \vdots$$

Extend this pattern and examine which will be the last line of this pattern.

Pattern 5: There is another interesting fact connected with 9.

$$0 \times 9 + 1 = 1$$
$$1 \times 9 + 2 = 11$$
$$12 \times 9 + 3 = 111$$
$$123 \times 9 + 4 = 1111$$
$$1234 \times 9 + 5 = 11111$$
$$12345 \times 9 + 6 = 111111$$
$$\vdots \quad \vdots \quad \vdots$$
$$123456789 \times 9 + 10 = 1111111111$$

Pattern 6: In association with certain numbers, 8 plays a remarkable part.

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 98765$$
$$\vdots \quad \vdots \quad \vdots$$
$$123456789 \times 8 + 9 = 987654321$$

Pattern 7: Let us enjoy the architecture of the following patterns:

$$11 \times 11 = 121$$
$$111 \times 111 = 12321$$
$$1111 \times 1111 = 1234321$$
$$11111 \times 11111 = 123454321$$
$$111111 \times 111111 = 12345654321$$
$$1111111 \times 1111111 = 1234567654321$$
$$11111111 \times 11111111 = 123456787654321$$
$$111111111 \times 111111111 = 12345678987654321$$

Pattern 8:

$$12345679 \times 9 = 111111111$$
$$12345679 \times 18 = 222222222$$
$$12345679 \times 27 = 333333333$$
$$12345679 \times 36 = 444444444$$
$$12345679 \times 45 = 555555555$$
$$12345679 \times 54 = 666666666$$
$$12345679 \times 63 = 777777777$$

Pattern 9:

$$(1 \times 8) - 1 = 07$$
$$(21 \times 8) - 1 = 167$$
$$(321 \times 8) - 1 = 2567$$
$$(4321 \times 8) - 1 = 34567$$
$$\vdots \quad \vdots \quad \vdots$$

Pattern 10:

$$9 \times 6 = 54$$
$$99 \times 66 = 6534$$
$$999 \times 666 = 665334$$
$$9999 \times 6666 = 66653334$$
$$\vdots \quad \vdots \quad \vdots$$

Pattern 11:

$$\frac{11-2}{9} = 1$$

$$\frac{111-3}{9} = 12$$

$$\frac{1111-4}{9} = 123$$

$$\frac{11111-5}{9} = 1234$$

⋮ ⋮ ⋮

Whole Numbers as Geometrical Patterns

Whole numbers as line segments

If we represent 1 as		
Then we may represent 2 as		1 = 1 line segment
We may represent 3 as		1 + 2 = 3 line segments
We may represent 4 as		1 + 2 + 3 = 6 line segments
We may represent 5 as		1 + 2 + 3 + 4 = 10 line segments and so on

On observing the above, we find a pattern:

The number of line segments formed by 2 points = $1 = \frac{2 \times 1}{2}$

The number of line segments formed by 3 points = $3 = \frac{3 \times 2}{2}$

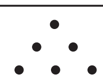
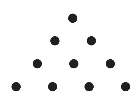
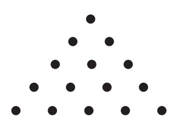
The number of line segments formed by 4 points = $6 = \frac{4 \times 3}{2}$

The number of line segments formed by 5 points = $10 = \frac{5 \times 4}{2}$

We form the rule to find the number of line segments obtained on representing a whole number as points:

$$\text{Number of line segments} = \frac{(\text{Whole number}) \times (\text{Whole number} - 1)}{2}$$

Whole numbers as triangles

We can represent 1 as	
We can represent 3 as	
We can represent 6 as	
We can represent 10 as	
We can represent 15 as	

The whole numbers which can be represented as triangles described above are called *triangular numbers*.

On observing it closely, we find:

First triangular number = $1 = \frac{1 \times 2}{2} = \frac{1 \times (1+1)}{2}$

Second triangular number = $3 = \frac{2 \times 3}{2}$

$= \frac{2 \times (2+1)}{2}$

Third triangular number = $6 = \frac{3 \times 4}{2} = \frac{3 \times (3+1)}{2}$

Fourth triangular number = $10 = \frac{4 \times 5}{2}$

$= \frac{4 \times (4+1)}{2}$ and so on.

Therefore,

For any whole number, the corresponding triangular number = $\frac{\text{Whole number} \times (\text{Whole number} + 1)}{2}$

Whole numbers as squares or rectangles

We can represent 1 as	•
We can represent 4 as	• • • •
We can represent 6 as	• • • • • •
We can represent 8 as	• • • • • • • •
We can represent 9 as	• • • • • • • • •
We can represent 10 as	• • • • • • • • • •

On observing carefully, we find the numbers 1, 4, 9, ... represent squares and 6, 8, 10, ... represent rectangles.

So, 1, 4, 9, 16, ... are called *square numbers* and 6, 8, 10, 12, 14, ... are called *rectangular numbers*.

Exercise 2.4

1. Solve the following.

(a) 527 + 99 (b) 65 × 99 (c) 3468 – 999 (d) 123 × 999

2. State if the following numbers form a triangular pattern, square pattern or rectangular pattern.

(a) 4 (b) 10 (c) 16 (d) 21

3. Observe the patterns and answer the following questions.

(a)
9 × 9 + 7 = 88
98 × 9 + 6 = 888
987 × 9 + 5 = 8888

(i) What will be the next step?
(ii) How can we write 888888 using the above pattern?

(b)
1 × 9 – 1 = 08
21 × 9 – 1 = 188
321 × 9 – 1 = 2888
4321 × 9 – 1 = 38888

(i) What will be the fifth step?
(ii) How can we write 68888888 using the above pattern?

(c)
7 × 7 = 49
67 × 67 = 4489
667 × 667 = 444889
6667 × 6667 = 44448889

(i) What will be the next step?
(ii) How can we write 444444888889 using the above pattern?

(d)
$\frac{1}{10} = 0.1$

$\frac{1}{10 \times 10} = 0.01$

$\frac{1}{10 \times 10 \times 10} = 0.001$

$\frac{1}{10 \times 10 \times 10 \times 10} = 0.0001$

(i) What will be the sixth step?
(ii) How can we express 0.0000001 using the above pattern?

4. **Observe the pattern and fill in the blanks.**

(a)
$37 \times 3 = 111$
$37 \times 6 = 222$
$37 \times \ldots = 333$
$37 \times 12 = \ldots\ldots$

(b)
$6 \times 7 = 42$
$66 \times 67 = 4422$
$666 \times 667 = 444222$
$6666 \times 6667 = \ldots\ldots$
$66666 \times \ldots\ldots = 4444422222$

(c)
$121 = \dfrac{22 \times 22}{1+2+1}$
$12321 = \dfrac{333 \times 333}{1+2+3+2+1}$
$\ldots\ldots = \dfrac{4444 \times 4444}{1+2+3+4+3+2+1}$
$123454321 = \ldots\ldots$

Competency Based Exercise

21st CS

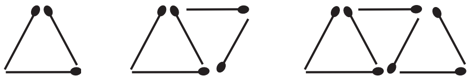
1. **Tick (✓) the correct answer.**
   (a) If the sum of two distinct whole numbers is even, then their difference must be:
   (i) odd (ii) even (iii) 0 (iv) 1
   (b) The predecessor of the largest 8-digit number that can be formed by using the digits 8, 2, 5, 7, 3, 0, 1 and 9 only once is:
   (i) 98753209 (ii) 98753210 (iii) 97853210 (iv) 98573210
   (c) The product of a non-zero whole number and its predecessor is always:
   (i) an even number (ii) an odd number (iii) a prime number (iv) divisible by 3
   (d) If a whole number is divided by another non-zero whole number, then the remainder is always smaller than:
   (i) quotient (ii) divisor (iii) dividend (iv) 0
   (e) A mobile number consists of ten digits. If the first four digits in order are 9, 9, 7 and 9, then the smallest mobile number formed by using only one digit twice from the digits 8, 3, 5, 6, 0 is:
   (i) 9979300865 (ii) 9979005683 (iii) 9979003568 (iv) 9979003300

2. **Complete the following chart for property of whole numbers.**

Property	+	–	×	÷
Closure	Sum of two whole numbers is a whole number.			
Commutative		$a - b \neq b - a$	$a \times b = b \times a$	
Associative	$a + (b + c) = (a + b) + c$			
Identity			1 is multiplicative identity, *i.e.*, $a \times 1 = a$.	

(**Note:** *a*, *b* and *c* are whole numbers.)

3. If 11,100 is divided by a whole number and the quotient is 198 and the remainder is 12, then find the divisor.
4. Find the greatest whole number which always divides the product of predecessor and successor of an odd whole number other than 1.
5. The sum of nine consecutive whole numbers is 99. What could be the largest of these whole numbers?
6. **Check whether the following numbers are triangular, rectangular or square number.**
   (a) 6 (b) 9 (c) 15 (d) 24
7. **Observe the given pattern and fill in the blanks.**



______ , ______ , ______ .

8. **Simplify:**
   (a) $144 \div 3 \text{ of } 4 - 2(18 - \overline{10 + 6})$ (b) $40 - [15 + \{80 \div 8(2 + 3)\}]$

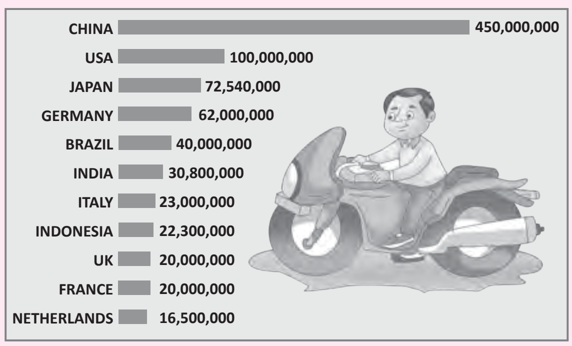
Challenge! 21st CS

1. Siya picked the newspaper and she found that page 13 was missing. Back page of the newspaper was numbered 20. What other pages of the newspaper were missing?
2. If successor of 52, predecessor of 69, predecessor of the predecessor of 36 and successor of the successor of 57 are added, then what will be the sum?

Let's Work in Mind 21st CS

1. What would be the product 25 × 99?
2. What is the sum 428 + 999?
3. What is the difference 1234 – 99?
4. Which is the smallest among 2 + 2, 2 × 2, 2 ÷ 2 and 2 – 2?
5. Whole numbers are closed for all the four operations. (True/False)

CASE STUDY CC



1. Rahul calculated the total bikes in China and India by writing 450,000,000 + 30,800,000 while Rama calculated it by writing 30,800,000 + 450,000,000 and got the same answer. Which property of addition of numbers is indicated here?
2. Rajan calculated the total number of bikes in UK, France and Netherlands by adding as number of bikes in UK + number of bikes in France + number of bikes in Netherlands.
   Use associative property of addition of whole numbers and rewrite the above addition by changing brackets to get the same answer.
3. The sum of the total number of bikes in any two countries will result in number of bikes. Which property of whole numbers is indicated here?

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A)** : 56 + 39 is a whole number.
   Reason (R) : Whole numbers are closed with respect to addition.
2. **Assertion (A)** : 13(5 + 8) = 13 × 5 + 13 × 8
   Reason (R) : Multiplication is distributive over addition.
3. **Assertion (A)** : 575 + 699 is an even number.
   Reason (R) : The sum of two odd numbers is an odd number.
4. **Assertion (A)** : 69,866 can be rounded off to 69,870 to the nearest tens.
   Reason (R) : 66 is closer to 70 than 60.
5. **Assertion (A)** : ₹1,50,125 can be rounded off to ₹1,50,000.
   Reason (R) : The number 1,50,125 is rounded off to the nearest hundreds to get 1,50,000.
6. **Assertion (A)** : 5 + 0 = 0 + 5 = 5
   Reason (R) : 0 is additive identity.
7. **Assertion (A)** : 10 + 27 = 27 + 10
   Reason (R) : Associative property of addition.
8. **Assertion (A)** : 887 × 10 × 461 + 361 × 8870 = 887 × 10 ÷ (461 + 361)
   Reason (R) : Distributive property of multiplication over addition.
9. **Assertion (A)** : 8 × 5 is a whole number where 8 and 5 are whole numbers.
   Reason (R) : Whole numbers are closed with respect to addition.
10. **Assertion (A)** : Every whole numbers has a successor.
    Reason (R) : On adding 1 to given whole number, we get the next number.
11. **Assertion (A)** : The predecessor of a 2-digit number can be a one-digit number.
    Reason (R) : 10 – 1 = 9

3 Playing with Numbers

What Learners Will Achieve

- define factors, multiples and their properties.
- know about prime, composite, even, odd, co-prime and twin prime numbers.
- find the prime and composite numbers using the Sieve of Eratosthenes.
- test divisibility of given numbers by 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11.
- find the HCF and LCM of two or more numbers using different methods.
- understand the relationship between HCF and LCM of two numbers.
- apply the concept of HCF and LCM in real life problems.

Warm-up

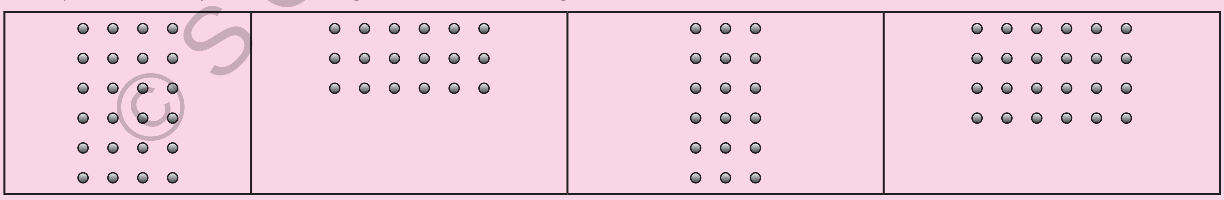
What we already know

- A *factor* of a number is an exact divisor of that number.
- Number of factors of a given number are finite.
- A number whose ones digit is 0, 2, 4, 6 or 8 is an *even* number. Alternatively, a number which is divisible by 2 is an *even* number.
- A number which is not an even number is called an *odd* number.
- A number is a multiple of each of its *factors*.
- All those numbers other than 1 which have exactly two factors, 1 and the number itself, are called *prime* numbers.
- A number with more than two factors is called a *composite* number.

Now, try to solve the following.

Sheeba and Ashu are playing with beads. Sheeba has 18 beads while Ashu has 24 beads. Both of them are trying to arrange their beads in rectangular form as shown below.

Can you identify who arranged the following?



Try to arrange the beads in different ways and complete the following:

1. 18 = ___3___ × ___6___ = ________ × ________ = ________ × ________

1, 2, 3, 6, 9 and 18 are the factors of ________.

2. 24 = ___4___ × ___6___ = ________ × ________ = ________ × ________ = ________ × ________

1, 2, 3, 4, 6, _____, 12 and 24 are the factors of ________.

3. ________, ________, ________ and ________ are the common factors of 18 and 24.

INTRODUCTION

The number concepts discussed in this chapter has several real life applications like the one given below.

A designer of a new production line at a local ball-bearing manufacturing plant predicts that the number of ball bearings produced each minute is divisible by 5. The next day a total of 1620 ball bearings were produced in 9 minutes.

Does this support the designer's prediction?

After learning the concepts in this chapter, you will be able to make interesting predictions like the one designer has made. Your predictions may be more accurate with knowledge of the concepts learnt in this chapter.

FACTORS AND MULTIPLES

- When two or more numbers are multiplied together, each number is called a **factor** of the product.

  For example, 4 × 3 = 12. So, numbers 4 and 3 are the factors of 12.

- **A factor of a number is an exact divisor of that number.**

  For example, 15 = 5 × 3. Here, 15 is divisible by both 5 and 3. So, 5 and 3 are divisors or factors of 15. Each factor is a divisor too.

- **A number is a multiple of each of its factors**.

  For example, 6 is a multiple of both 2 and 3. The numbers 2, 4, 6, 8, ... are all multiples of 2. Similarly, 5, 10, 15, ... are all multiples of 5.

  In fact, there are infinitely many multiples of a number.

Note

Usually, we talk about factors and multiples only in the case of natural numbers. So, for a number in this chapter, we shall mean a natural number.

Interesting Facts about Factors and Multiples

Factors

- There are finite number of factors of a given number.

  Illustration 1: The number 18 has six factors, which are 1, 2, 3, 6, 9 and 18.

- 1 is a factor of every number.

  Illustration 2: 10 = 1 × 10; 8 = 1 × 8, etc.

- Every number is a factor of itself.

  Illustration 3: 15 = 1 × 15

- Every factor is either less than or equal to the given number.

  Illustration 4: Factors of 12 are 1, 2, 3, 4, 6 and 12, which are less than or equal to 12.

Multiples

- Multiples of a number are infinite.

  Illustration 5: 12 has infinite number of multiples such as 12, 24, 48, ...

- Every number is a multiple of 1.

  Illustration 6: 2 = 2 × 1, 10 = 10 × 1, 1500 = 1500 × 1, etc.

- Every number is a multiple of itself.

  Illustration 7: 5 = 5 × 1, 7 = 7 × 1.

- Multiple of a number is either equal to or greater than the given number.

  Illustration 8: Multiples of 13 are 13, 26, 39, ..., which are greater than or equal to 13.

TYPES OF NUMBERS

Even Numbers

A number which is divisible by 2 is called an **even number**. In other words, a number which is a multiple of 2 is called an **even number**.

The ones digit of an even number is 0, 2, 4, 6 or 8.

For example, 30, 46, 178, 4032, 5874, etc., are even numbers.

Odd Numbers

A number that is not an even number is said to be an **odd number**. In other words, a number which is not a multiple of 2 is called an **odd number**.

The ones digit of an odd number is 1, 3, 5, 7 or 9.

For example, 13, 79, 225, 441, 537, etc., are odd numbers.

DID YOU KNOW?

The last date when every digit was odd was 19. 11. 1999 and the next such date will be 1.1.3111.

Prime Numbers

A natural number, other than 1, which has exactly two factors, 1 and the number itself is called a **prime number**.

For example, 2, 3, 5, 7, 11, 13, ... are prime numbers.

Remember

- There are infinitely many prime numbers.
- The smallest prime number is 2.
- The number 1 is neither prime nor composite.
- There is no largest prime number.

Composite Numbers

A natural number with more than two factors is called a **composite number**.

For example, 4, 6, 8, 9, 14, ... are composite numbers.

Thus, composite numbers are natural numbers greater than 1 which are not prime.

There are infinitely many composite numbers. The smallest composite number is 4. In fact, 2 is the only number which is both even and prime. All other even numbers are composite numbers.

Let us observe 221 = 13 × 17.

221 is a composite number and it is a product of two prime numbers.

Note

1. Every natural number greater than 1 can be written as a product of prime numbers.
2. Every prime number, other than 2, must be an odd number, since 2 is a factor of every even number.

Skill Check

- When a number is multiplied by another number, the result is called a ______.
- 2, 3, 5, 7, 11 are prime numbers because:
  (a) they are divisible by 2.
  (b) they have more than two factors.
  (c) they are not divisible by 2.
  (d) they have only two factors.
- The smallest prime number greater than 23 is ______.
- The number having exactly one factor is ______.

Co-prime Numbers

Two numbers having only 1 as a common factor are called **co-prime** numbers.

Illustration 1: 4 and 15 are co-primes.

Justification: 4 = 1 × 4 = 2 × 2.

Factors of 4 are 1, 2 and 4.

15 = 1 × 15 = 3 × 5. Factors of 15 are 1, 3, 5 and 15. Common factor is 1. Therefore, 4 and 15 are co-primes.

Remember

Co-prime numbers may be or may not be prime numbers. However, at least one of the two co-prime numbers must be an odd number.

Illustration 2: 18 and 24 are not co-primes.

Justification: 18 = 1 × 18 = 2 × 9 = 3 × 6.

Factors of 18 are 1, 2, 3, 6, 9 and 18.

24 = 1 × 24 = 2 × 12 = 3 × 8 = 4 × 6.

Factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

Common factors are 1, 2, 3 and 6.

Therefore, 18 and 24 are not co-primes.

Twin Primes

Two prime numbers whose difference is 2 are called **twin primes**.

Illustration 3:

3 and 5 are twin primes. (5 − 3 = 2)
5 and 7 are twin primes. (7 − 5 = 2)
11 and 13 are twin primes. (13 − 11 = 2)
17 and 19 are twin primes. (19 − 17 = 2)
3 and 7 are not twin primes. (Why?)

Remember

A set of three consecutive prime numbers differing by 2 is called a **prime triplet**. The set 2, 3, 5 is the only prime triplet known so far.

Perfect Numbers

A number for which the sum of all its factors is equal to **twice** the number is called a **perfect number**.

Illustration 4:

(a) Consider the number 6. Factors of 6 are 1, 2, 3 and 6.

Sum of its factors = $1 + 2 + 3 + 6 = 12 = 2 \times 6$

Therefore, 6 is a perfect number.

(b) Consider the number 28. Factors of 28 are 1, 2, 4, 7, 14 and 28.

Sum of its factors = $1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 \times 28$

Therefore, 28 is a perfect number.

(c) Consider the number 16. Factors of 16 are 1, 2, 4, 8 and 16.

Sum of its factors = $1 + 2 + 4 + 8 + 16 = 31 \neq 2 \times 16$.

Therefore, 16 is not a perfect number.

Remember

6 is the smallest perfect number.

Think!

What is the next perfect number? 496, 8128, ...

Sieve of Eratosthenes

A Greek mathematician **Eratosthenes** who lived in 3rd century BCE gave the following method for finding prime numbers. This method is known as the **Sieve of Eratosthenes**.

Step 1: List all the natural numbers from 1 to 50, in rows of ten numbers, as shown here.

Step 2: Cross out 1 (since 1 is neither prime nor composite). The next number is 2. Circle 2 and cross out all other multiples of 2 since they are not prime.

Step 3: The next available number is 3. Circle 3 and cross out all other multiples of 3 since they are not prime.

Step 4: The next available number is 5. Circle 5 and cross out all other multiples of 5 that are not crossed out yet.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~

If we continue the process, we will obtain the above table in which 1 and all the composite numbers are crossed out and the prime numbers are circled. Therefore, all the prime numbers up to 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

Skill Check

- The number of prime numbers between 1 and 20 is ____.
- The smallest pair of twin prime numbers is ____.
- The only even prime number is ____.
- Which pair of numbers represent twin prime numbers?
  (a) 1, 3 (b) 5, 11 (c) 6, 8 (d) 3, 5

Testing a Number as Prime or Composite

Procedure: To test whether a number is *prime* or *composite*, list all prime numbers whose *square* is less than the given number.

- If the given number is *divisible* by *any* of these prime numbers, it is a *composite number*.
- If the given number is not *divisible* by any of these prime numbers, it is a *prime number*.

Let us study some more examples.

Ex. 1. Test whether 57 is a prime number or not.

Sol. $7^2 = 49 < 57$ and $8^2 = 64 > 57$

Prime numbers less than 8 are 2, 3, 5 and 7. We observe that 2 does not divide 57 but 3 divides 57. So, 57 is not a prime number. It is a composite number.

Ex. 2. **Test whether 41 is a prime or not.**

Sol. $6^2 = 36 < 41$ and $7^2 = 49 > 41$

Prime numbers less than 7 are 2, 3 and 5.

We observe that 2 does not divide 41, 3 does not divide 41 and 5 does not divide 41. So, 41 is a prime number.

Ex. 3. **Express 18 as the sum of two odd primes.**

Sol. 18 = 5 + 13 or 7 + 11

Ex. 4. **Express 21 as the sum of three odd primes.**

Sol. 21 = 3 + 5 + 13 or 3 + 7 + 11

Ex. 5. **Write five pairs of prime numbers less than 20 whose sum is divisible by 5.**

Sol. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

Now, 2 + 3 = 5 which is divisible by 5.

2 + 13 = 15 which is divisible by 5.

3 + 7 = 10 which is divisible by 5.

3 + 17 = 20 which is divisible by 5.

7 + 13 = 20 which is divisible by 5.

11 + 19 = 30 which is divisible by 5.

13 + 17 = 30 which is divisible by 5.

Hence, choice of any five pairs from above is an answer. One of the answers is (2, 3), (2, 13), (3, 17), (11, 19) and (13, 17).

Observe!

221 = 37 + 41 + 43 + 47 + 53

221 is written as the sum of five consecutive prime numbers.

Also, 221 = 11 + 13 + 17 + 19 + 23 + 29 + 31 + 37 + 41.

221 is written as the sum of nine consecutive prime numbers.

Can you think of any other number?

Exercise 3.1

1. **Write all the factors of the following numbers.**
   (a) 30 (b) 42 (c) 68 (d) 96
2. **Determine whether 9 is a factor or not of the following numbers.**
   (a) 987 (b) 198 (c) 1428
3. **Write the first five multiples of each of the following.**
   (a) 4 (b) 7 (c) 16 (d) 23 (e) 28
4. **Identify whether the given numbers are prime or composite.**
   (a) 27 (b) 43 (c) 249 (d) 113 (e) 307
5. **Express each of the following numbers as the sum of two odd primes.**
   (a) 36 (b) 52 (c) 84
6. **Express each of the following numbers as the sum of three odd primes.**
   (a) 51 (b) 93 (c) 107
7. **Write the greatest prime number between the given numbers.**
   (a) 21 and 35 (b) 17 and 79 (c) 22 and 47
8. **Write the prime numbers between the given numbers.**
   (a) 40 and 10 (b) 25 and 45
9. Write 6 pairs of prime numbers less than 25 whose sum is divisible by 5.
10. List all the twin prime numbers between 51 and 100.
11. The numbers 13 and 31 are prime numbers. Both have the same digits 1 and 3. Find such pairs of prime numbers up to 100.
12. Which is the smallest number that must be added to an odd number to make it even?
13. How many pairs of prime numbers are there which can be used to express 20 as the sum of two prime numbers?
14. Find the total number of 2-digit prime numbers in which both the digits are also prime numbers.

TESTS FOR DIVISIBILITY OF NUMBERS

A number is said to be divisible by another number, if on division the remainder is zero.

76 is divisible by 4. Since 76 ÷ 4 = Q 19 R 0
(Quotient 19, Remainder 0)

81 is not divisible by 6. Since 81 ÷ 6 = Q 13 R 3
(Quotient 13, Remainder 3)

To find whether a number is divisible by another number, we usually perform actual division and see whether the remainder is zero or not. But some rules can save the time where we do not need to do actual division.

Let us learn divisibility test for numbers 2 to 10.

Divisibility Tests for 2, 3 and 5

Divisibility by 2

A number is divisible by 2 if its ones digit is 0, 2, 4, 6 or 8, *i.e.,* even.

Illustration 1:

- 254, 1250, 72, 356 are divisible by 2 as their ones digits 4, 0, 2 and 6 respectively are even.
- 17, 351, 457, 785 are not divisible by 2 as their ones digits are not even.

Divisibility by 3

A number is divisible by 3 if the sum of its digits is divisible by 3.

Illustration 2: Check if 123 and 245 are divisible by 3.

- Consider the number 123. The sum of the digits of 123 = 1 + 2 + 3 = 6.

  Here, 6 is divisible by 3. So, 123 is divisible by 3.
- Consider the number 245. The sum of its digits is 2 + 4 + 5 = 11.

  Here, 11 is not divisible by 3. So, 245 is not divisible by 3. Check by actual division.

Divisibility by 5

A number is divisible by 5, if its ones digit is either 0 or 5.

Illustration 3:

- Consider the number 115. The ones digit in 115 is 5. So, it is divisible by 5. The number 1110 is divisible by 5 as its ones digit is 0.
- Consider the number 523. The ones digit in 523 is neither 0 nor 5. So, it is not divisible by 5. It can also be verified by actual division that 523 is not divisible by 5.

Divisibility Tests for 6, 9 and 10

Divisibility by 6

A number is divisible by 6, if it is divisible by both 2 and 3.

Illustration 4: 3204 is divisible by 2, since its ones place digit is 4 which is even.

3204 is also divisible by 3, since the sum of its digits is 3 + 2 + 0 + 4 = 9, which is divisible by 3.

So, 3204 must be divisible by 6.

It can now be verified by actual division that 3204 is divisible by 6 as well.

Divisibility by 9

A number is divisible by 9, if the sum of its digits is divisible by 9.

Illustration 5:

- The sum of the digits of 387 is 3 + 8 + 7 = 18, which is divisible by 9. So, the number 387 is divisible by 9.
- The sum of the digits of the number 2719 is 2 + 7 + 1 + 9 = 19, which is not divisible by 9. So, the number 2719 is not divisible by 9. Check by actual division.

Divisibility by 10

A number is divisible by 10, if its ones digit is 0.

Illustration 6: The numbers 20, 50, 100, 150, 1000, 10000 are all divisible by 10 as ones digit of these numbers is 0.

Divisibility Tests for 4 and 8

Divisibility by 4

A number is divisible by 4 if the last two digits (that is, digits in the tens and ones places) are zeros or form a number which is divisible by 4.

Illustration 7:

- The numbers 908, 45700, 87764 are divisible by 4 since the last two digits form the numbers 08, 00, 64 respectively which are *divisible* by 4.

- The number 5134 is not divisible by 4 since the last two digits form the number 34, which is *not divisible by* 4.

Divisibility by 8

A number is divisible by 8, if the last three digits (that is, the digits in the ones group) are either all zeros or form a number divisible by 8.

Illustration 8:

- 75,000 is divisible by 8 since the last three digits are all zeros.
- 34,824 is divisible by 8 since the last three digits form the number 824 which is divisible by 8.
- 34,058 is not divisible by 8 since the last three digits form the number 58, which is not divisible by 8.

Divisibility Test for 11

A number is divisible by 11 if the difference of the sum of the digits in odd places and the sum of its digits in even places, is either 0 or divisible by 11.

Ex. 6. Test whether the following numbers are divisible by 11.

(a) 30,81,331 (b) 1265 (c) 1471

Sol. (a) In the number 30,81,331;

The sum of its digits in the odd positions(places) is 3 + 8 + 3 + 1 = 15.

The sum of its digits in the even positions (places) is 0 + 1 + 3 = 4.

The difference between 15 and 4 is 11, which is divisible by 11.

Thus, the given number 30,81,331 is divisible by 11.

(b) In the number 1265,

The sum of its digits at the odd places = 1 + 6 = 7.

The sum of its digits at the even places = 2 + 5 = 7.

Difference = 7 – 7 = 0.

Thus, the number 1265 is divisible by 11.

(c) In the number 1471, the sum of its digits at the odd places = 1 + 7 = 8 and the sum of its digits at the even places = 4 + 1 = 5. Difference = 8 – 5 = 3, which is not divisible by 11.

So, 1471 is not divisible by 11.

Alternate Method of checking the divisibility by 11:

Subtract the last digit from the remaining leading truncated number. If the result is divisible by 11, then so was the first number.

Apply this rule over and over again as necessary.

Illustration 9:

To check whether 19,151 is divisible by 11 or not, follow the given steps:

Step 1: 191**5** – 1 = 191**4**

Step 2: 191 – 4 = 18**7**

Step 3: 18 – 7 = 11 ← Divisible by 11

Thus, 19,151 is divisible by 11.

Skill Check

- Which lowest number must be subtracted from 115 so that it is exactly divisible by 9?
- Is the numbers 1702 divisible by 11?
- For what value of digit *p*, the number 467*p*96 is divisible by 11?

Divisibility Test for 7

From the given number, remove the digit at ones place. Multiply the removed digit by 5 and add the product so obtained to the truncated number. If the number so obtained is divisible by 7, then the given number is divisible by 7. (If the number obtained is large, repeat the process till we arrive at a 2-digit or a 1-digit number).

Illustration 10:

Let the given number be 61,838.

- Removing the digit at ones place, the truncated number becomes 6183.

  6183 8 ← Remove
  + 40
  6223

  Adding 8 × 5 = 40 to this gives 6223.
- Again removing the digit at ones place, the truncated number becomes 622. Adding 3 × 5 = 15 to this gives 637.

  622 3 ← Remove
  + 15
  637

- Removing the digit at ones place, the truncated number becomes 63.

  63 7 ← Remove
  + 35
  98

  Adding 7 × 5 = 35 to this gives 98.
- Removing the digit at ones place, the truncated number becomes 9.

  9
  + 40
  49 ← Divisible by 7

  Adding 8 × 5 = 40 to this gives 49, which is divisible by 7.

Thus, 61,838 is divisible by 7.

DID YOU KNOW?

The birthday of Mahendra Singh Dhoni, famous Indian cricketer falls on 7th of July, since the date is 7/7, he wears the jersey number 7 in all matches.

Ex. 7. Check if the given number 6837 is divisible by 7.

Sol. We remove the digit 7 (at ones place) to get the truncated number 683. Multiply the removed digit 7 by 5 to get 7 × 5 = 35. Add this to the truncated number to get 683 + 35 = 718.

683
+ 35
718

Repeat the above process. Digit removed is 8, truncated number becomes 71.

Product of 8 and 5 = 8 × 5 = 40.

71
+ 40
111 ← Not divisible by 7

Add to 71 to get 71 + 40 = 111. 111 is not divisible by 7.

Thus, 6837 is not divisible by 7.

Alternate Method of checking the divisibility by 7:
Double the last digit and subtract from the number so formed by the rest of the digits. If the difference is 0 or divisible by 7, then the number is divisible by 7 otherwise not. (If the difference is too big, you can do this process again.)

Illustration 11:

553; 3 + 3 = 6; 55 – 6 = 49

Since 49 is divisible by 7, so 553 is also divisible by 7.

Some More Divisibility Rules

Consider the number **24**. It is divisible by 6 and factors of 6 are 1, 2, 3 and 6. We see that the number 24 is also divisible by 1, 2, 3 and 6.

Take another number **30**. It is divisible by 10 and factors of 10 are 1, 2, 5 and 10. We see that the number 30 is also divisible by 1, 2, 5 and 10.

From this, we can say that:

Rule 1: If a number is divisible by another number (say *x*), then it is divisible by each of the factors of that number (*x*).

Number 60 is divisible by 3 and 5, which are **twin primes.**

60 is also divisible by 3 × 5 or 15.

Similarly, number 80 is divisible by 4 and 5, which are **co-primes**.

80 is also divisible by 4 × 5 or 20.

From this, we may conclude that:

Rule 2: If a number is divisible by two twin prime or co-prime numbers, then it is also divisible by their product.

Numbers 15 and 25 are both divisible by 5. Now, look at the number 15 + 25 = 40, which is also divisible by 5.

Numbers 20 and 40 are both divisible by 4 and their sum 20 + 40 = 60 is also divisible by 4.

From this, we can say that:

Rule 3: If two given numbers are divisible by a number, then their sum is also divisible by that number.

Numbers 15 and 6 are both divisible by 3. Their difference, *i.e.*, 15 – 6 = 9 is also divisible by 3.

Numbers 20 and 40 are both divisible by 4 and their difference, *i.e.*, 40 – 20 = 20 is also divisible by 4.

From this, we can say that:

Rule 4: If two given numbers are divisible by a number, then their difference is also divisible by that number.

Ex. 8. **Insert the smallest digit in the blank space in the number 245_72, so that the number is divisible by (a) 3, (b) 9.**

Sol. Sum of the digits of the given number = 2 + 4 + 5 + _____ + 7 + 2 = 20 + ____

(a) The next number (smallest) divisible by 3 is 21, that means 20 + __1__ = 21.
So, the digit in the blank space would be 1.

(b) The number greater than 20 divisible by 9 is 27 (smallest) that means 20 + __7__ = 27.
So, the digit in the blank space would be 7.

Ex. 9. **Write a digit in the blank space of the number 92__389, so that the number is divisible by 11.**

Sol. In the number 92__389,
the sum of the digits at odd places
= 9 + _____ + 8 = 17 + ____
and the sum of the digits at even places
= 2 + 3 + 9 = 14.
Difference between these sums
= 17 + ____ − 14 = 3 + _____
We know that the number will be divisible by 11, if the difference is 0 or a multiple of 11, *i.e.*, 11, 22, etc.
Here, it should be 11.
That means 3 + __8__ = 11.
Thus, the required digit is 8.

Exercise 3.2

1. **Tick (✓) the correct answer.**
   (a) If a number is divisible by 14, then it is always divisible by:
   (i) 2 and 3 (ii) 3 and 4 (iii) 10 and 4 (iv) 7 and 2
   (b) The smallest number which is divisible by all the numbers from 1 to 10 is:
   (i) 4040 (ii) 2520 (iii) 4545 (iv) 4050
   (c) If a number is divisible by 2 and 3, then by which other number will the number be always divisible?
   (i) 4 (ii) 5 (iii) 6 (iv) 9
   (d) The sum of two consecutive odd numbers is always divisible by:
   (i) 6 (ii) 3 (iii) 4 (iv) 10
2. **Determine whether the following numbers are divisible by 2, 4 or 8.**
   (a) 236 (b) 4,37,890 (c) 64,84,244 (d) 63,72,400
3. **Determine whether the following numbers are divisible by 3, 6 or 9.**
   (a) 201 (b) 3784 (c) 2745 (d) 5460
4. **Determine whether the following numbers are divisible by 5 or 10.**
   (a) 21,335 (b) 1552 (c) 64,240
5. **Determine whether the following numbers are divisible by 11 or not:**
   (a) 21,084 (b) 87,835 (c) 1,11,10,011
6. A number is divisible by both 7 and 9. By which other two numbers will that number be divisible?
7. A number is divisible by both 6 and 7. By which other two numbers will that number be divisible?
8. Using divisibility tests, determine whether the following numbers are divisible as directed:
   (a) 11,040 by 20 (b) 9504 by 44 (c) 4340 by 35
   (d) 14,560 by 8 (e) 10,824 by 22 (f) 25,110 by 45

9. The designer of the new production line at a local nut-bolt manufacturing plant predicts that the number of nut-bolts produced each minute is divisible by 5. The next day a total of 1539 nut-bolts are produced in 9 minutes. Does this support the designer's prediction?

10. Write the (a) smallest digit and (b) greatest digit, in the blank space of 4765__2, so that the number is divisible by 3.

11. **Write the smallest digit in the given blank spaces so that the numbers are divisible by 9.**
(a) 219__35 (b) __25732

12. **Write the smallest digit in the given blank spaces so that the numbers are divisible by 11.**
(a) 325__9215 (b) 8212__5

13. A number is divisible by both 5 and 6. By which other two numbers will that number be divisible?

PRIME FACTORISATION

A number is said to be factorised, if it can be written as a product of two or more numbers.

For example, 12 can be factorised in many ways,

$1 \times 12, \quad 2 \times 6, \quad 3 \times 4, \quad 2 \times 2 \times 3$

Each of these is called a *factorisation* of 12.

However, the factorisation $2 \times 2 \times 3$ is special as all the factors are prime numbers.

Such a factorisation is called **prime factorisation**.

"*Every composite number has a unique prime factorisation.*"

This is known as prime factorisation property or the "**Fundamental Theorem of Arithmetic**."

Prime Factorisation Method

Step 1: In prime factorisation method, division of a number starts with the smallest prime number, *i.e.*, 2. If the number is not divisible by 2, move to the next prime number, namely 3 and so on.

Step 2: If the number is divisible by 2, then write the quotient and repeat the above process again.

Step 3: Continue this process by moving on the next prime number until the quotient comes out to be 1.

Step 4: Write the number as the product of prime factors (numbers) so obtained by repeating a prime number as many times in the product as the number of times it appears in the division process.

Illustration 1: Let us determine the prime factorisation of the number 72.

- 72 is divisible by the smallest prime number 2, yielding a quotient of 36.
- Now, divide 36 by 2 and get the quotient 18 and so on to save time, do not rewrite the division problem, but simply keep dividing each quotient until the quotient is 1.

2	72
2	36
2	18
3	9
3	3
	1 ← Stop here.

- Since 9 is not divisible by 2, we now consider the next prime number 3, which divides 9.

Thus, the prime factorisation of 72 is the product of all the successive divisors. That is,

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

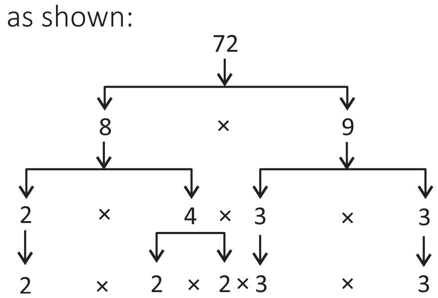
Tree Method

Another method to find the prime factorisation of a number is the factor tree method.

Illustration 2: Consider the number 72. Express it as a product of any two factors.

$$72 = 8 \times 9$$

Now, factorise 8 and 9 separately. Observe the process as shown:



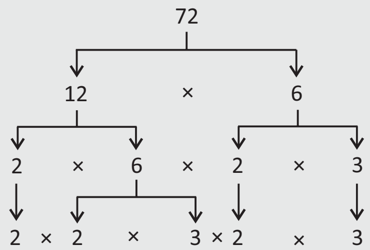
Thus, prime factorisation of 72 $= 2 \times 2 \times 2 \times 3 \times 3$.

Watch Your Step!

Do not use 1 as a factor in the factor tree, as it is neither prime nor composite.

Note

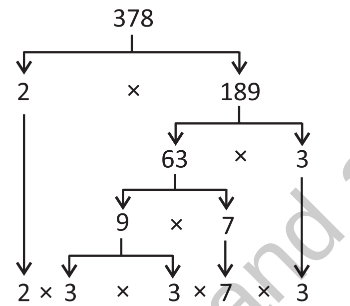
It is important to note that there can be more than one factor tree for a given number. However, the prime factorisation obtained from each factor tree will be the same. Consider another factor tree for 72. Thus, 72 = 2 × 2 × 3 × 2 × 3 = 2 × 2 × 2 × 3 × 3, the same as before.



Let us study some more examples.

Ex. 10. Express 378 as a product of prime factors using the factor tree method.

Sol.



Thus, 378 = 2 × 3 × 3 × 7 × 3
= 2 × 3 × 3 × 3 × 7

Ex. 11. Write the greatest 4-digit number and express it as a product of prime factors.

Sol. The greatest 4-digit number = 9999

On factorising 9999, we have

3	9999
3	3333
11	1111
101	101
	1

Thus, the prime factorisation of 9999 = 3 × 3 × 11 × 101. 101 is a prime number as it is not divisible by 2, 3, 5 and 7 (we need not to check by 11 since $11^2 > 101$).

Ex. 12. Write the smallest 5-digit number and express it as product of prime factors.

Sol. The smallest 5-digit number is 10,000. On factorising, we have

2	10000
2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

Thus, the prime factorisation of 10,000
= 2 × 2 × 2 × 2 × 5 × 5 × 5 × 5

Alternate Method:

10,000 = 10 × 10 × 10 × 10
= 2 × 5 × 2 × 5 × 2 × 5 × 2 × 5
= 2 × 2 × 2 × 2 × 5 × 5 × 5 × 5

Exercise 3.3

1. **Tick (✓) the correct answer.**

(a) The prime factors of 24 are:

(i) 2 × 6 × 2 (ii) 2 × 2 × 2 × 3 (iii) 4 × 3 × 2 (iv) 6 × 4 × 1

(b) Which factor(s) is(are) not included in the prime factorisation of a composite number?

(i) 0 and 1 (ii) 1 and the number itself

(iii) the number itself (iv) the prime numbers

(c) The prime factorisation of the smallest 3-digit number is:

(i) 2 × 5 × 5 × 2 (ii) 1 × 2 × 5 × 5 × 2

(iii) 1 × 2 × 5 × 5 × 2 × 100 (iv) 2 × 5 × 5 × 2 × 100

(d) The smallest number having four different prime factors is:
(i) 219 (ii) 216 (iii) 215 (iv) 210

(e) The smallest number whose prime factors are 2, 3, 5 and 11 is:
(i) 220 (ii) 660 (iii) 550 (iv) 330

2. Express the following numbers as a product of prime factors, using the tree method.
(a) 90 (b) 749 (c) 625 (d) 1480 (e) 6182

3. Write the greatest 6-digit number and express it as a product of prime factors.

4. Write the smallest 4-digit number and express it as a product of prime factors.

HIGHEST COMMON FACTOR (HCF)

The **Highest Common Factor (HCF)** of two or more given numbers is the largest number that divides all of them completely. In other words, if all the factors of all the given numbers are written, then the largest of these factors which is common to all the numbers is their HCF.

Now, we shall discuss four different methods for finding the HCF of numbers.

Listing of Factors Method

The highest common factor of numbers can be found by listing all the factors of each number and finding the highest factor that is common to all the lists.

Ex. 13. Find the HCF of 12 and 18, by listing factors.

Sol. Let us find the factors of the numbers 12 and 18.
Factors of 12 are 1, 2, 3, 4, **6**, 12.
Factors of 18 are 1, 2, 3, **6**, 9, 18.
The common factors in the two lists are 1, 2, 3 and 6. The highest amongst them is 6.
Therefore, HCF of 12 and 18 is 6.
We also write it as HCF (12, 18) = 6.
Finding the HCF by this method has a drawback. Some large numbers have many factors and listing them all can be difficult.

Since HCF of two or more numbers is the greatest number that divides each of them exactly, it is also called the **Greatest Common Divisor (GCD)**.

Prime Factorisation Method

Ex. 14. Find the HCF of 36 and 126 by prime factorisation method.

Sol. To find the highest common factor of 36 and 126, write the prime factorisation of each.

2	36
2	18
3	9
	3

2	126
3	63
3	21
	7

36 = 2 × 2 × 3 × 3

126 = 2 × 3 × 3 × 7

The HCF is the product of the common prime factors.

So, the HCF is 2 × 3 × 3 = 18.

Short Division Method

Using the method of prime factorisation to find the HCF requires lots of calculations, especially if the numbers are large. Given below is another method which is usually referred to as the **Short Division Method**.

To use this method with two numbers, divide both the numbers by any **common** prime number factor. Repeat this step with the quotients and continue in the same manner until the quotients have no common factor. The highest common factor is the product of all the divisors.

Ex. 15. Find the HCF of 72 and 96, using the short division method.

Sol. HCF of 72 and 96
= 2 × 2 × 2 × 3
= 24

2	72, 96
2	36, 48
2	18, 24
3	9, 12
	3, 4

Long Division Method (or Continued Division Method)

To find the HCF of two given numbers, follow these steps:

Step 1: Divide the larger number by the smaller one. If the remainder is 0, the smaller number is the HCF.

Step 2: If the remainder is non-zero, divide the smaller number by the remainder and get a new remainder.

Step 3: Repeat the process of dividing the preceding remainder by the new remainder till the next new remainder is 0.

Step 4: The last divisor is the required HCF.

Ex. 16. Find the HCF of 360 and 456.

Sol. Clearly, 456 > 360, so first we divide 456 by 360.

Here, the last divisor = 24.

Therefore, HCF (360, 456) is 24.

```
360) 456 (1
    −360
     96) 360 (3
        −288
         72) 96 (1
            −72
             24) 72 (3
                −72
                  0
```

Note

This method can be used for finding the HCF of more than two numbers also.

For example, for finding the HCF of 360, 456 and 210, we first find the HCF (360, 456) = 24. Then, we find the HCF (24, 210) as shown alongside:

```
24) 210 (8
   −192
    18) 24 (1
       −18
        6) 18 (3
          −18
            0
```

Thus, the required HCF is 6.

Skill Check

Find the HCF of:

(a) 36, 48 and 54

(b) 11, 33, 44 and 88

(c) 144, 180 and 192

Let Us Do AIL

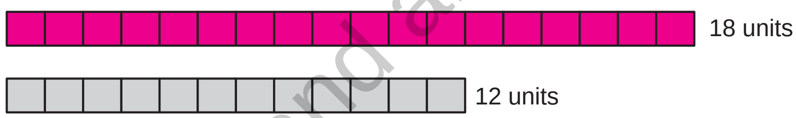
Objective: To find HCF using paper strip

Materials required: Paper strips of unit length (two different colours), a pair of scissors

Procedure: To find the HCF of two given numbers, cut one strip of length 18 units and another strip of length 12 units.

Let us try to find the HCF of 12 and 18.

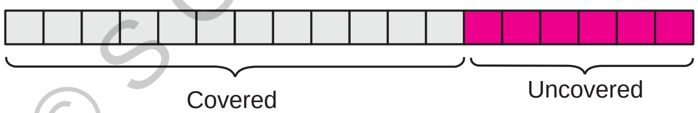
Step 1: Take a strip of length 18 units and another strip of length of 12 units.




```
12) 18 (1
   −12
     6
```

Cover the larger strip with the smaller strip.

The strip of length 12 units covers a part of strip of length 18 units.



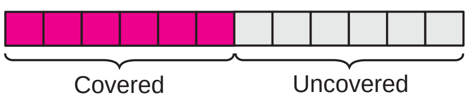
We observe that 6 units of 18 units length strip remain uncovered.

Cut this strip of 6 units. 

Step 2: Now, repeat step 1 with strips of lengths 6 units and 12 units.

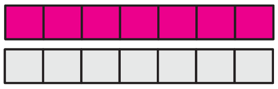
Cover the strip of 12 units length with strip of 6 units length.

```
6) 12 (2
  −12
    0
```



Observe that 6 units of 12 units length strip remain uncovered.

Step 3: Another strip of 6 units will completely cover the remaining portion.



(Both will cover completely each other.)

We observe that, no part of strip of 6 units length are left uncovered.

So, HCF = 6.

Conclusion: HCF of 12 and 18 is 6.

> **Note**
>
> It is not necessary that the process may end after step 3. We may have to continue further steps.

Properties of HCF

- The HCF of two or more numbers is not greater than any of the given numbers.
  For example, HCF of 8 and 12 is 4. Observe that 4 < 8 and 4 < 12.
- The HCF of two consecutive numbers is always 1.
  For example, HCF of 15 and 16 is 1; HCF of 100 and 101 is 1.
- The HCF of two co-prime numbers is 1.
  For example, HCF of 7 and 20 is 1.
- The HCF of two consecutive even numbers is 2.
  For example, HCF of 12 and 14 is 2.
  (Factors of 12 = 2 × 2 × 3, factors of 14 = 2 × 7)
- The HCF of two consecutive odd numbers is 1.
  For example, HCF of 25 and 27 is 1.
  (Factors of 25 = 1, 5, 25 and factors of 27 = 1, 3, 9, 27)

Skill Check

- The HCF of two co-prime numbers is __________ .
- The HCF of two consecutive even numbers is always __________ .
- The HCF of prime number 2 and a non-zero even whole number is __________ .
- The HCF of two 2-digit prime numbers is __________.

Let us study some more examples.

Ex. 17. Find the common factors of 12, 20 and 28.

Sol. We know that

12 = 1 × 12 = 2 × 6 = 3 × 4

20 = 1 × 20 = 2 × 10 = 4 × 5

and 28 = 1 × 28 = 2 × 14 = 4 × 7

Factors of 12 are 1, 2, 3, 4, 6 and 12.

Factors of 20 are 1, 2, 4, 5, 10 and 20.

Factors of 28 are 1, 2, 4, 7, 14 and 28.

Thus, the common factors of 12, 20 and 28 are 1, 2 and 4.

Ex. 18. There are 252 apples and 576 oranges. These fruits are to be arranged in heaps containing the same number of fruits. Find the greatest number of fruits possible in each heap.

Sol. Fruits are to be arranged in heaps so that each heap has the same number of fruits. So, number of fruits in each heap = HCF of 252 and 576.

```
252) 576 (2
    −504
     72) 252 (3
        −216
          36) 72 (2
             −72
               0
```

Using continued division method, HCF of 252 and 576 = 36

Therefore, the greatest number of fruits possible in each heap = 36

Ex. 19. Find the greatest number which divides 280 and 1245 leaving remainders 4 and 3 respectively.

Sol. The greatest number which divides 280 and 1245 leaving remainders 4 and 3 respectively. So, the required number will be the HCF of (280 – 4) and (1245 – 3) *i.e.*, 276 and 1242.

By short division method,

2	276, 1242
3	138, 621
23	46, 207
	2, 9

∴ HCF (276, 1242) = 2 × 3 × 23 = 138

Thus, the required number is 138.

Ex. 20. Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of both the tankers when used an exact number of times.

Sol. The required container should have the capacity that can measure the quantity of petrol in two tankers exactly. That means this capacity will be the exact divisor of 850 and 680. Also, it should be maximum, so we have to find the HCF of 850 and 680.

By prime factorisation,

2	850
5	425
5	85
	17

2	680
2	340
2	170
5	85
	17

850 = 2 × 5 × 5 × 17

680 = 2 × 2 × 2 × 5 × 17

∴ HCF (850, 680) = 2 × 5 × 17 = 170

Thus, the capacity of the required container must be 170 litres.

Ex. 21. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Find the longest tape which can measure the three dimensions of the room exactly.

Sol. Given, length = 8 m 25 cm = (8 × 100 + 25) cm = 825 cm, breadth = 6 m 75 cm = (6 × 100 + 75) cm = 675 cm and height = 4 m 50 cm = (4 × 100 + 50) cm = 450 cm

The required length of the tape should be longest and an exact divisor of 825 cm, 675 cm and 450 cm.

Therefore, we need to find the HCF of 825, 675 and 450.

Using continued division method, first we find the HCF of 825 and 675.

675) 825 (1
−675
150) 675 (4
−600
75) 150 (2
−150
0

∴ HCF (825, 675) = 75

Now, find the HCF of 75 and 450.

75) 450 (6
−450
0

∴ HCF (450, 75) = 75

Thus, HCF (825, 675, 450) = 75

Therefore, the required length of the tape is 75 cm.

Exercise 3.4

1. **Determine the common factors of the given numbers.**
   (a) 15 and 25 (b) 28 and 42 (c) 15, 20 and 30 (d) 20, 30 and 40
2. **Determine the HCF of the given numbers by listing factors.**
   (a) 12 and 54 (b) 18 and 40 (c) 8, 12 and 20 (d) 16, 40 and 56
3. **Determine the HCF of the given numbers using prime factorisation method.**
   (a) 30 and 45 (b) 63 and 42 (c) 144, 162 and 180 (d) 324, 180 and 120
4. **Determine the HCF of the given numbers using short division method.**
   (a) 50 and 140 (b) 63 and 81 (c) 105 and 140
5. There are 119 apples and 153 oranges. These fruits are to be arranged in heaps containing the same number of fruits. Find the greatest number of fruits possible in each heap.
6. Two tankers contain 650 litres and 780 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.
7. Find the greatest number which divides 135, 245 and 385 leaving remainder 5 in each case.
8. The length, breadth and height of a room are 768 cm, 576 cm and 832 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

9. Two ropes 16 m and 20 m long are to be cut into small pieces of equal length. What will be the maximum length of each piece?

10. An origami artist wants to cut squares from a hand painted sheet of paper which measures 210 cm wide by 330 cm long. She wants all the squares to be of the same size. What size of the square should she cut so that she gets the largest possible square without wasting any paper?

LOWEST (LEAST) COMMON MULTIPLE

We are now familiar with factors and multiples of numbers. In the previous section, we have learnt to find the HCF of two or more numbers. Now, we shall learn how to find the **common multiples** and **least common multiple** of two or more numbers.

Common Multiples

Let us observe the multiples of more than one number.

Multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, **36**, ...

Multiples of 6 are 6, **12**, 18, **24**, 30, **36**, 42, 48, ...

Multiples of 12 are **12**, **24**, **36**, 48, 60, 72, 84, ...

Comparing these three lists, we find that the numbers **12**, **24**, **36**, ... are present in every list. The numbers which are present in each of the above lists are known as common multiples of 4, 6 and 12. Therefore, **common multiples** of 4, 6 and 12 are **12, 24, 36,** ... .

Lowest Common Multiple

The lowest common multiple (LCM) of two or more given numbers is the lowest of their common multiples. In the above lists, the LCM of 4, 6 and 12 is 12.

Finding LCM

Now, we will discuss various methods to find the LCM of two or more numbers.

Listing of multiples method

Step 1: List the multiples of each of the given numbers.

Step 2: Find the least multiple common to all the lists.

Ex. 22. Find the LCM of the numbers 12 and 15.

Sol. Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, ...

Multiples of 15 are 15, 30, 45, 60, 75, 90, 105, 120, ...

First two common multiples in the two lists are **60** and **120**. The lowest or the least amongst them is 60. Therefore, LCM of 12 and 15 is 60. We also write it as LCM (12, 15) = 60.

Prime factorsation method

Step 1: Write the prime factorisation for each of the given numbers.

Step 2: Write the product of each prime factor with the maximum number of times it occurs in each of the given numbers.

Ex. 23. Find the LCM of 18, 20 and 24.

Sol. By prime factorisation, we have

$18 = 2 \times \boxed{3 \times 3}$

$20 = 2 \times 2 \times \boxed{5}$ (Occurring maximum)

$24 = \boxed{2 \times 2 \times 2} \times 3$

Thus, the LCM of 18, 20 and 24

$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Short division method (or Common division method)

Step 1: Divide by a prime number which is a common divisor of at least two of the given numbers and carry forward the number(s) which are not divisible.

Step 2: Repeat Step 1 with the quotient and the undivided numbers and continue the process until no two numbers have any common divisor.

Step 3: Find the product of all of the divisors and the final quotients.

Ex. 24. Find the LCM of 15, 14 and 24.

Sol.

2	15, 14, 24	← Divide by 2, since 2 divides 14 and 24. Carry forward 15 with the quotients.
3	15, 7, 12	← The next prime number which divides at least two of 15, 7 and 12 is 3. Divide by 3.
	5, 7, 4	← Stop, since no two of the numbers 5, 7 and 4 have a common divisor.

Thus, the LCM of 15, 14 and 24

$= 2 \times 3 \times 5 \times 7 \times 4 = 840$

Properties of LCM

- The LCM of the numbers is not less than any of the given numbers.
  For example,

2	12, 16, 40
2	6, 8, 20
2	3, 4, 10
	3, 2, 5 ← Stop

$\therefore$ LCM = 2 × 2 × 2 × 2 × 3 × 5 = 240

LCM of 12, 16, 40 = 240 and 240 is not less than either of 12, 16 or 40.

- The LCM of two co-prime numbers is equal to their product.
  For example, consider the numbers 15 and 16. Note that 15 and 16 are co-primes.

15 = 3 × 5

16 = 2 × 2 × 2 × 2

(15 and 16 have no common prime factors.)

$\therefore$ LCM (15, 16) = 2 × 2 × 2 × 2 × 3 × 5
= 240 = 15 × 16

Relation between Two Numbers and their HCF and LCM

Consider the numbers 36 and 48.

The prime factorisation of 36 = 2 × 2 × 3 × 3 and that of 48 = 2 × 2 × 2 × 2 × 3

HCF of 36 and 48 = 2 × 2 × 3 = 12

LCM of 36 and 48 = 2 × 2 × 2 × 2 × 3 × 3 = 144

Product of the given numbers = 36 × 48 = 1728

Product of HCF and LCM = 12 × 144 = 1728

Therefore, we can write it as follows:

> The product of two numbers
> = HCF of the numbers × LCM of the numbers

Skill Check

- What is the LCM of 4, 12 and 15?
- If two numbers have 5 as their HCF, then which of the following numbers can be their LCM?
  (a) 31 (b) 32 (c) 33 (d) 35
- If the HCF of two numbers is 16 and their product is 6400, then their LCM is ______.
- If the HCF of two numbers *x* and *y* is 1, then their LCM is ______.

Let us study some more examples.

Ex. 25. Write the 7th to 12th multiples of 4.

Sol. Required multiples are 7 × 4 = 28, 8 × 4 = 32, 9 × 4 = 36, 10 × 4 = 40, 11 × 4 = 44 and 12 × 4 = 48.

Ex. 26. Find the LCM of 96, 108 and 180, using the prime factorisation method.

Sol.

2	96
2	48
2	24
2	12
2	6
3	3
	1

2	108
2	54
3	27
3	9
3	3
	1

2	180
2	90
3	45
3	15
5	5
	1

96 = 2 × 2 × 2 × 2 × 2 × 3 (2 occurs maximum 5 times)

108 = 2 × 2 × 3 × 3 × 3 (3 occurs maximum 3 times)

180 = 2 × 2 × 3 × 3 × 5 (5 occurs maximum 1 time)

Thus, the LCM of 96, 108 and 180

$= \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{\text{Maximum 5 times}} \times \underbrace{3 \times 3 \times 3}_{\text{Maximum 3 times}} \times \underset{\text{Maximum 1 time}}{5}$

= 4320

Ex. 27. The product of two numbers is 1200 and their HCF is 10. Find the LCM of the numbers.

Sol. Product of the two numbers = 1200,
HCF of the numbers = 10
We know that,
Product of the two numbers = HCF × LCM
So, 1200 = 10 × LCM

$\Rightarrow \quad \text{LCM} = \dfrac{1200}{10} = 120$

Thus, the LCM of the numbers is 120.

Ex. 28. The HCF of two numbers is 18 and their LCM is 108. If one of the numbers is 54, find the other number.

Sol. We know that for any two numbers,
Product of the two numbers = HCF × LCM
If one number = 54, then the other number

$= \dfrac{\text{HCF} \times \text{LCM}}{\text{Given number}} = \dfrac{18 \times 108}{54} = 36$

Thus, the other number is 36.

Ex. 29. Find the least number which when divided by 6, 15 and 18 leaves a remainder 5 in each case.

Sol. The least number which can be completely divided by 6, 15 and 18 is the LCM of 6, 15 and 18.

2	6, 15, 18
3	3, 15, 9
	1, 5, 3

LCM of 6, 15 and 18 = 2 × 3 × 5 × 3 = 90

But we need the least number that leaves a remainder 5 in each case.

Therefore, the required number is 5 more than 90.

Thus, the least number that leaves remainder 5 in each case = 90 + 5 = 95.

Ex. 30. Four gongs strike at intervals of 6 minutes, 16 minutes, 18 minutes and 20 minutes respectively. At what earliest possible time will they strike together again, if they start simultaneously at 12 noon?

Sol. First, we find the LCM of 6, 16, 18 and 20.

2	6, 16, 18, 20
2	3, 8, 9, 10
3	3, 4, 9, 5
	1, 4, 3, 5

LCM = 2 × 2 × 3 × 4 × 3 × 5 = 720

The gongs will strike again together after every 720 minutes = $\frac{720}{60}$ hours = 12 hours.

They start simultaneously at 12 noon, the earliest they will all strike together again is at 12 noon + 12 hours = 12 midnight.

Ex. 31. Determine the largest 3-digit number exactly divisible by 8, 10 and 12.

Sol. To find a number divisible by 8, 10 and 12, we find the LCM of 8, 10 and 12.

2	8, 10, 12
2	4, 5, 6
	2, 5, 3

LCM of 8, 10 and 12

= 2 × 2 × 2 × 3 × 5 = 120

So, the largest 3-digit number which is exactly divisible by 8, 10 and 12 will be a multiple of 120 and it is equal to 120 × 8 = 960.

Exercise 3.5

1. **Find the multiples as directed:**
   (a) 5th multiple of 20 (b) 8th to 11th multiples of 30
   (c) 3rd common multiple of 6, 9 and 15
2. **Using the listing of multiples method, find the LCM of:**
   (a) 21 and 14 (b) 24 and 16 (c) 60, 75 and 150
3. **Determine the LCM of the given numbers using prime factorisation method.**
   (a) 216, 72 and 270 (b) 144, 48 and 180 (c) 120, 160 and 240
4. **Determine the LCM of the given numbers using short division method.**
   (a) 21, 16 and 18 (b) 27, 10 and 12 (c) 60, 75 and 150
5. Determine the LCM, if the product of two numbers is 1250 and their HCF is 25.
6. The HCF of two numbers is 19 and their LCM is 228. If one of the numbers is 57, find the other number.
7. Three bells ring at regular intervals of 20 minutes, 36 minutes and 48 minutes respectively. At what time will they ring together again next, if they start simultaneously at 12 noon?
8. Three friends go for a morning walk and step off together from the same spot. Their steps measure 40 cm, 45 cm and 48 cm respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?

9. Find the least number which when divided by 10, 15 and 20 leaves a remainder 4 in each case.
10. Four gongs strike at intervals of 30 minutes, 60 minutes, 90 minutes and 150 minutes respectively. At what time will they strike together again next, if they start simultaneously at 12 noon?
11. The traffic lights at three different road crossings change after every 35 seconds, 42 seconds and 70 seconds respectively. If they change simultaneously at 9 a.m., at what time will they change simultaneously next?
12. Determine the largest 3-digit number exactly divisible by:
(a) 6, 10 and 12 (b) 10, 15 and 18 (c) 4, 8 and 18
13. Find the smallest 4-digit number which is exactly divisible by 32, 36, 40 and 45.
14. Find the greatest 6-digit number which is exactly divisible by 15, 18 and 24.
15. Which is the least number of plants I should buy, if I wish to plant them in equal rows of 4, 5 or 6?

Competency Based Exercise

21st CS

1. **Tick (✓) the correct answer.**
   (a) The smallest 4-digit number divisible by 4 and formed by using the digits 2, 5, 6 and 3 only once is:
   (i) 5236 (ii) 3256 (iii) 2356 (iv) 3526
   (b) The least 5-digit number exactly divisible by 16, 18, 24 and 30 is:
   (i) 10,440 (ii) 10,340 (iii) 10,080 (iv) 10,000
   (c) In a colony of 110 blocks of flats, a school van stops at every sixth block while the school bus stops at every tenth block. If they start from the entrance of the colony, then the blocks on which both of them stop together are :
   (i) 12, 60, 90 (ii) 30, 60, 90 (iii) 30, 70, 110 (iv) 30, 80, 110
   (d) Which of the following numbers is divisible by 11?
   (i) 40,91,810 (ii) 77,77,777 (iii) 2,22,22,222 (iv) 33,33,333
   (e) If the number 2795*84 is divisible by 44, then the digit at hundreds place is:
   (i) 4 (ii) 5 (iii) 6 (iv) 7
   (f) A 6-digit number of the form ABCABC is always divisible by:
   (i) 2, 3 and 5 (ii) 3, 5 and 7 (iii) 5, 7 and 11 (iv) 7, 11 and 13
   (g) The LCM of the two co-prime numbers a and b is equal to:
   (i) $a + b$ (ii) $a - b$ (iii) $a \div b$ (iv) $a \times b$
2. Find the greatest number which divides 1055, 756 and 432, leaving the remainders 5, 6 and 7 respectively.
3. The HCF and LCM of two numbers x and y are respectively 27 and 2079. If x is divided by 9, the quotient is 21, then find y.

4. This year my age is a multiple of 7. Next year, it will be a multiple of 5. If I am above 20 years old but less than 80 years, then what is my present age?
5. If coffee is to be packed in 100 g, 200 g, 250 g and 500 g packets, then find the least quantity of coffee (in kg) to make any kind of packets from it.
6. Find the least number which when divided by 15, 18 and 25 leaves a remainder 2 in each case.
7. Three planets complete a revolution around the Sun once in every 200 days, 250 days and 300 days respectively. If they are in a line at a particular moment, then find the minimum number of days when they will again be in the same positions.
8. Renu was asked to add 14 to a certain number and then divide the result by 4. Instead, she first added 4 and then divided by 14, getting the result as 5. Had she followed the instructions correctly, by how much would her previous result have differed from the correct result?

Challenge! 21st CS

1. Sumitra is reading a storybook. The product of the page numbers opened in front of her is 600. What are the page numbers?
2. Tushar tries to catch a rabbit. They are 180 m apart. For every 10 m Tushar runs, the rabbit jumps 7 m. How much distance Tushar must run to catch the rabbit?
3. Find the two numbers whose sum is 100 and product is 1771.
4. Find the three numbers whose sum is 100 and product is 35,964.

Let's Work in Mind 21st CS

1. Sohan divides a number by 124 and gets the remainder 62. If Swati divides the same number by 31, then what will be the remainder?
2. If a number gives a remainder 3 on dividing by both 10 and 12, then what will be the smallest such number?
3. Find the least number which when increased by 15 is exactly divisible by 30 and 40.
4. When five numbers out of the six numbers 2, 3, 5, 7, 9 and 11 are multiplied, the product is 2310. What is the left out number?
5. Find the smallest number which when divided by 49 and 14 leaves the remainder 1 in each case.

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

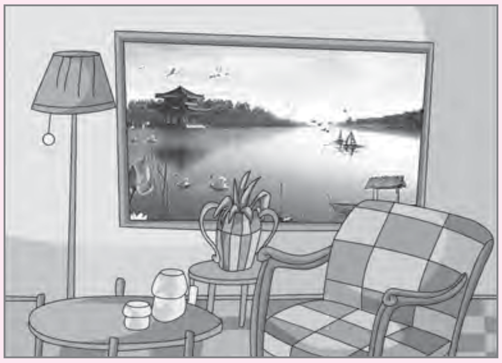
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A)** : 24 is not a factor of 12.
   Reason (R) : Every factor is either less than or equal to the given number.
2. **Assertion (A)** : 21 and 23 are twin primes.
   Reason (R) : 21 and 23 are prime numbers and their difference is 2.
3. **Assertion (A)** : 4 and 15 are co-primes.
   Reason (R) : Two numbers having only 1 as a common factor are called co-prime numbers.
4. **Assertion (A)** : 115 is an even number.
   Reason (R) : Even numbers are multiple of 2.
5. **Assertion (A)** : HCF of 21 and 23 is 1.
   Reason (R) : The HCF of two consecutive odd numbers is 1.
6. **Assertion (A)** : HCF of 41 and 43 is 1.
   Reason (R) : HCF of two consecutive even numbers is 1.
7. **Assertion (A)** : 7 and 20 are co-primes.
   Reason (R) : 7 and 20 are prime numbers.
8. **Assertion (A)** : 369 is divisible by both 3 and 9.
   Reason (R) : A number which is divisible by 9 is always divisible by 3.
9. **Assertion (A)** : 369 is divisible by both 3 and 9.
   Reason (R) : Both 3 and 9 are factors of 369.
10. **Assertion (A)** : 729 is divisible by both 3 and 9.
    Reason (R) : Sum of the digits of 729 is 18, which is divisible by both 3 and 9.
11. **Assertion (A)** : 17 and 19 are twin primes.
    Reason (R) : Two prime numbers whose difference is 2 are called twin primes.

CASE STUDY

CC

Some paintings of the artists Rajat, Vikram and Victor were selected by the hotel owner to enhance the Hotel Decor. Hotel rooms are numbered from 1 to 200. The owner gave the following instructions to the hotel manager:



- Each room should have the painting by Rajat.
- Each room with room number divisible by 3 shall have the painting by Vikram.
- Each room with room number ending with 0 or 5 shall have the painting by Victor.

1. How many rooms will have paintings by Vikram?
2. How many rooms will have paintings by Rajat?
3. To calculate the number of rooms with the paintings of all the three artists, which concept is applicable – Finding HCF or LCM.
4. In how many rooms the paintings of all the three artists will be there?

4 Basic Geometrical Ideas

What Learners Will Achieve

- develop understanding of basic terms of geometry.
- measure and compare the line segments.
- understand angle as revolution and measure in degrees.
- understand acute angles and obtuse angles with respect to right angle.
- understand reflex angles with respect to straight angle.

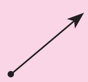
Warm-up

What we already know

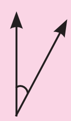
- A point determines a location or a position. A point is represented by a dot and capital letters are used to name the points.
- A plane is any flat and smooth surface which extends endlessly in all the directions.
- A line segment corresponds to the shortest distance between the two points in a plane.
- A line segment extended endlessly in both (opposite) sides (directions) is called a line.
- A ray is a part of a line which starts from a point called the initial point and extends in one direction.
- An angle is a plane figure formed by two rays with a common initial point.

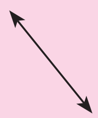
Now, try to solve the following.

1. What do the following figures represent?

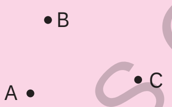
(a) 

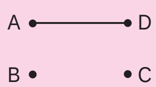
(b) 

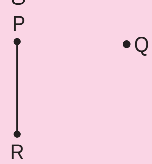
(c) 

(d) 

2. Join the following points in order and recognise the figure so formed.

(a) 

(b) 

(c) 

3. How many line segments do you find in the above figures?

DID YOU KNOW?

CC

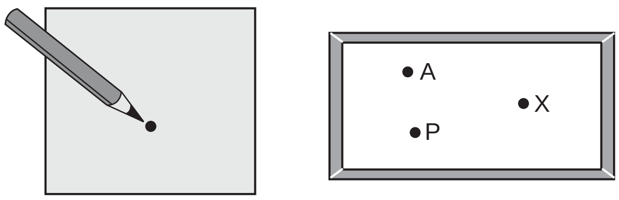
The term 'Geometry' is taken from the Greek word 'Geometron', 'Geo' means 'Earth' and 'Metron' means 'Measurement'. Thus, 'Geometry' means measurement of the earth (land).

That is how it began in Egypt and many other parts of the world. Floods in the river Nile would submerge (cover with water) neighbouring fields. After the floods were gone, people of the area were not able to know where their fields were. As the land was very fertile (good for growing crops), they would often fight and claim other's field as their own. Methods had to be devised so that questions concerning the shape, size, relative position and other measurements of the piece of land could be answered. Geometry answers all these questions and more.

BASIC CONCEPTS AND TERMS OF GEOMETRY

Point

As soon as the tip of a pen or pencil comes in contact with paper, a point is obtained. A point determines a location. A point is represented by a dot. A point has no dimensions such as length, width or thickness. Capital letters such as A, B, P, X, etc., are used to name the points.



(a) (b)

Fig. 4.1

Plane

- A plane is any flat and smooth surface which extends endlessly in all the directions.
- A plane has two dimensions called length and width (or breadth) but has no thickness.

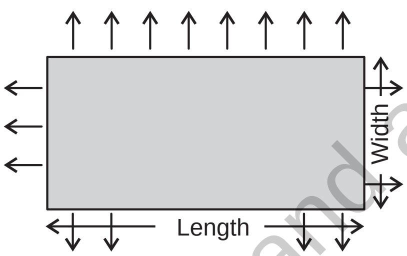


Fig. 4.2

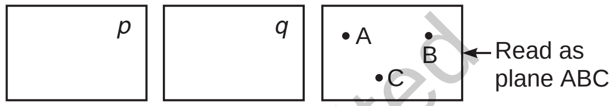
- A plane has no boundary.
- The surface of the **top of a table**, a **sheet of paper**, a **wall**, **floor** or **ceiling** of a room are all examples of a portion of a plane.

Table (a) | Sheet of paper (b) | Blackboard (c)

Fig. 4.3

We cannot draw a plane on a paper, only a part of it can be drawn on it. We draw figures such as triangles, circles or rectangles in a plane. We call them **plane figures**.

- A plane can be named by writing a single letter of the English alphabet as *p*, *q*, etc.; read as plane *p*, plane *q*, *etc.*, or by taking three different points, say A, B, C in the plane (not lying on a line) as shown in Fig. 4.4.



(a) (b) (c)

Fig. 4.4

Line Segment

Consider two points A and B on a plane. The straight path from A to B is called the line segment AB. It is denoted by $\overline{AB}$. A and B are called the end points of the line segment. The line segment AB is the same as line segment BA.

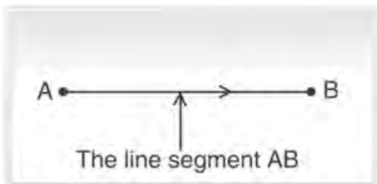


Fig. 4.5

Line

- A line segment extended endlessly in both sides (opposite directions) is called a line.

Fig. 4.6

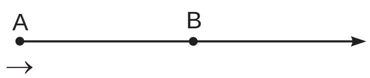
- A line segment AB extended in both sides and marked by arrows at the two ends represents a line AB. It is denoted by $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$. (The arrows indicate that it can be extended indefinitely in both the directions).
- A line has no end points.
- A line is endless. So, it cannot be drawn on a paper. We can draw a part of a line only.
- A line has no definite length. It has no breadth or thickness.

- A line has infinitely many points.
- We use small case letters *l*, *m*, *n*, *etc.*, to denote a line.

Ray

A **ray** is part of a line which starts from a point called the ***initial point*** and extends indefinitely in other side. Sun's rays are the most common example of a ray. (The Sun's rays begin at the Sun and travel endlessly). In other words, a line segment extended endlessly in one direction is called a *ray*. A ray has no definite length.

The initial point is used along with one more point on the ray to represent a **ray**. The initial point is written first to describe a ray.



$\overrightarrow{AB}$ is a ray with A as the initial point.

(a)

$\overrightarrow{AL}$ and $\overrightarrow{AM}$ are different names of the same ray with A as the initial point.

(b)

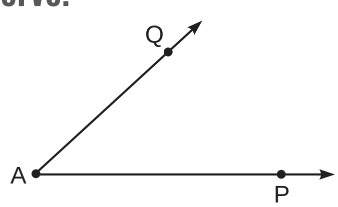
Fig. 4.7

Note

We have used three symbols $\overline{AB}$, $\overleftrightarrow{AB}$ and $\overrightarrow{AB}$ to denote line segment AB, line AB and ray AB respectively. However, in practice, we usually use the same symbol AB to denote all the above three. It will be clear from the context, which of the three we are referring to.

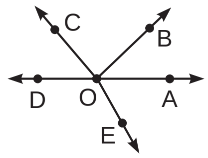
Observe!

(a)



$\overrightarrow{AP}$ and $\overrightarrow{AQ}$ are two different rays having the same initial point A.

(b)



Infinitely many rays can be drawn from a given point, say O, as the initial point, in different directions.

PAIR OF LINES

Intersecting Lines

Two lines are called **intersecting lines** if they have one common point.

- Lines AB and XY are intersecting lines because they have one common point P [Fig. 4.8 (a)].
- P is called the **point of intersection** of the given lines [Fig. 4.8 (a)].
- Three or more lines are said to be **concurrent lines** if all of them pass through the same point.

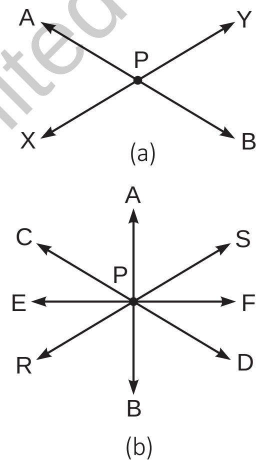


Fig. 4.8

In Fig. 4.8 (b), lines CD, EF and RS are concurrent lines and their common point P is called the **point of concurrence**.

Remember

When more than two lines pass through a common point known as the **point of concurrence**, lines are called **concurrent lines**.

Think!
How many lines can be drawn through a point?

Skill Check

- The number of lines that can be drawn passing through the two given points is __________.
- The roof of your classroom is an example of a __________.
- Which of the following has no end points?
  (a) Line segment (b) Ray (c) Line

Parallel Lines

Let us observe the ruled printed lines on an exercise book (Fig. 4.9).

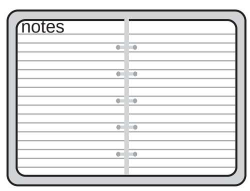


Fig. 4.9

We observe that:

- the distance between any pair of lines is always the same.
- if the lines could be produced through any distance beyond the page of the book, they would never meet.

Such pairs of lines are called **parallel lines**. In other words, we can say that:

Two lines in a plane which do not intersect each other at any point, when produced indefinitely in either direction, are called ***parallel lines***.

In Fig. 4.10, lines l and m are parallel lines. So, we write $l \parallel m$.

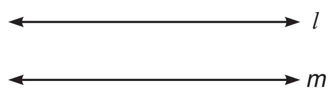
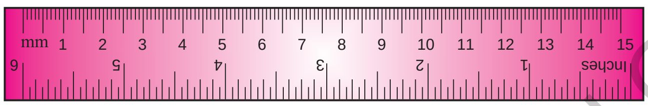


Fig. 4.10

Opposite edges of a ruler, rails of a railway line, opposite edges of a blackboard are some common examples of parallel lines [Fig. 4.11].



Ruler

(a)

Railway line (b) Blackboard (c)

Fig. 4.11

Perpendicular Lines

Two lines AB and CD are mutually **perpendicular** (written as $AB \perp CD$), when they intersect at right angles (Fig. 4.12).

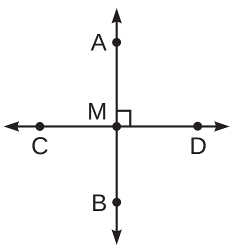
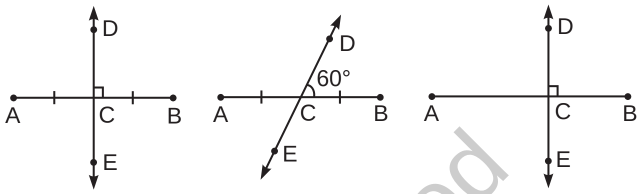


Fig. 4.12

If perpendicular line AB intersects the line segment CD at its mid-point M, then AB is called the **perpendicular bisector** of CD.

In the Fig. 4.13 (a), AC = CB and measure of $\angle DCB = 90°$. So, $\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{AB}$.



(a) (b) (c)

Fig. 4.13

In the Fig. 4.13 (b),

AC = CB, but $\angle DCB = 60°$.

So, $\overleftrightarrow{DE}$ is a bisector of AB but not perpendicular bisector of $\overline{AB}$.

In the Fig. 4.13 (c), $\angle DCB = 90°$, but AC > BC.

So, $\overleftrightarrow{DE}$ is perpendicular to AB but not a bisector of $\overline{AB}$.

Let us study some more examples.

Ex. 1. In Fig. 4.14,

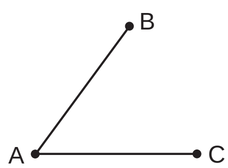


Fig. 4.14

(a) count the number of marked points.
(b) count the number of line segments.

Sol. (a) The points are A, B and C.
Thus, the number of points is 3.

(b) The line segments are AB and AC.
Thus, the number of line segments is 2.

Ex. 2. In Fig. 4.15,

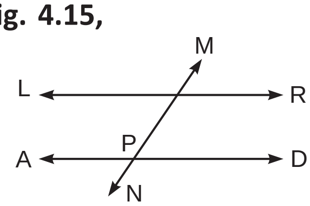


Fig. 4.15

(a) **lines LR and AD are _____.**

(b) **lines AD and MN are _____.**

Sol. (a) Lines LR and AD do not meet at any point.

Thus, LR and AD are *parallel lines*.

(b) Lines AD and MN are *intersecting* lines because they have one common point of intersection P.

Ex. 3. How many rays are there in the given Fig. 4.16?

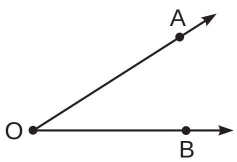


Fig. 4.16

Sol. There are two rays OA and OB in the given figure.

Ex. 4. Name the line segment(s) and ray(s) in Fig. 4.17.

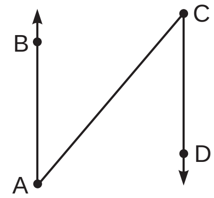


Fig. 4.17

Sol. We know that, a ray is a part of a line which starts from a point and extends to one side and line segment has two end points.

Thus, there are two rays AB and CD and one line segment AC.

Ex. 5. Describe the given (see Fig. 4.18) perpendicular lines in symbolic form.

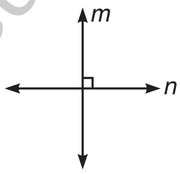


Fig. 4.18

Sol. Line *m* is perpendicular to line *n*.

Symbolic form: $m \perp n$ or $n \perp m$

Skill Check

- Are perpendicular lines intersecting lines?
- Do all intersecting lines represent perpendicular lines?

Let Us Do

EL

Objective: To represent the following using paper folding:

(a) perpendicular lines (b) intersecting lines (c) parallel lines

Materials required: Fluorescent square sheets

Procedure: (a) To represent a perpendicular line from a point on a given line.

Step 1: Fold a square sheet along one of the edges, say along length of the square. Unfold it. Draw a line on crease. Let this be the given line. [See Fig. 4.19 (a) and (b).]

Step 2: Make any two points A and B on the line to get line segment AB. Take a third point C on it. [See Fig. 4.19 (c) and (d).]

Step 3: Fold the sheet through the point C along the other edge, *i.e.*, along breadth of the square sheet.

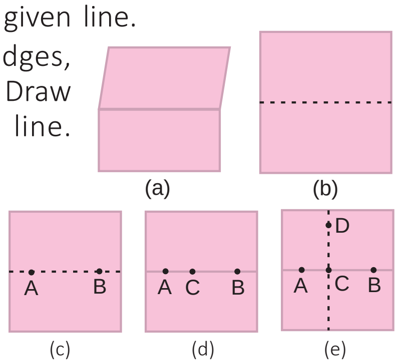


Fig. 4.19

Unfold the sheet and draw a line on the crease obtained. Mark any point D on this line. Thus, line CD represents a line perpendicular to line AB. [See Fig. 4.19 (e).]

(b) To represent two intersecting lines.

In the above case, after step 2, fold the sheet along point C in any direction. Unfold the sheet and draw line on the crease obtained (see Fig. 4.20).

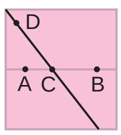


Fig. 4.20

Mark any point D on this line. Thus, Line CD is representing the line intersecting with line AB at point C.

(c) To represent two parallel lines.

Step 1: Take a rectangular sheet. Make one fold along one of the edges, *i.e.*, along the length. Make another fold along other edge of the same length.

Step 2: Unfold the sheet and draw lines on both the creases. Mark the points A and B on one line and C and D on the other line. Thus, lines AB and CD represent parallel lines. [See Fig. 4.21 (a) and (b).]

Step 3: Repeat the procedure along the other edge too.

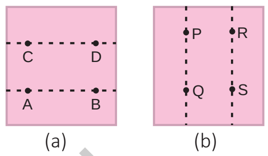
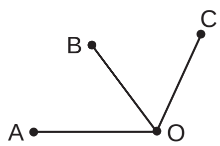
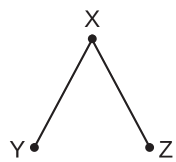


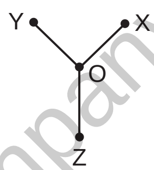
Fig. 4.21

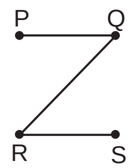
Exercise 4.1

1. For each of the following figure, count the number of marked points and line segments:

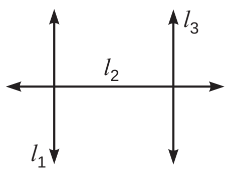
(a) 

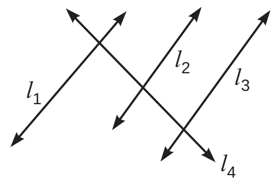
(b) 

(c) 

(d) 

2. Fill in the blanks for the following figures:

(a) 

(b) 

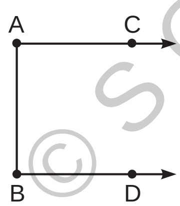
(a) (i) lines l_1 and l_3 are ______.

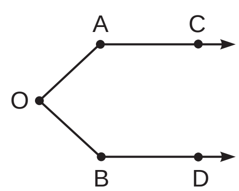
(ii) lines l_1 and l_2 are ______.

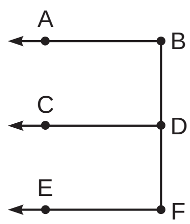
(b) (i) lines l_1 and l_4 are ______.

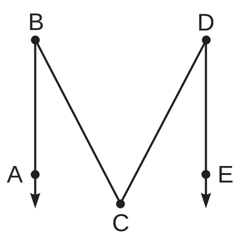
(ii) lines l_3 and l_2 are ______.

3. Name the line segment(s) and ray(s) in the given figures.

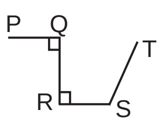
(a) 

(b) 

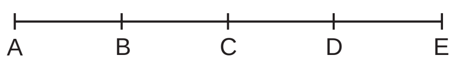
(c) 

(d) 

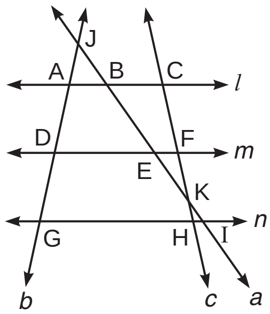
4. Identify the perpendicular line segments in the given figure.



5. Find the number of line segments in the given figure.



6. **In the given figure, find the following:**
   (a) Lines whose point of intersection is B.
   (b) Lines whose point of intersection is G.
   (c) All pairs of parallel lines.
   (d) All points lying on the line *a*.
   (e) Point of intersection of line *l* and *b*.



7. **Answer the following questions:**
   (a) How many rays are there in Fig. 1?
   (b) Name the line segments in Fig. 2.
   (c) Write the concurrent lines and concurrent points in Fig. 3.

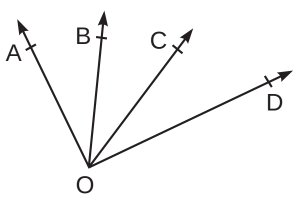


Fig. 1

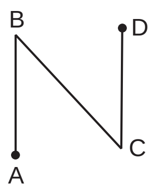


Fig. 2

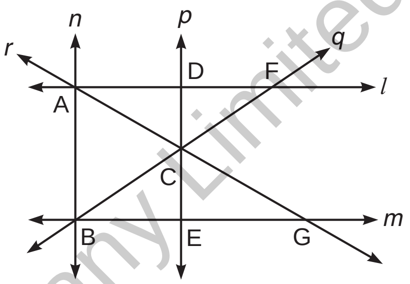
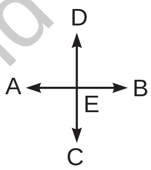


Fig. 3

8. Differentiate between line segment, line and ray.
9. In the given figure, how many rays are there in a line starting from point O. Name them.

10. In the given figure, name the lines and their point of intersection.



COMPARING AND MEASURING LINE SEGMENTS

In this section, we shall learn how to compare and measure line segments.

Comparing by Observing

- Look at the two line segments in Fig. 4.22 (a).

  By just looking at them, we can say that AB is longer than CD.
- But this is not the case everytime. Consider the line segments in Fig. 4.22 (b). This time, by just looking at them, we cannot say which one is longer.

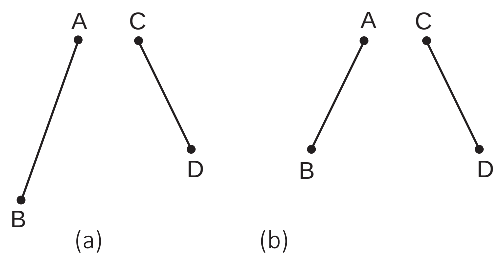


Fig. 4.22

- We can compare two line segments by using tracing paper, and using a divider and a ruler.

Comparing by Tracing

We can also compare two line segments, say MN and RS by tracing one of them on a tracing paper and placing it on the other. So, the line segments can be easily compared.

(a)

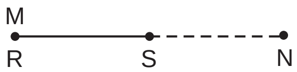
1. If MN overlaps completely on RS, then $\overline{MN} = \overline{RS}$.

(b)

2. If $\overline{MN}$ falls short on $\overline{RS}$, then $\overline{MN} < \overline{RS}$.

(c)

3. If $\overline{MN}$ is longer than $\overline{RS}$, then $\overline{MN} > \overline{RS}$.



(d)

Fig. 4.23

Measuring a Line Segment Using a Ruler

Illustration 1:

- For measuring the length of a line segment AB, place the zero mark of the ruler at the starting point of the line segment, *i.e.*, at A.

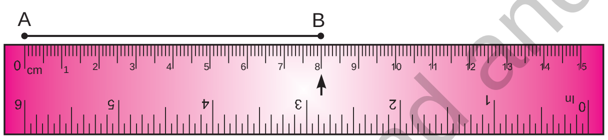


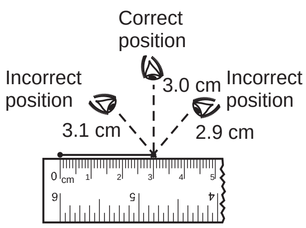
Fig. 4.24

- Keep the ruler parallel to and along the line segment. Read the mark against the end point of the line segment, *i.e.*, at B. This gives the length of AB.

  Thus, AB = 8 cm.

Watch Your Step! LS

To get the correct reading, the eye should be just vertically above the mark. Otherwise, errors can happen due to angular viewing.



Comparing by measurement

We can also compare two line segments by their measurements. For this, we need a ruler or a ruler and a divider. We already know how to measure the length of given line segments. So, to compare the two line segments, follow these steps:

Step 1: Measure the lengths of the line segments with the help of a ruler.

Step 2: Compare the lengths of their (line segments) measurements.

Illustration 2. To find out which line segment PQ or RT has a greater length, let us take their measurements with the help of a ruler.

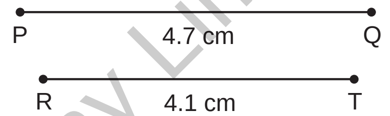


Fig. 4.25

We have,

PQ = 4.7 cm and RT = 4.1 cm

Now, taking measurements of the two line segments, clearly, PQ > RT.

Measuring a line segment using a ruler and a divider

Illustration 3:

Given, line segments AB and CD whose lengths are to be measured.

Fig. 4.26 (a)

Step 1: Place the divider's one pointer at A and the other pointer at B.

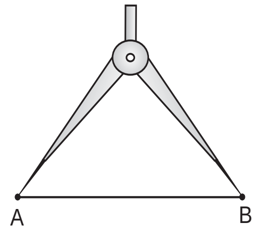


Fig. 4.26 (b)

Step 2: Without disturbing the opening, lift the divider and place it on the ruler in such a way that one pointer of the divider is at the zero mark of the ruler.

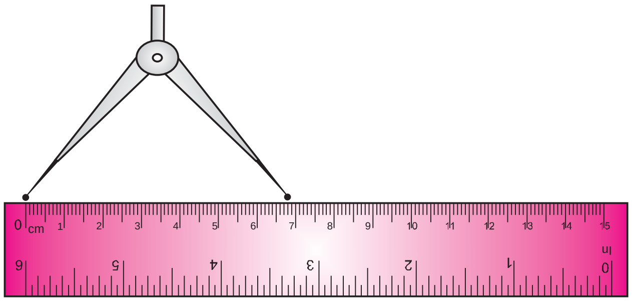


Fig. 4.26 (c)

Step 3: Read the mark against the other pointer. This gives the length of AB.

Thus, AB = 6.8 cm.

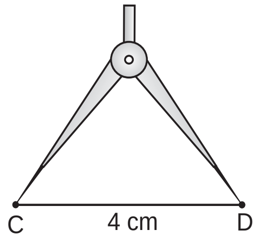


Fig. 4.26 (d)

Similarly, the measure of line segment CD = 4 cm.

Clearly, AB > CD.

Comparing Line Segments Using Only Divider

Illustration 4: Let AB and CD be two line segments.

Fig. 4.27 (a)

To examine which one is longer, follow these steps:

Step 1: Place the divider's one pointer at A and the other at B.

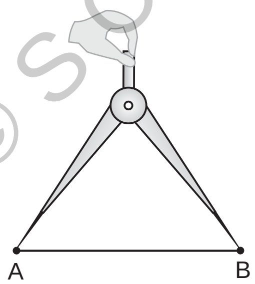


Fig. 4.27 (b)

Step 2: Now, without disturbing the opening, lift the divider and place it on the line segment CD with one pointer at C.

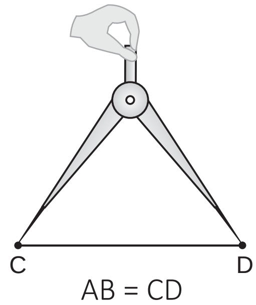


Fig. 4.27 (c)

We find that the other pointer of the divider is exactly on D. So, we can say that the lengths of the line segments AB and CD are equal.

In some cases, we observe that the other point may also lie before or ahead point D. In those cases, we can observe as follows:

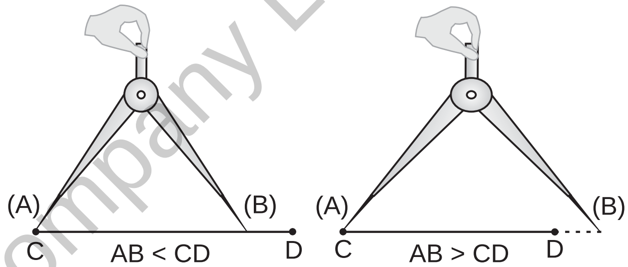


Fig. 4.27 (d)

Let us study some more examples.

Ex. 6. Draw four line segments XY, AL, XP and ST. Without actually measuring their lengths, identify the longest and the shortest line segment.

Sol. The line segment AL seems to be the longest one, while XP seems to be the shortest line segment.

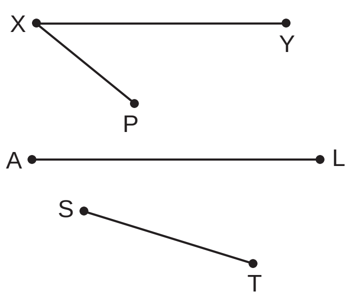


Fig. 4.28

Ex. 7. If B is the mid-point of $\overline{AC}$ and C is the mid-point of $\overline{BD}$, where A, B, C and D lie on a straight line. Prove that AB = CD.

Sol.

Fig. 4.29

Since B is the mid-point of $\overline{AC}$,

AB = BC. ... (*i*)

Again C is the mid-point of BD,

BC = CD. ... (*ii*)

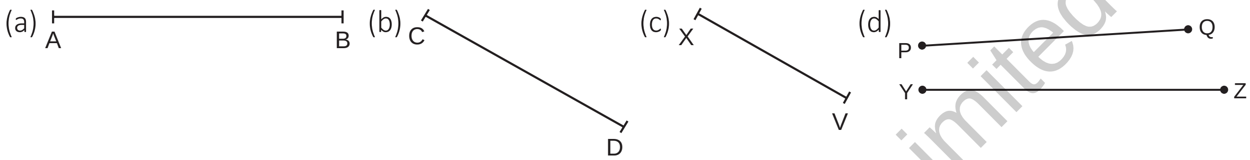
Thus, from (*i*) and (*ii*), we have

AB = CD.

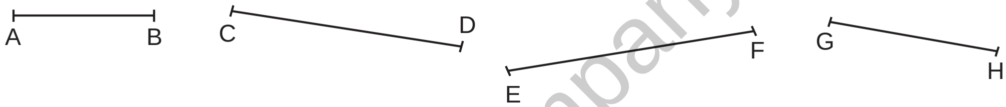
Hence, proved.

Exercise 4.2

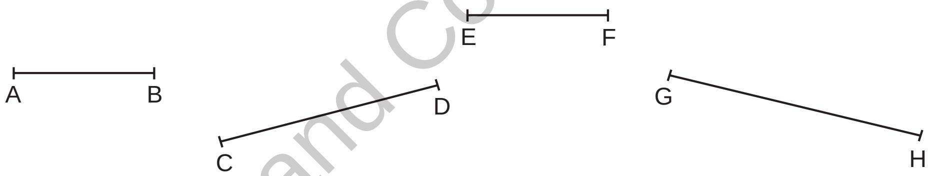
1. **Measure the following line segments using a ruler and a divider.**



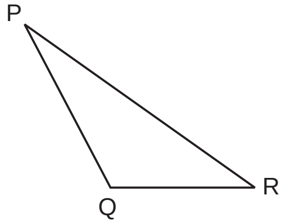
2. **Measure the following line segments using a ruler and a divider and write them in ascending order of their lengths.**



3. **Without measuring their lengths, identify the longest line segment.**



4. Draw any line segment AB. Mark a point C on AB lying between A and B. Verify that AC = AB – CB.
5. If P, Q and R are three collinear points such that PQ = 6 cm, QR = 2 cm and PR = 8 cm, which one point lies between the other two?
6. Measure the sides of the given triangle. Check if the sum of the lengths of any two sides is greater than the third side.



ANGLES

An angle (symbol: ∠) is formed by two rays having a common initial point, called **vertex** [Fig 4.30 (a)]. The rays forming the angle are called its **arms** or **sides**.

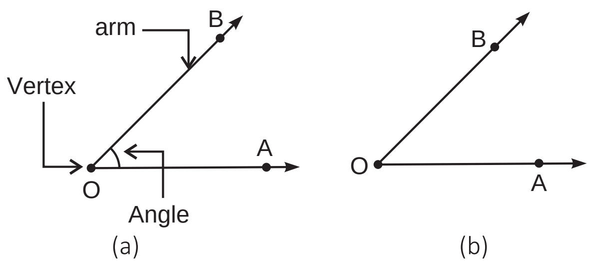


Fig. 4.30

Naming an Angle

Any two points, one on each of the two arms of an angle, along with the vertex can be used to name the angle.

The angle shown in Fig. 4.30 (b) is written as ∠AOB and read as angle AOB.

We observe that the common initial point, the vertex O, is written in the middle.

Watch Your Step!

It also can be written as ∠BOA but writing ∠ABO or ∠BAO is incorrect.

Alternative ways of naming an angle

An angle may also be denoted by the single letter as its vertex or by some different letter or symbol designated for this purpose. For example, ∠QOP; ∠POQ; ∠O; ∠x are different names of the angle shown in Fig. 4.31.

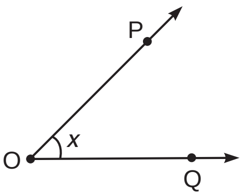


Fig. 4.31

Regions of an Angle

An angle has three regions associated with it.

Interior (or inside) region

The part of the plane which is within the arms of an angle, produced indefinitely, is called the **interior** of the angle.

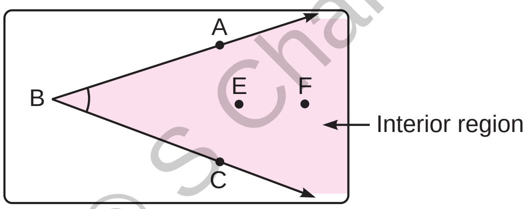


Fig. 4.32 (a)

Points E and F are in the interior of the angle ABC [Fig. 4.32 (a)].

Boundary (or on) region

Points G and H are on the **angle** (boundary) itself [Fig. 4.32 (b)].

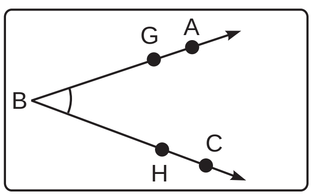


Fig. 4.32 (b)

Exterior (or outside) region

The points in the plane which neither lie in the interior nor on the arms, are called exterior points and those points constitute the **exterior region** of the angle. Points I and J are in the exterior of the angle ABC [see Fig. 4.32 (c)].

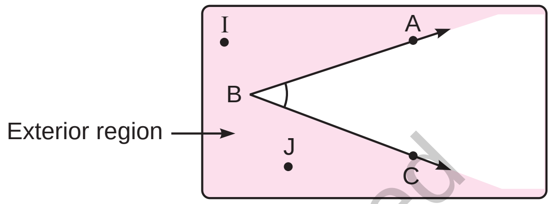


Fig. 4.32 (c)

Let us study some more examples.

Ex. 8. **Identify all the line segments and all the angles in the Fig. 4.33.**

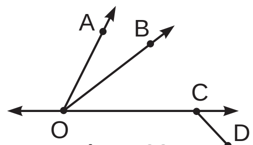


Fig. 4.33

Sol. There are four line segments in the given figure, *i.e.*, OA, OB, OC and CD.

There are four angles in the figure, *i.e.*, ∠AOB or ∠BOA, ∠AOC or ∠COA, ∠BOC or ∠COB and ∠OCD or ∠DCO.

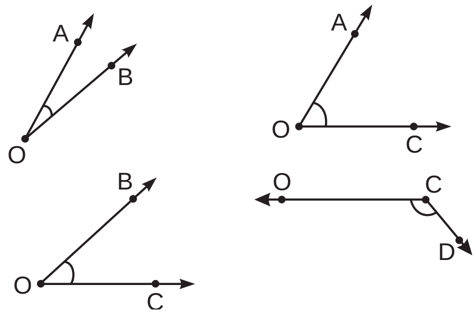


Fig. 4.34

Ex. 9. **In Fig. 4.35, name the marked points:**

(a) in the interior of ∠EOB.

(b) in the exterior of ∠COE.

(c) on ∠COB.

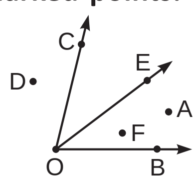


Fig. 4.35

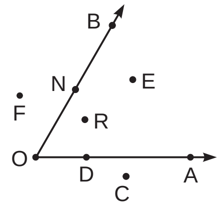
Sol. (a) The points in the interior of ∠EOB are A and F.

(b) The points in the exterior of ∠COE are A, B, D and F.

(c) The points on ∠COB are B, C and O.

Exercise 4.3

1. Look at the given figure, write the name of the points which are:

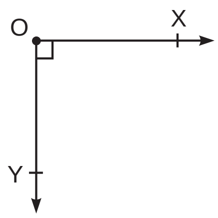


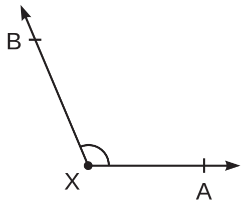
(a) on ∠AOB. ____________________

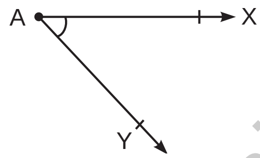
(b) in the interior of ∠AOB. ____________________

(c) in the exterior of ∠AOB. ____________________

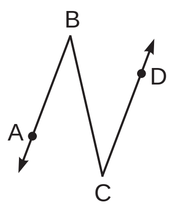
2. In each of the following, name the vertex and the arms of the given angle and hence name the angle.

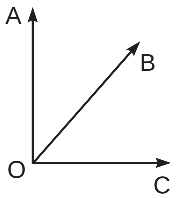
(a) 

(b) 

(c) 

3. How many angles are there in the given figures? Name them.

(a) 

(b) 

MEASURING ANGLES AND TYPES OF ANGLES

Now, we shall learn how to measure an angle and classify angles according to their measurements.

Measuring an Angle

Consider the ray OA in Fig. 4.36 (a). If we rotate it about the point O so that it takes the final position OB, we get an ∠AOB with O as vertex and rays OA and OB as its arms.

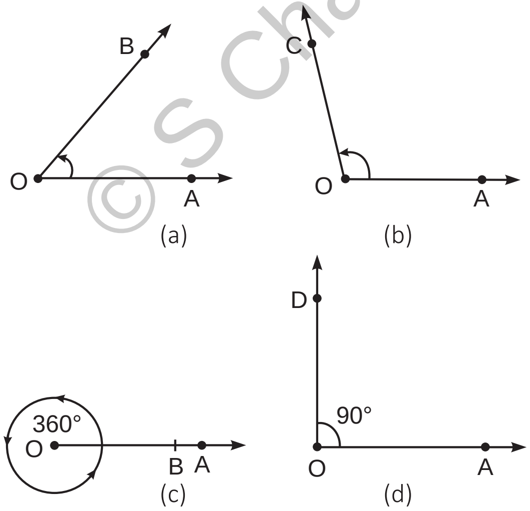
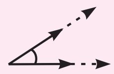


Fig. 4.36

> **Remember**
>
> The size of the arms of an angle does not affect its measure.
>
> 

On rotating further as shown in Fig. 4.36 (b), we get an ∠AOC.

On observing the two angles, we say that ∠AOC is greater than ∠AOB (opening between the two arms of ∠AOC is more).

The **magnitude** or **measure** of an angle is the amount of rotation from one arm to the other. If two angles have different inclinations between their arms, then we say that these two angles have different **magnitudes** or **measures**.

The standard unit of measurement of an angle is **degree** (°). It is denoted by a small circle.

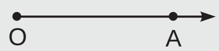
Consider a ray OA. On turning it in the same direction (clockwise or anticlockwise), it becomes coincident with its initial position OA after a complete turn [Fig. 4.36 (c)]. This one complete turn is called a **revolution**. The angle for one revolution is assigned a measure of 360° and is called a **complete angle**.

So, $1° = \frac{1}{360}$ of one complete revolution.

If a ray OA takes a quarter turn $\left(\frac{1}{4}\text{turn}\right)$ [Fig. 4.36 (d)], then we say that the ray has moved through an angle of $\frac{360°}{4} = 90°$ and 90° is the measure of this angle.

Note

If the ray OA does not rotate at all, then the angle so formed is called an angle of measure 0°.



Protractor

A **protractor**, shown in Fig. 4.37, is an instrument used for measuring a given angle or for constructing an angle of the given magnitude.

It is semicircular in shape and is usually made of transparent plastic so that markings are visible when it is placed over arms of an angle.

It has degree marks on the curved edge from 0° to 180° on the inner scale and from 180° to 0° on the outer scale which enables us to read the measure of an angle or construct an angle of the given measure.

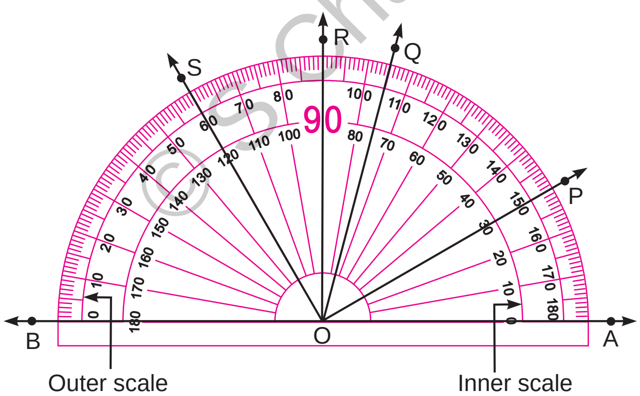


Fig. 4.37

It is clear from Fig. 4.37 that the angle AOB at the centre in a semicircle is one-half of the complete revolution $= \frac{1}{2} \times 360° = 180°$. The angle around the centre of a circle (complete angle) is 360°.

Measuring an Angle, using Protractor

To measure an ∠AOB, follow these steps:

Step 1: We place the protractor so that its centre is exactly on the vertex O of the angle and the base line lies along the arm OA (Fig. 4.38).

Step 2: Starting with 0° mark of the scale lying on OA, we read the mark (anticlockwise for inner scale) through which the arm OB passes. It passes through 60° (Fig. 4.38).

Thus, ∠AOB = 60°.

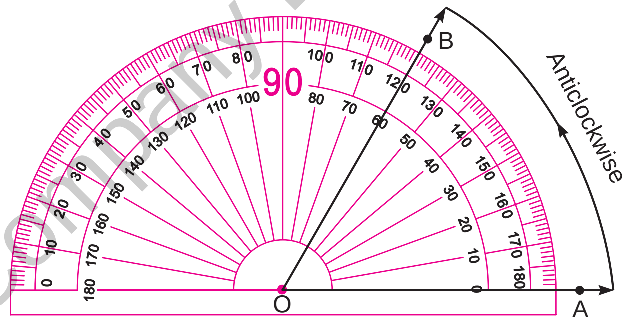


Fig. 4.38

Types of Angles

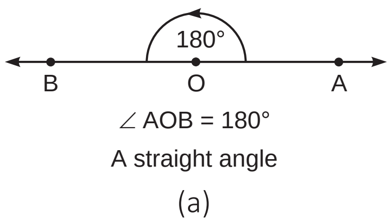
Straight and right angles

An angle of measure 180° is called a **straight angle** [Fig. 4.39 (a)].

An angle of measure 90° is called a **right angle** [Fig. 4.39 (b)].

Note

Two right angles form a straight angle.



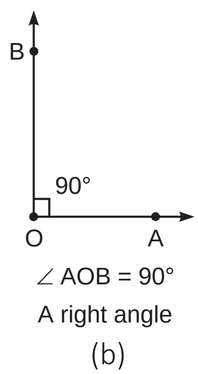
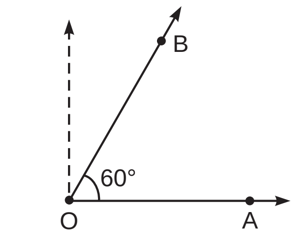


Fig. 4.39

Acute and obtuse angles

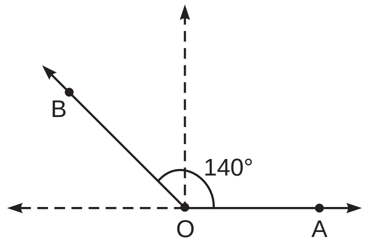
An angle less than 90° but greater than 0° is called an **acute angle** [Fig. 4.40 (a)].

An angle greater than 90° but less than 180° is called an **obtuse angle** [Fig. 4.40 (b)].



∠AOB = 60°. It is less than 90° but greater than 0°. It is an acute angle.

(a)



∠AOB = 140°. It is greater than 90° and less than 180°. It is an obtuse angle.

(b)

Fig. 4.40

Reflex angle

An angle which is greater than 180° but less than 360° is called a **reflex angle** (Fig. 4.41).

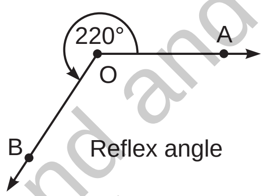
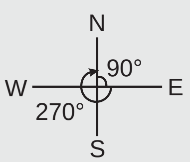


Fig. 4.41

DID YOU KNOW?

There is a right angle formed between the East and the North directions when we turn anticlockwise but there are 3 right angles formed on taking a clockwise turn.



Ex. 10. **In Fig. 4.42, recognise the types of angles by observing the angles.**

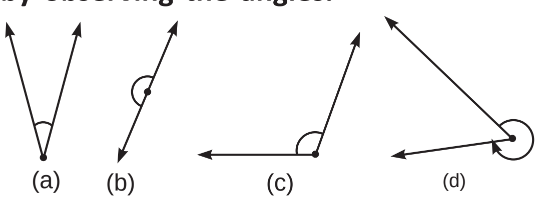


Fig. 4.42

Sol. (a) It is an acute angle.

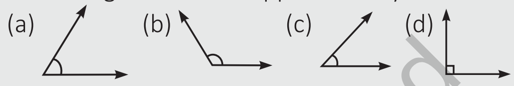
(b) It is a straight angle.

(c) It is an obtuse angle.

(d) It is a reflex angle.

Skill Check

- Which type of an angle is more than 180° but less than 360°?
- Which angle measures approximately 45°?



- Which of the following is true for the measure of shaded angle in the clock?

  (a) less than 90°
  (b) greater than 90° but less than 180°
  (c) measures 140°
  (d) greater than 180° but less than 360°
- If the measure of an angle is 120°, what kind of angle is this?

Let us study some more examples.

Ex. 11. **Identify the type of angle given in Fig. 4.43.**

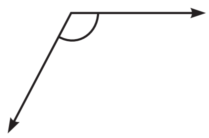


Fig. 4.43

Sol. Observe the given angle.

Clearly, the angle is more than 90° but less than 180°.

Thus, the given angle is an obtuse angle.

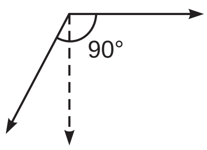


Fig. 4.44

Ex. 12. **In the Fig. 4.45, count the number of acute angles and right angles.**

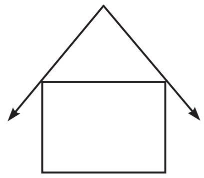


Fig. 4.45

Sol. On inspecting the given figure, we have 5 acute angles in the figure: ∠5, ∠6, ∠7, ∠8, ∠9, all are < 90°.

4 right angles: ∠1, ∠2, ∠3, ∠4, all are equal to 90°.

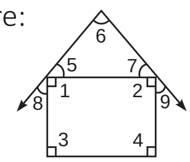


Fig. 4.46

Ex. 13. Find the number of right angles turned through by the hour hand of a clock when it goes from 3 to 6.

Sol. Angle turned by the hour hand from 3 to 6

$= \frac{1}{4}$ of one revolution $\left(\frac{3}{12} \text{ or } \frac{1}{4}\right)$

$= \frac{1}{4} \times 360°$ (since one revolution = 360°)

= 90° or one right angle.

Thus, the hour hand turned through one right angle in going from 3 to 6.

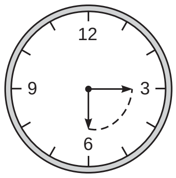


Fig. 4.47

Ex. 14. In Fig. 4.48, measure the angles, using a protractor.

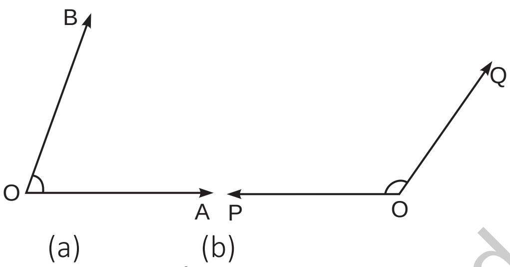


Fig. 4.48

Sol. (a) To measure ∠AOB, we place the protractor so that its centre is exactly on the vertex O of the angle and the base line lies along the arm OA.

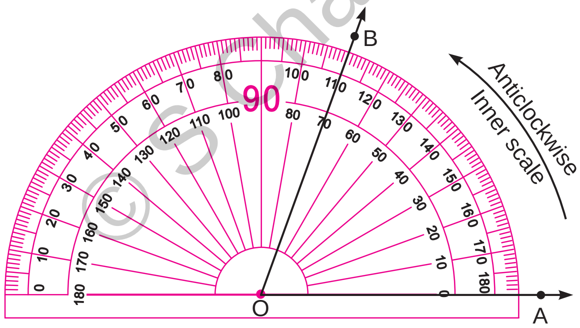


Fig. 4.49 (a)

Starting with 0° mark of the inner scale lying on OA, we read the mark through which the arm OB passes. Observe that it is 70°.

Thus, ∠AOB = 70°.

Watch Your Step!

∠AOB ≠ 110°, as reading the arm OB [see Fig. 4.49 (a)] passes through 110° is incorrect as we move anticlockwise from $\overrightarrow{OA}$. So, read the mark on the inner scale.

(b)

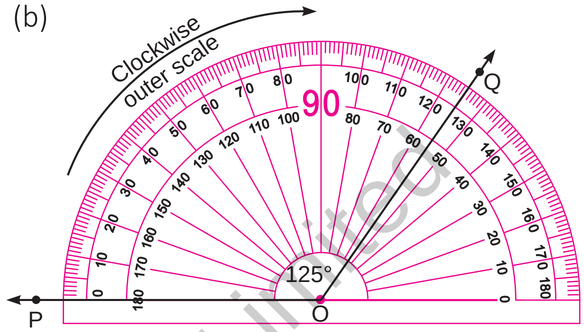


Fig. 4.49 (b)

We start from the arm OP of ∠POQ and read the angle on the outer scale. We observe that $\overrightarrow{OQ}$ aligns with 125° on the outer scale.

Thus, ∠POQ = 125°.

Note

∠POQ = 125° is correct as arm OP is on the left side of O under base line. So, we read the mark on the outer scale.

Ex. 15. How many right angles do you make, if you start facing the South and turn clockwise to the West?

Sol. Starting from the South and turning clockwise, after a right angle, we face the West.

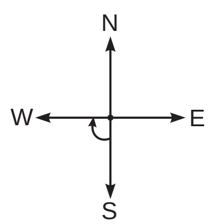


Fig. 4.50

So, only one right angle movement is made.

Ex. 16. Where will the hour hand of a clock stop, if it starts from 6 and turns through 1 right angle?

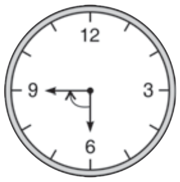


Fig. 4.51

Sol. From 6, through 1 right angle, *i.e.,* it makes $\frac{1}{4}$ of a revolution.

So, hour hand of the clock will stop at 9.

Ex. 17. What part of a revolution have you turned through, if you stand facing the South and turn clockwise to face the East?

Sol. Angle turned = 90° + 90° + 90° = 270°

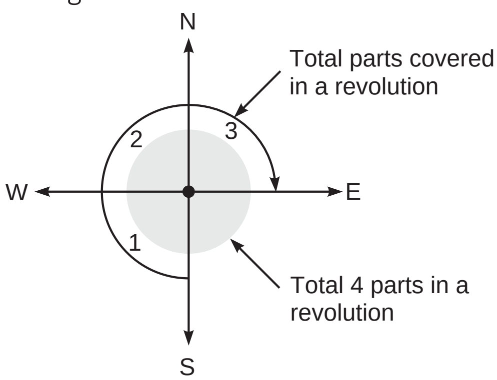


Fig. 4.52

$$360° = 1 \text{ revolution}$$

$$1° = \frac{1}{360} \text{ part of a revolution}$$

$$270° = \frac{1}{360} \times 270 \text{ part of a revolution}$$

$$= \frac{27}{36} = \frac{3}{4} \text{ part of a revolution}$$

Thus, the part of a revolution when we stand facing the South and turn to the East clockwise is $\frac{3}{4}$ of a revolution.

Ex. 18. Which direction will you face, if you start facing the East and make $1\frac{1}{2}$ revolutions clockwise?

Sol. From Fig. 4.53, it is clear that after revolving, $1\frac{1}{2}$ revolutions clockwise, we will face the West direction.

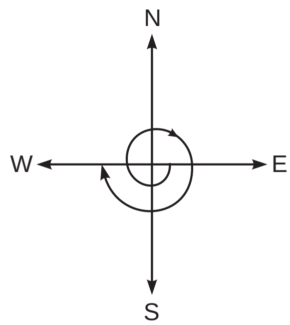
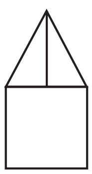
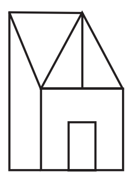


Fig. 4.53

Exercise 4.4

1. **In the following figures, count the number of acute angles and right angles.**

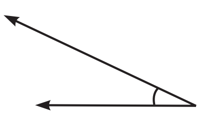
   (a) 

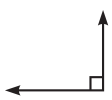
   (b) 

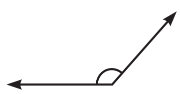
2. **Find the number of right angles turned through by the hour hand of a clock in each of the following cases:**

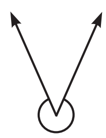
   (a) when it goes from 2 to 8.
   (b) when it goes from 10 to 1.
   (c) when it goes from 12 to 9.

3. **Identify the following types of angles.**

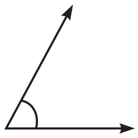
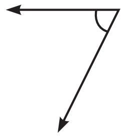
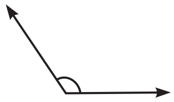
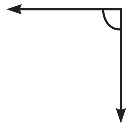
   (a) 

   (b) 

   (c) 

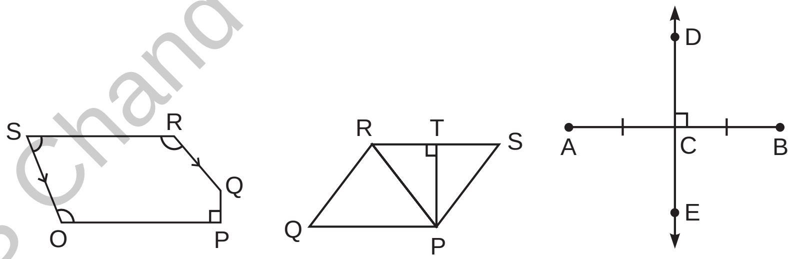
   (d) 

4. **Measure and state the kind of each of the following angles.**

(a) 
(b) 
(c) 
(d) 

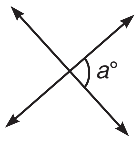
5. **What fraction of a revolution does the hour hand of a clock turn through, when it goes from:**
(a) 2 to 8? (b) 3 to 6? (c) 4 to 10? (d) 5 to 8?
6. **Where will the hour hand of a clock stop, if it:**
(a) starts at 5 and makes $\frac{1}{2}$ of a revolution? (b) starts at 8 and makes $\frac{3}{4}$ of a revolution?
7. **Which direction will you face, if you start facing:**
(a) the West and make $\frac{1}{2}$ of a revolution anticlockwise?
(b) the West and make $1\frac{1}{2}$ revolutions anticlockwise?

8. **How many right angles do you make, if you start facing:**
(a) the North and turn clockwise to the West?
(b) the South and turn anticlockwise to the East?
9. Arpita estimated the back of her armchair to be at an angle of 125° to the seat of the chair. What type of an angle has she estimated it to be?
10. **Answer the following questions:**
(a) Rehman stood facing the North, then turned to his right to face directly the East. What kind of angle did he trace?
(b) Mohan is facing the North. He turns and faces the South. What kind of angle has he made in this turn?
11. **In the following diagrams, name the right angles and the obtuse angles.**



(a) (b) (c)

12. What type of an angle is marked at the intersection of the two lines?



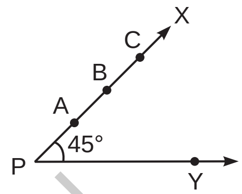
13. Estimate the size of the smaller angle between the two arms (hour and minute) of the clock.

Competency Based Exercise

21st CS

1. Tick (✓) the correct answer.

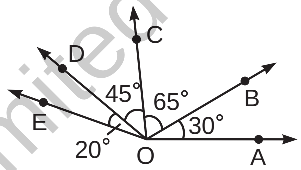
(a) In the given figure, if a point A is shifted to point B along the ray PX such that PB = 2PA, then the measure of ∠BPY is:



(i) less than 45° (ii) greater than 45°

(iii) 45° (iv) 90°

(b) The number of obtuse angles in the given figure is:



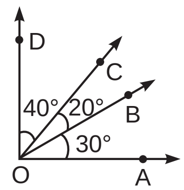
(i) 2 (iii) 3

(iii) 4 (iv) 5

(c) A bicycle wheel has 60 spokes spread evenly across the wheel. The angle between the two consecutive spokes is:

(i) 10° (ii) 8° (iii) 7° (iv) 6°

(d) In the given figure, the number of angles is:



(i) 3 (ii) 4

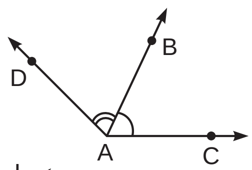
(iii) 5 (iv) 6

(e) If the sum of two angles is greater than 180°, then which of the following is not possible for the two angles?

(i) One reflex angle and one acute angle (ii) Two obtuse angles

(iii) Two right angles (iv) One obtuse angle and one acute angle

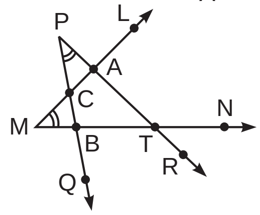
(f) In the given figure, the common part between ∠BAC and ∠DAB is:



(i) line segment AB (ii) ray AB

(iii) line AB (iv) side AB

(g) In the given figure, number of common points marked on the two angles RPQ and LMN are:

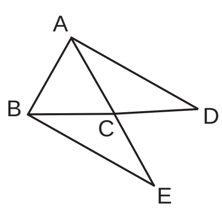


(i) 3 (ii) 4

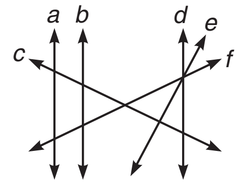
(iii) 5 (iv) 6

2. Find the number of lines passing through five points such that no three of them are collinear.

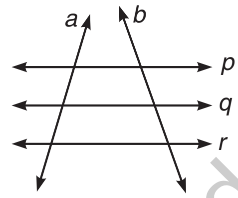
3. Count the number of line segments in the given figure.



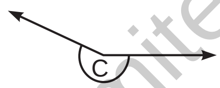
4. In the given figure, identify the number of pairs of parallel lines.



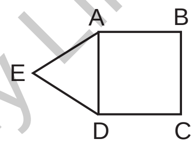
5. In the given figure, find the number of pairs of intersecting lines.



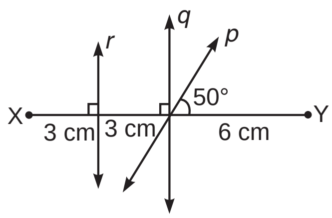
6. What is the best estimate of the angle marked C in the figure 210°, 180°, 90°, 60°?



7. In the given figure, name the angles which are greater than 90° but less than 180°.

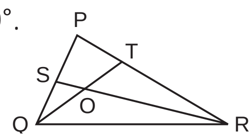


8. In the given figure, identify the perpendicular bisector of the given line segment XY. What do you conclude?

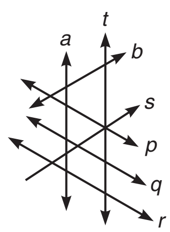


Challenge! 21st CS

1. In the given figure, find the number of angles with measure less than 180°.

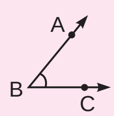


2. Identify the concurrent lines in the given figure.



Let's Work in Mind 21st CS

1. What can be the minimum number of points of intersection of 5 lines in a plane?
2. Do a horizontal line and a vertical line always intersect at right angles?
3. Are the measure of ∠ABC and ∠CBA in the given figure the same?



4. What part of a revolution have you turned through, if you stand facing the South and turn clockwise to face the East?

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

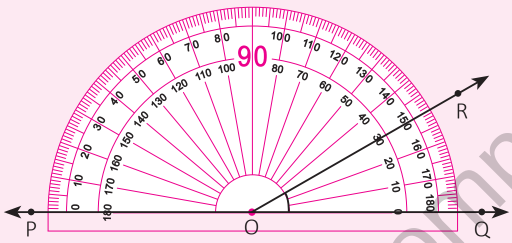
(d) Assertion (A) is false but Reason (R) is true.

1. Assertion (A) : 

is a line segment.

Reason (R) : Ray is a line segment with two initial points.

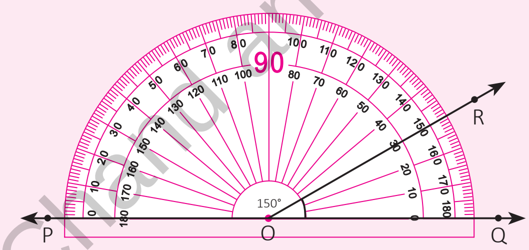
2. Assertion (A) :



∠POR = 150°

Reason (R) : Angle around the centre of the semicircle is 180°.

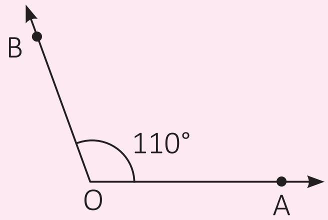
3. Assertion (A) :



∠POR = 150°

Reason (R) : ∠POQ = 180° and ∠POR + ∠QOR = ∠POR.

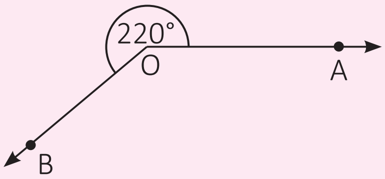
4. Assertion (A) :



∠AOB is an obtuse angle.

Reason (R) : Angle measuring between 90° and 180° is an obtuse angle.

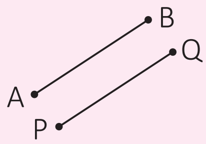
5. Assertion (A) :



∠AOB is an obtuse angle.

Reason (R) : ∠AOB is more than 90°.

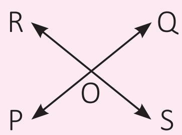
6. Assertion (A) :



AB and PQ are parallel lines.

Reason (R) : Two line segments in the same plane always parallel.

7. Assertion (A) :



O is the point of intersection of RS and PQ.

Reason (R) : Two lines in a plane always intersect at one point.

CASE STUDY

CC

1. In the given picture, mark all the rays.
2. How many pairs of intersecting lines or rays are there?
3. What is the angle between two lines, when the position of two hands is marked by a horizontal line?
4. In one of the mudras in the given picture, leg and hand movement shown are completing one circle and in the other mudra it is completing half circle. The complete turn of the body movements is called one revolution and the angle covered is _______ degree, while the half turn covers angle of measure _______ degrees.

5 Understanding Elementary Shapes

What Learners Will Achieve

- build up the concept of plane shapes.
- classify and name the triangles on the basis of sides and angles.
- classify and name the quadrilaterals on the basis of sides, angles and diagonals.
- form quadrilaterals using set squares.
- classify and name the polygons on the basis of number of sides and angles.
- know about circle and its related terms.

Warm-up

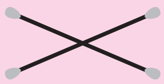
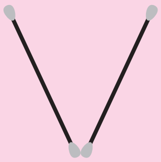
What we already know

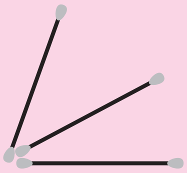
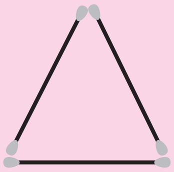
- A point has no dimension. That means it does not have length, breadth and height.
- A line segment that joins two points in a straight way has one dimension, *i.e.,* length.
- Some plane figures like triangles, quadrilaterals, circles, etc., have two dimensions, *i.e.,* length and breadth.

Now, try to solve the following.

1. Here some earbud sticks are kept in different ways. Can you say which geometrical figures do they represent?

(a) 1 stick (i)  (ii) 

(b) 2 sticks (i)  (ii)  (iii) 

(c) 3 sticks (i)  (ii) 

2. Which shape can you make using:

(a) 4 earbud sticks? (b) 6 earbud sticks?

GEOMETRICAL SHAPES

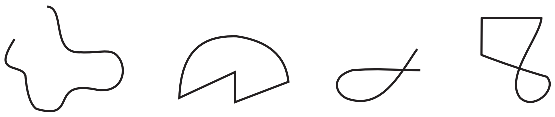
Once we know some basic concepts and terms in geometry like point, line, plane, angle, etc., our next step is to use them in understanding different geometrical shapes such as polygons, circles, etc. In this chapter, we shall learn these elementary shapes.

Curve

Any drawing on a paper done with a pencil or pen without lifting it, is a **curve**.

Curves in everyday usage means 'not straight', but in mathematics a curve can be straight too.

Observe the following curves.



(a) (b) (c) (d)

Fig. 5.1

We observe that in Fig. 5.1 (a) and (b), the curve does not cross itself whereas in (c) and (d), the curve crosses itself.

The curve that does not cross itself is called a **simple curve** [see Fig. 5.1 (a) and (b)].

Closed curve

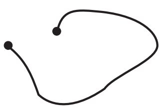
A curve that has no end point and which completely encloses a certain area is called a **closed curve** [Fig. 5.2].

Closed curves

Fig. 5.2

Open curve

A curve that is not closed is called an **open curve** [Fig. 5.3].

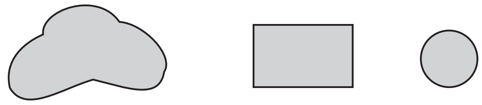


Open curve

Fig. 5.3

Simple closed curve

A curve that is **simple** as well as closed is called a **simple closed curve** [Fig. 5.4].

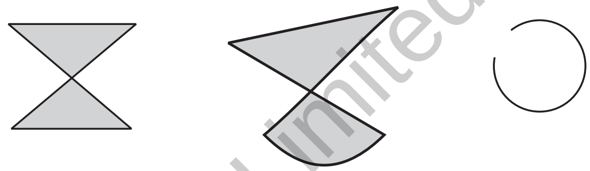


Simple closed curves

Fig. 5.4

Not simple closed

Observe the given figures.



(a) (b) (c)

Not simple closed curves

Fig. 5.5

We find that the Figs. 5.5 (a), (b) are closed, but not simple. And the Fig. 5.5 (c) is simple but not closed.

Regions of a Curve

A simple closed curve has the following three regions associated with it:

Interior (inside) region

The part of the plane which consists of all points such as E and F is called the interior region of the simple closed curve [Fig. 5.6].

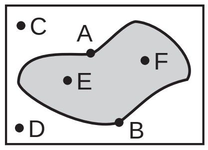


Fig. 5.6

Boundary (on) region

The part of the plane which consists of points such as A and B, which lie on the simple closed curve, forms the boundary of the curve [see Fig. 5.6].

Remember

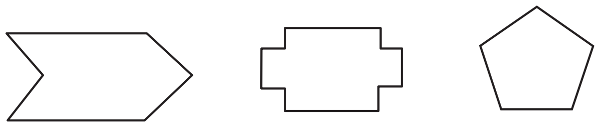
The interior of a simple closed curve together with its boundary is called its region.

Exterior (outside) region

The part of the plane which consists of all points such as C and D is called the exterior region of the curve [see Fig. 5.6].

Polygons

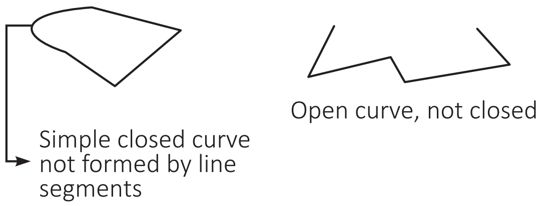
A polygon is a "simple closed curve" formed by three or more **line segments**.



Polygons

Fig. 5.7

Simple closed curves formed by line segments only.



Not Polygons

Fig. 5.8

Sides and vertex

- The line segments forming the polygon are called the **sides** of the polygon. The point of intersection of the sides is called its **vertex**.

  In Fig. 5.9, points A, B, C, D, E and F are the **vertices** (plural of vertex) of the polygon ABCDEF, while AB is a side. Similarly, AF, FE, ED, DC and CB are also sides.

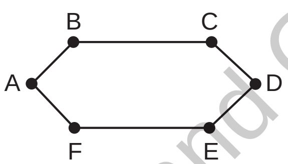
  

  Fig. 5.9

- Any two sides of a polygon having a common end point are called its **adjacent sides**.

  In Fig. 5.9, AB and BC is a pair of adjacent sides, whereas AB and CD are **not** adjacent sides. The end points of the same side of a polygon are called the **adjacent vertices**.

  The vertices E and D are adjacent, whereas vertices E and C are not adjacent vertices.

- A polygon can be named by listing its vertices in consecutive order, either clockwise or anticlockwise.

  In Fig. 5.9, ABCDEF is a polygon, whereas ACEDBF is not the polygon as shown in Fig. 5.9.

Diagonals

The **diagonal** of a polygon is a line segment whose end points are two non-adjacent vertices.

In Fig. 5.10 (a), AC is a diagonal of the polygon ABCD.

In Fig. 5.10 (b), EC is a diagonal of the polygon ABCDE. Similarly, AD is also a diagonal. What about AC? Is it also a diagonal?

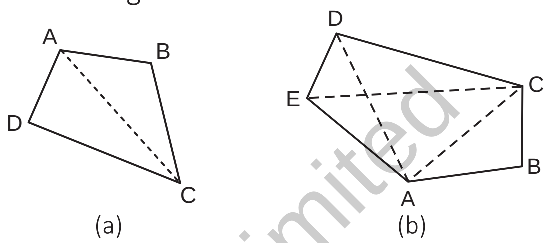


Fig. 5.10

Note

- BD is also diagonal in Fig. 5.10 (a).
- BD and BE are also diagonals in Fig. 5.10 (b).

Convex polygon

A polygon whose each angle measures less than 180° is called a **convex polygon**. Look at the polygon ABCD. All angles, *i.e.,* ∠A, ∠B, ∠C and ∠D are less than 180°. Also, line segment joining any two points in its interior lies entirely in the interior. Thus, ABCD is a convex polygon.

Can you say why polygon EFGHI is not a convex polygon? What kind of polygon it is?

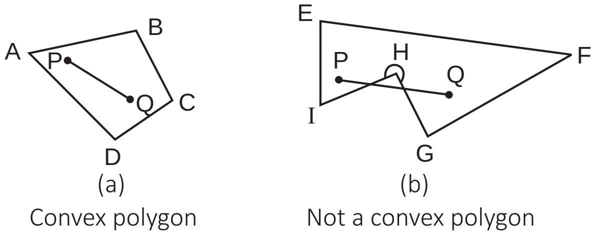


Fig. 5.11

Note

Unless stated otherwise, by a 'polygon', we shall always mean a convex polygon.

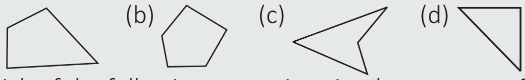
Concave polygon

If at least one interior angle of a polygon is more than 180°, then the polygon is called a **concave polygon**. For example, Fig. 5.11 (b) is a concave polygon.

Skill Check

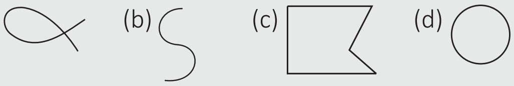
- Simple closed curves which are entirely made up of line segments in a plane are called _______.
- Which of the following polygons is not a convex polygon?

(a) (b) (c) (d)



- Which of the following curves is a simple open curve?

(a) (b) (c) (d)



Types of Polygon

According to the number of sides

We already know that a polygon is a simple closed figure made by three or more line segments. So, we name a polygon according to the number of sides.

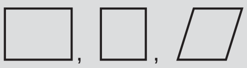
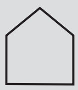
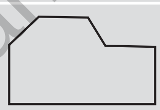
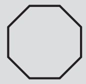
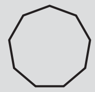
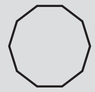
Number of Sides	Polygon	Shape
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Septagon	
8	Octagon	
9	Nonagon	
10	Decagon	

Table 5.1

We can also classify polygons on the basis of the length of their sides and their angle measures as regular and irregular polygons.

Regular and irregular polygons

A polygon having all sides of equal length and all angles of the same measure is called a **regular polygon**. Fig. 5.12 shows regular polygons.

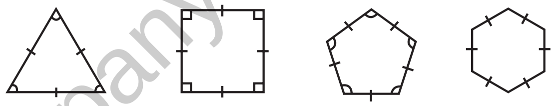


Fig 5.12

A polygon having sides or angles of different (not equal) lengths or measure, is called an **irregular polygon**. Fig. 5.13 shows irregular polygons.

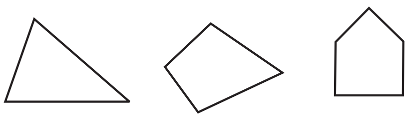


Fig 5.13

Ex. 1. Identify the polygon ABCDEF shown in Fig. 5.14. Also, say if it is regular or irregular.

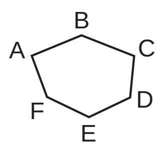


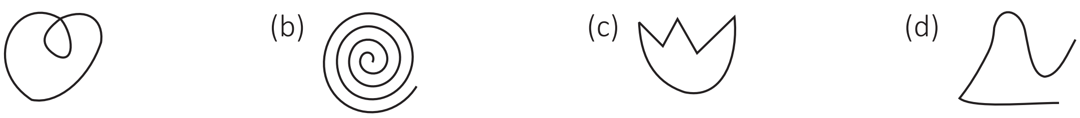
Fig. 5.14

Sol. ABCDEF is a hexagon as it has 6 sides AB, BC, CD, DE, EF and FA. It is an irregular hexagon as it has unequal sides and angles.

Exercise 5.1

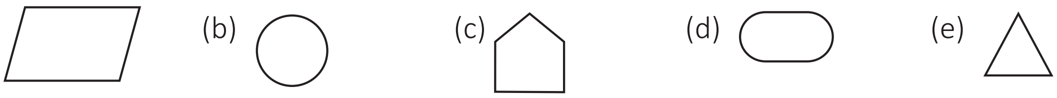
1. **Determine whether the given curves are open or closed.**

(a) (b) (c) (d)



2. **Which of the following are polygons?**

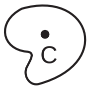
(a) (b) (c) (d) (e)



3. **Give an example from the daily life for each of the following.**

(a) Curved boundary (b) Linear boundary (c) Polygon (d) Not a convex polygon

4. **Identify the position of the points with respect to the curve.**

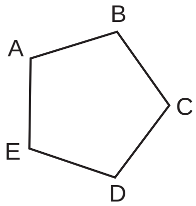
(a) 

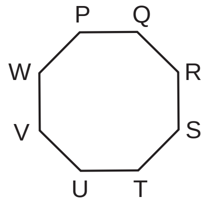
(b) 

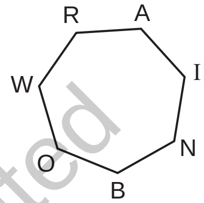
(c) 

(d) 

5. **Name the polygons shown below. Also, say if they are regular or irregular.**

(a) 

(b) 

(c) 

TRIANGLE

A triangle is a polygon having three sides. It is usually named by its vertices, taken in clockwise or anticlockwise order.

The symbol 'Δ' is used to denote the word triangle. ΔABC is read as triangle ABC.

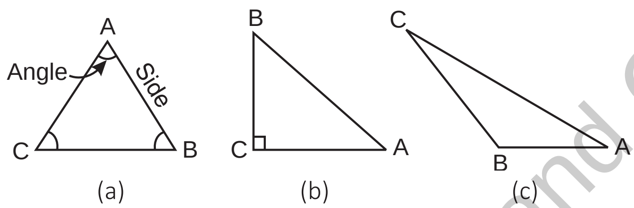


Fig. 5.15

The three points A, B and C are called the **vertices** of triangle ABC. AB, BC and CA are its three **sides**. ∠BAC, ∠BCA and ∠ABC are the three **angles** of triangle ABC.

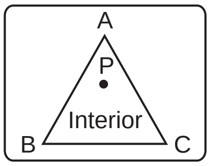
Three angles and three sides of a triangle taken together are called **six elements** (**or parts**) of the triangle.

Regions of a Triangle

A triangle has three regions associated with it.

Interior region

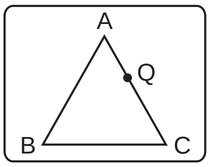
The part of the plane which consists of all points such as P, is called the **interior** of the triangle [Fig. 5.16 (a)].



(a)

Boundary (on) region

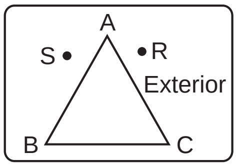
The part of the plane which consists of all points such as Q, forms the **triangle** (boundary) itself [Fig. 5.16 (b)].



(b)

Exterior region

The part of the plane which consists of all points such as S and R, is called the **exterior** of the triangle [Fig. 5.16 (c)].



(c)

Fig. 5.16

Triangular region

The interior of the ΔABC together with the boundary of ΔABC, is called the **triangular region** of ΔABC.

Median of a Triangle

A line segment joining a vertex to the mid-point of its opposite side in a triangle is called a **median** of the triangle.

In Fig. 5.17, D, E and F are the mid-points of sides BC, CA and AB respectively. Therefore, AD, BE and CF are the medians of ΔABC.

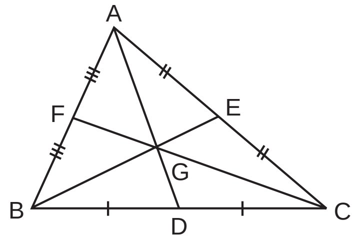


Fig. 5.17

A median divides a triangle into two equal parts.

> **Note**
>
> Point of concurrence of three medians of a triangle is called **centroid**. In Fig. 5.17, point G is the centroid of the ΔABC.

Altitude of a Triangle

The perpendicular drawn from the vertex of a triangle to its opposite side is called an altitude. In Fig. 5.18, PS is perpendicular to side QR from vertex P, so PS is an altitude.

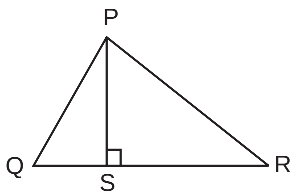


Fig. 5.18

Skill Check

- How many altitudes can a triangle have?

Ex. 2. **In Fig. 5.19,**

(a) _______ is a median of ΔXYZ.

(b) _______ is an altitude of ΔXYZ.

(c) Name all the triangles.

(d) Name the quadrilateral, if any.

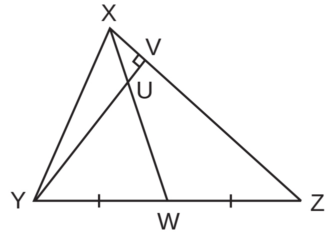


Fig. 5.19

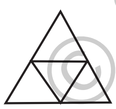
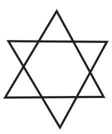
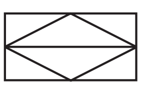
Sol. (a) Given, YW = WZ. So, W is the mid-point of side YZ. Thus, XW is a median of the ΔXYZ.

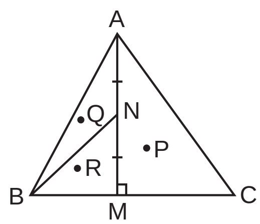
(b) YV is perpendicular to side XZ. So, YV is an altitude of ΔXYZ.

(c) ΔXUV, ΔXUY, ΔUYW, ΔXYW, ΔXWZ, ΔXYV, ΔYVZ, ΔXYZ are the triangles in the given figure.

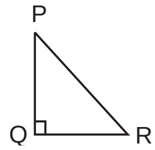
(d) WUVZ is a quadrilateral.

Exercise 5.2

1. **Fill in the blanks.**
   (a) A triangle has _____ parts, _____ sides and _____ angles.
   (b) The line segment joining the mid-point of a side to its opposite vertex in a triangle is called the _____.
   (c) The boundary together with interior of a triangle is called the _____ region.
2. **Count the number of triangles in the given figures.**
   (a)  (b)  (c) 
3. Label and name the sides of the triangles in Q2.
4. **In the given figure:**
   (a) name the altitude of ΔABC.
   (b) name the median of ΔABM.
   (c) name the points in the interior of ΔABM.
   (d) name the points in the exterior to ΔBMN.
   (e) how many triangles are there in all? Name them.



5. Mark any three points on the triangular region, one point in the exterior, two points in the interior of the given ΔPQR.



CLASSIFICATION OF TRIANGLES

Recall that a triangle is a polygon. It has three sides and three angles. Triangles can be classified in two ways:

1. Classification on the Basis of Angles

There are three types of triangles, classified on the basis of their angles.

Acute-angled triangle

A triangle in which all the three angles are less than 90° is called an **acute triangle** or **acute-angled triangle**.

In Fig. 5.20, ΔABC is an acute-angled triangle.

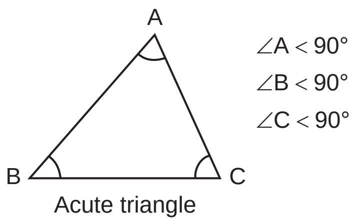


Fig. 5.20

Right-angled triangle

A triangle in which one of the angles is 90° and other two are acute angles is called a **right triangle** or **right-angled triangle**.

In Fig. 5.21, ΔPQR is a right-angled triangle, right-angled at Q.

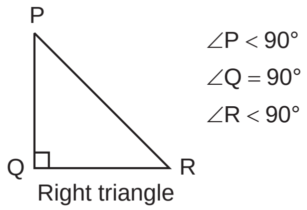


Fig. 5.21

The sides forming the right angle are called *legs* and the side opposite to the right angle is called the *hypotenuse.*

Thus, QP and QR are the legs of the triangle PQR and PR is its hypotenuse.

Obtuse-angled triangle

If one of the angles of a triangle is greater than 90° and the other two are acute angles then the triangle is called an **obtuse triangle** or **obtuse-angled triangle**.

In Fig. 5.22, ΔXYZ is an obtuse-angled triangle.

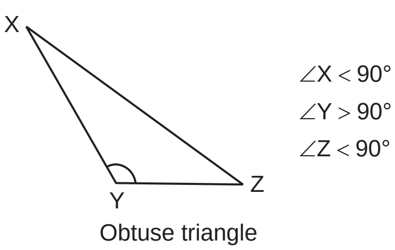
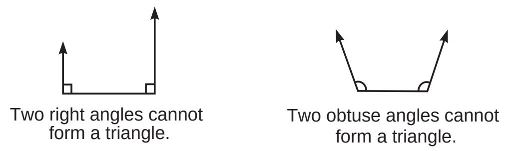


Fig. 5.22

Watch Your Step!

A right triangle can have only one of its angles equal to a right angle and an obtuse triangle can have only one of its angles as an obtuse angle. Why?



Skill Check

- Which set of three angles does not describe an acute-angled triangle?
  (a) 85°, 15°, 80° (b) 90°, 40°, 50°
  (c) 70°, 40°, 70° (d) 45°, 55°, 80°
- The maximum number of right angle(s), a triangle has, is:
  (a) one (b) two (c) three (d) none

2. Classification on the Basis of Sides

There are three types of triangles, classified on the basis of their sides.

Equilateral triangle

A triangle having all sides equal is called an **equilateral triangle**.

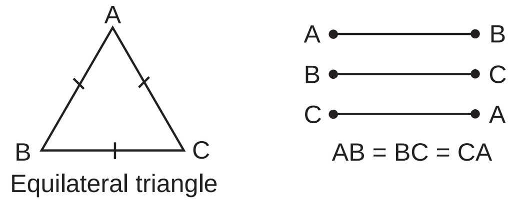


Fig. 5.23

In Fig. 5.23, ΔABC is an equilateral triangle.

Note

An equilateral triangle has all angles equal and measuring 60° each. In ΔABC, ∠A = ∠B = ∠C = 60°.

Isosceles triangle

A triangle having two sides equal is called an **isosceles triangle**.

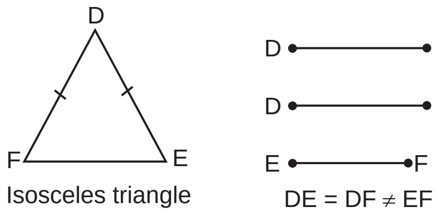


Fig. 5.24

In Fig. 5.24, ΔDEF is an isosceles triangle.

> **Note**
>
> An isosceles triangle has equal angles opposite to equal sides. In ΔDEF, ∠E = ∠F as DF = DE.

Scalene triangle

A triangle having no two sides equal is called a **scalene triangle**.

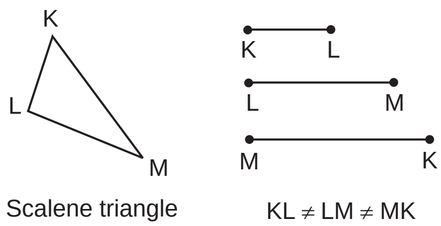


Fig. 5.25

In Fig. 5.25, ΔKLM is a scalene triangle.

Angle Sum Property of a Triangle

Draw two or more triangles and measure their angles. Then find the sum of the angles of the triangles. What do you observe?

The sum of interior angles of a triangle is 180°.

In ΔABC, ∠A + ∠B + ∠C = 180°.

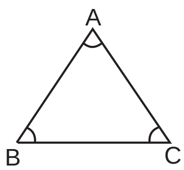


Fig. 5.26

> **Note**
>
> - A triangle can have a measure of angles as 60°, 50°, 70° but cannot have 100°, 90°, 10° or 50°, 45°, 60°.
> - The perimeter of a triangle ABC is AB + BC + CA.

Let us study some more examples.

Ex. 3. Identify the following triangles, based on their angles.

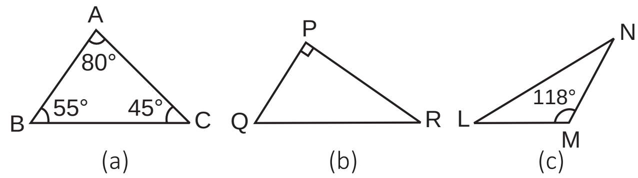


Fig. 5.27

Sol. (a) ΔABC is an acute-angled triangle as measure of each angle is less than 90°.

(b) ΔQPR is a right-angled triangle as ∠P = 90°.

(c) ΔLMN is an obtuse-angled triangle as ∠M = 118° > 90°.

Ex. 4. Name the following triangles in two different ways.

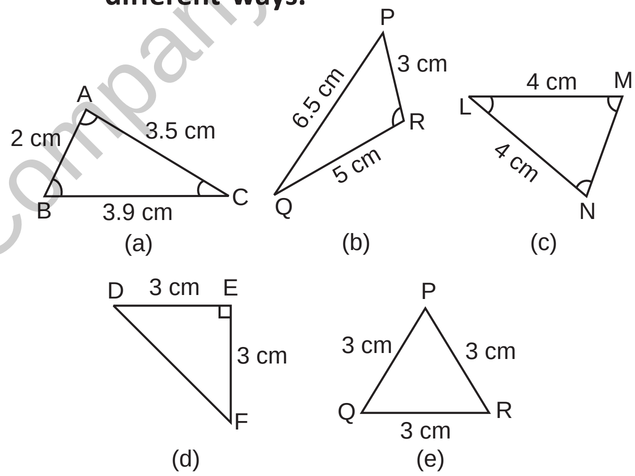


Fig. 5.28

Sol. (a) ΔABC is a scalene triangle as AB ≠ BC ≠ AC. It is also an acute-angled triangle.

(b) ΔPQR is a scalene triangle as PQ ≠ QR ≠ PR. It is also an obtuse-angled triangle as ∠R > 90°.

(c) ΔLMN is an isosceles triangle as LM = LN. It is also an acute-angled triangle.

(d) ΔDEF is a right triangle as ∠E = 90°, ΔDEF is also an isosceles triangle as DE = EF.

(e) ΔPQR is an equilateral triangle as all of its sides are equal. It is also an acute-angled triangle as each angle of an equilateral triangle is 60°.

Exercise 5.3

1. Fill in the blanks.

(a) The sides of a scalene triangle are of _________ lengths.

(b) The sum of angles of a triangle is _________.

(c) Each angle of an equilateral triangle measures _________.

(d) The angles opposite to equal sides of an isosceles triangle are _________.

(e) A triangle whose sides are of different measures is known as _________ .

(f) Hypotenuse is _________ side of the right-angled triangle.

2. Match the following.

Features	Types of triangle
(a) 3 equal sides	(i) obtuse-isosceles triangle
(b) 2 equal sides	(ii) right-angled triangle
(c) 3 acute angles	(iii) acute-scalene triangle
(d) one right angle	(iv) acute-angled triangle
(e) one obtuse angle with 2 equal sides	(v) isosceles triangle
(f) all acute angles with all different sides	(vi) equilateral triangle

3. Classify the following triangles according to the measure of sides.

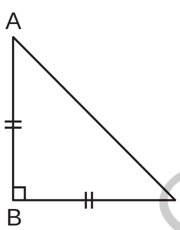
(a) 8 cm, 8 cm, 8 cm (b) 18 cm, 22 cm, 27 cm

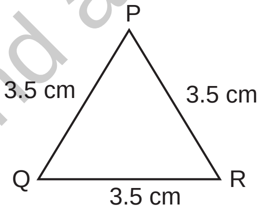
(c) 5 cm, 6 cm, 8 cm (d) 8 cm, 12 cm, 8 cm

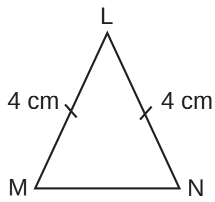
4. Classify the following triangles according to the measure of angles.

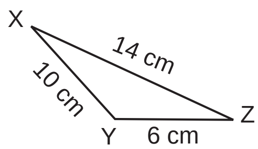
(a) 20°, 50°, 110° (b) 60°, 60°, 60° (c) 15°, 75°, 90° (d) 50°, 65°, 65°

5. Identify the following triangles in two different ways.

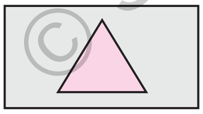
(a)


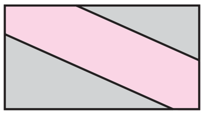
(b)


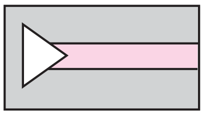
(c)


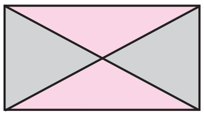
(d)


6. How many triangles do you see in the following flags?

(a)


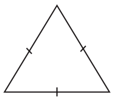
(b)


(c)


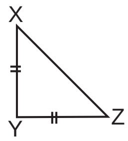
(d)


7. Answer the following questions.

(a) Name the figure given alongside.



(b) In the given figure, select the word that best describes triangle XYZ.



(c) In which triangle does one side form the hypotenuse?

QUADRILATERALS

A **quadrilateral** is a polygon having four sides and is usually named by its vertices taken in clockwise or counterclockwise (anticlockwise) order.

The quadrilateral in Fig. 5.29 is named as ADCB or ABCD.

In quadrilateral ABCD,

- AB, BC, CD and DA are its four sides.
- ∠A, ∠B, ∠C and ∠D are its four angles.
- AC and BD are its two diagonals.
- BC and CD are adjacent sides.
- AB and DC, AD and BC are pairs of opposite sides.

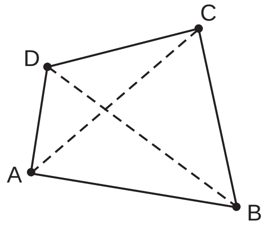


Fig. 5.29

Can you name the other pair of adjacent sides?

- AB and BC, BC and CD, CD and DA are adjacent sides.
- ∠D and ∠B are opposite angles.
- ∠A and ∠C are opposite angles.
- ∠A and ∠B are adjacent angles.

Other pairs of adjacent angles are ∠B and ∠C; ∠C and ∠D; ∠D and ∠A.

Interior and Exterior of a Quadrilateral

The region inside the quadrilateral is called *interior region* and that outside is called *exterior region* of the quadrilateral. The points lying on its sides are called on the boundary.

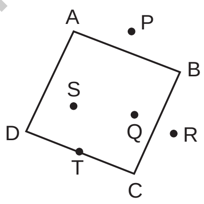


Fig. 5.30

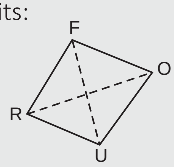
In Fig. 5.30, points Q and S are in interior, points P and R are in exterior and points A, B, C, T and D are on the boundary of the quadrilateral ABCD.

Note

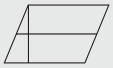
The quadrilateral region includes points on boundary and its interior.

Skill Check

- Identify the given figure and name its:
  (a) sides ___________
  (b) angles ___________
  (c) diagonals ________
  (d) vertices __________



- Count the number of quadrilaterals in the given figures.

(a)  (b)  (c) 

Types of Quadrilaterals

We already know that, a quadrilateral is a polygon with four sides. In this section, we shall discuss different types of quadrilaterals along with their properties.

Convex and concave quadrilaterals

In Fig. 5.31 (a), ABCD is a **convex quadrilateral** as its both diagonals AC and BD completely lie in its interior. Also, its each angle is less than 180°.

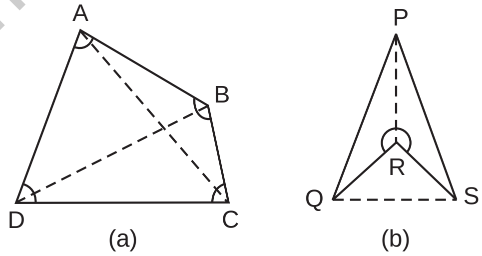


Fig. 5.31

In Fig. 5.31 (b), PQRS is a **concave quadrilateral** as the diagonal QS does not lie inside the quadrilateral. Also, ∠R is a reflex angle, *i.e.*, 180° < ∠R < 360°.

Parallelogram

A **parallelogram** is a quadrilateral in which both the pairs of opposite sides are parallel and equal.

In Fig. 5.32, AB is parallel to DC and AD is parallel to BC.

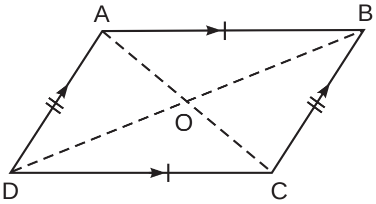


Fig. 5.32

Symbolically, it can be written as AB || DC and AD || BC.

The opposite sides of a parallelogram are also equal, *i.e.*, AB = DC, AD = BC.

Therefore, ABCD is a parallelogram.

Properties of a Parallelogram

- Opposite sides of a parallelogram are parallel and equal.
- Opposite angles are equal.
- Diagonals of a parallelogram need not be equal but they bisect each other at the point of intersection.

Rectangle

A **rectangle** is a parallelogram with each angle as a right angle.

In Fig. 5.33, PS || QR, PS = QR, SR || PQ, SR = PQ. ∠P = ∠Q = ∠R = ∠S = 90°.

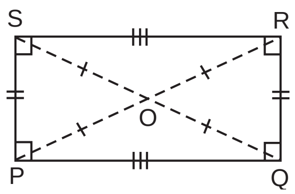


Fig. 5.33

Properties of a Rectangle

- Opposite sides are of equal length and are parallel to each other.
- Diagonals are equal and bisect each other.
- Each angle is a right angle.

Square

A **square** is a parallelogram in which all sides are equal and each angle is of 90°.

In Fig. 5.34, AB || DC, AD || BC, AB = BC = CD = DA and ∠A = ∠B = ∠C = ∠D = 90°.

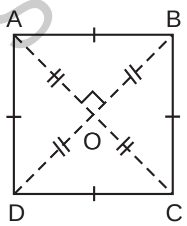


Fig. 5.34

> **Note**
>
> Observe that a square is both a rectangle and a parallelogram.

Properties of a Square

- Each angle is a right angle.
- All sides are of equal length.
- Diagonals are equal and perpendicular bisectors of each other.

Rhombus

A **rhombus** is a parallelogram with all four sides equal.

In Fig. 5.35, AB || DC, AD || BC, AB = BC = CD = DA.

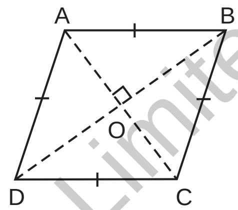


Fig. 5.35

> **Note**
>
> Every square is a rhombus, but every rhombus is not a square.

Properties of a Rhombus

- All sides are of equal length.
- Opposite angles are equal.
- Diagonals of a rhombus are perpendicular bisectors of each other.

Trapezium

A **trapezium** is a quadrilateral with one pair of parallel sides.

- Fig. 5.36 (a) shows a trapezium in which MN || PO.
- The parallel sides are called the **bases** of the trapezium.
- If the non-parallel sides are equal, then the trapezium is said to be an ***isosceles trapezium*** [see Fig. 5.36 (b)].

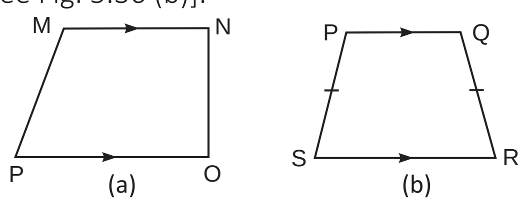


Fig. 5.36

In Fig. 5.36 (b), PQ || SR and PS = QR. So, PQRS is an isosceles trapezium.

Kite

A **kite** is a quadrilateral in which two pairs of adjacent sides are equal and diagonals intersect each other at right angle.

In Fig. 5.37, WX = WZ, YX = YZ, WY ⊥ XZ, *i.e.*, ∠WOZ = 90°. So, WXYZ is a kite.

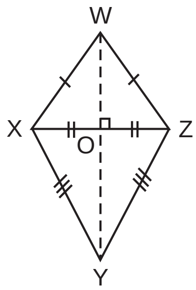
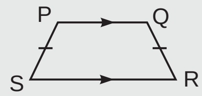


Fig. 5.37

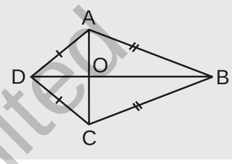
Observe, in Fig. 5.37,
WX = WZ and YX = YZ but WX ≠ YX and WZ ≠ YZ.
XZ ⊥ WY .
Also, OX = OZ but OW ≠ OY (why?)
Since OW ⊥ XZ, YO ⊥ XZ, ΔXWZ and ΔXYZ are isosceles triangles, but ΔYXW and ΔYZW are not isosceles. So, OW ≠ OY.

Skill Check

- In an isosceles trapezium PQRS, draw the diagonals PR and QS.



- Measure $\overline{PQ}$ and $\overline{QS}$ using scale and check, if $\overline{PQ} = \overline{RS}$.
- Using a protractor, measure ∠P, ∠Q, ∠R and ∠S. Check, if ∠P = ∠Q and ∠R = ∠S.
- In the kite ABCD, measure AO, CO, ∠DAB and ∠DCB. What do you observe?



Let Us Do

EL

Objective: To create a square and a rectangle using set squares

Materials required: 2 scalene set squares (90° – 60° – 30°); 2 Isosceles set squares (90° – 45° – 45°)

Procedure:

To create a square

Step 1: Take two isosceles set squares (90° – 45° – 45°) and join them along their longest edges.

Step 2: Trace the outline of the shape thus obtained on a drawing sheet. Observe, this shape is a square [see Fig. 5.38].

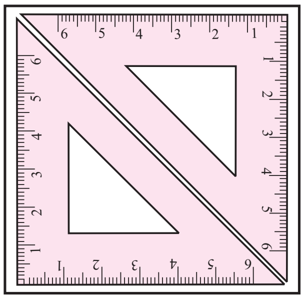
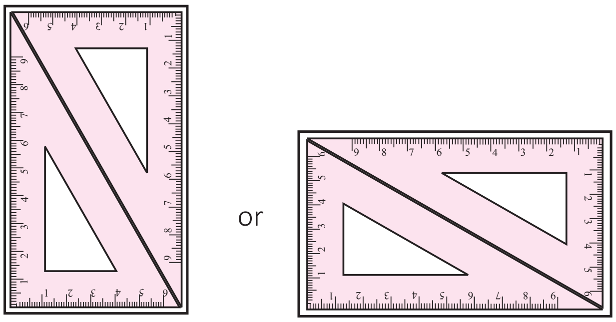


Fig. 5.38

To create a rectangle

Step 1: Join the two scalene set squares (90° – 30° – 60°) and join them along their longest edges.

Step 2: Trace the outline of the shape thus obtained on a drawing sheet. Observe, this shape is a rectangle [see Fig. 5.39 (a) and (b)].



(a) (b)

Fig. 5.39

Let us study some more examples.

Ex. 5. **In Fig. 5.40, AXYB is a quadrilateral. List the points in the interior of the quadrilateral.**

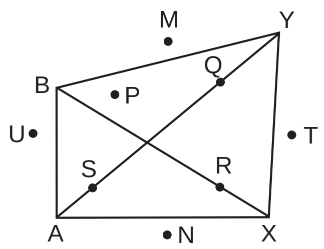


Fig. 5.40

Sol. P, Q, R and S are the points in the interior of the quadrilateral.

Ex. 6. **In Fig. 5.41, PQRS is a parallelogram. Name its parallel sides and diagonals.**

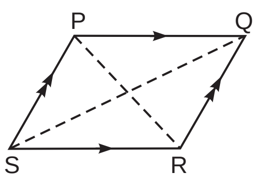


Fig. 5.41

Sol. PQ || SR and PS || QR. (Opposite sides are parallel.)

Diagonals are PR and QS.

Ex. 7. **Specify the type of quadrilateral ABCD, if the following information are given.**

AB || DC, AD || BC, ∠DAB = 60°

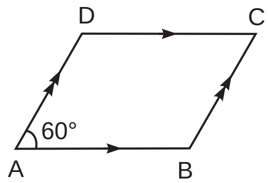


Fig. 5.42

Sol. Here, the opposite sides are parallel.

So, by definition, ABCD is a parallelogram.

Ex. 8. **Specify the type of quadrilateral ABCD on the basis of the following information:**

AB = BC, CD = DA, DB ⊥ AC, DA ⊥ AB, DC ⊥ CB, O being the point of intersection of the diagonals.

Sol. Since AB = BC, CD = DA, DB ⊥ AC, DA ⊥ AB and DC ⊥ CB, by definition, ABCD is a kite.

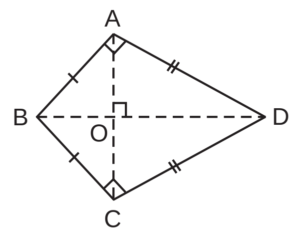
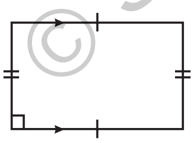
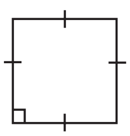


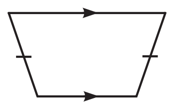
Fig. 5.43

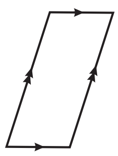
Exercise 5.4

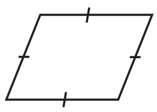
1. **Classify each of the following figures as square, parallelogram, rhombus, rectangle, kite, trapezium or isosceles trapezium.**

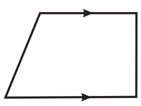
(a) 

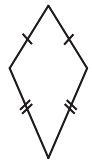
(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

2. Identify each of the following parallelograms.

(a) The adjacent sides are equal and the diagonals are also equal.

(b) The adjacent sides are not equal but the diagonals are equal.

(c) All sides are equal and the diagonals are not equal.

3. State True (T) or False (F) for the following statements.

(a) The diagonals of a rhombus are equal.

(b) The diagonals of a rectangle are perpendicular to each other.

(c) The diagonals of a parallelogram are equal.

(d) Kite has an unequal pair of adjacent sides with diagonals intersecting at 90°.

4. Fill in the blanks.

(a) A quadrilateral in which all sides and all angles are equal is known as a ___________.

(b) The diagonals of a rhombus bisect each other at ___________ angles.

(c) A quadrilateral whose one pair of opposite sides is parallel and the other pair of opposite sides is non-parallel is called a ___________.

(d) If the diagonals of a quadrilateral bisect each other at 90°, the quadrilateral is a ___________ .

5. In the given figure, a quadrilateral is shown. Name:

(a) the points lying in the interior region.

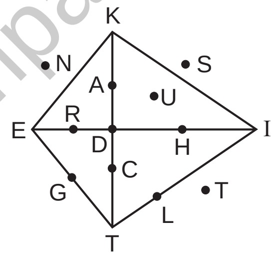
(b) the points lying on the boundary.

(c) two pairs of opposite sides.

(d) two pairs of opposite angles.

(e) two pairs of adjacent sides.

(f) two pairs of adjacent angles.



6. Identify the correct answer in each of the following questions.

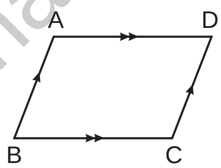
(a) Which of the word below could not be used to describe figure ABCD?

(i) rectangle

(ii) polygon

(iii) quadrilateral

(iv) parallelogram



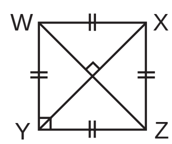
(b) In the given figure, WZ and XY are

(i) not equal

(ii) equal

(iii) horizontal

(iv) adjacent



(c) Which of the following best describes a rectangle which is not a square?

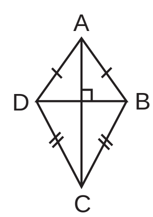
(i) Four equal sides

(ii) Four right angles and two opposite sides equal

(iii) Four equal angles

(iv) Four equal angles and two sides equal

7. In the given figure, AB = AD, AC ⊥ BD, CD = CB and ∠ABC = ∠ADC. Identify the figure ABCD.



8. In a quadrilateral PQRS, PS = QR and PQ || RS. Specify the type of quadrilateral.

CIRCLE

We very commonly see things such as chapati, wheel, ring, etc., in our daily life. When we trace these, we get a shape known as a **circle**.

Observe the spokes of a wheel (Fig. 5.44). Each spoke is joining the centre to a point on the wheel. A wheel represents a circle and the length of each spoke is equal *i.e.*, distance from the centre to the boundary of the wheel.

Fig. 5.44

Thus, a circle is a simple closed curve (Fig. 5.45). It is the set (collection) of all the points in a plane equidistant from a given point called its **centre** (C).

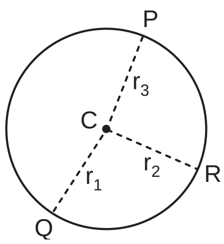


Fig. 5.45

In Fig. 5.45, we have

- P, Q and R are the three points on the circle.
- CQ = CR = CP, *i.e.*, $r_1 = r_2 = r_3$.

Parts of a Circle

Radius

The **radius** of a circle is a line segment joining the centre of the circle to a point on the circle.

In Fig. 5.46,

- CP is a radius.
- *r* is the length of this radius, since CP = *r*.
- CP = CQ = CR = *r*.
- CP, CQ and CR are the radii (plural of radius) of the given circle.

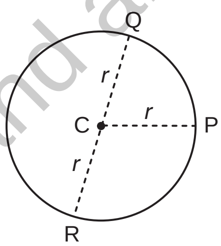
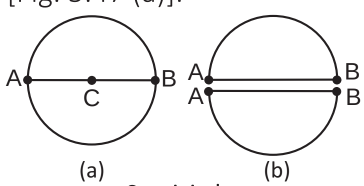


Fig. 5.46

Diameter

The **diameter** of a circle is a line segment joining two points on the circle and also passing through the centre [Fig. 5.47 (a)].



Semicircle
AB is a diameter of the circle.
Fig. 5.47

Diameter is double the length of the radius, *i.e.,*

Diameter = 2 × radius

The end points of a diameter divide the circle into two equal parts. Each part is called a "**semicircle**" [Fig. 5.47 (b)].

Thus, a semicircle is half of a circle.

Circumference

The distance moved around a circle once is called its **perimeter** or **circumference**.

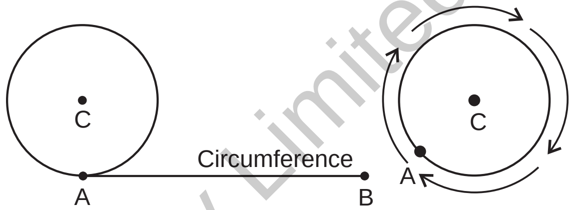


Fig. 5.48

> **Note**
>
> Circumference, diameter and radii are measured in linear units such as inches, centimetres, etc. A circle has many radii and many diameters, each passing through the centre.

Chord

The **chord** of a circle is a line segment joining any two points on it.

AB is a chord of the circle [Fig. 5.49 (a)].

If the chord passes through the centre of the circle, it is called its **diameter**.

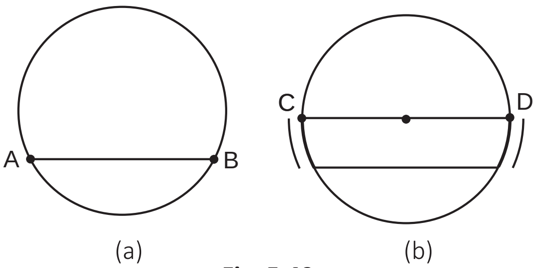


Fig. 5.49

CD is a diameter [Fig. 5.49 (b)].

> **Note**
>
> The diameter is the longest chord of a circle.

If A and B are two points on a circle, then we get an arc AB, written as $\overparen{AB}$.

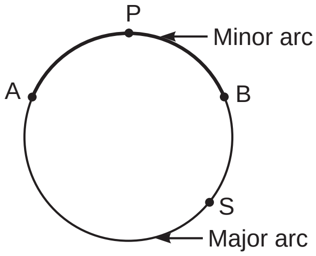


Fig. 5.50

In fact, any two points A and B of a circle divide it into two parts called **arcs** of the circle. Generally, the two parts are not equal [Fig. 5.50]. The smaller part is called the **minor arc** and the other one is called the **major arc**. In Fig. 5.50, $\overparen{APB}$ is the minor arc and $\overparen{ASB}$ is the major arc.

Segment and sectors

Look at Fig. 5.51 (a).

The chord AB divides the circle into two parts. Each part is called a segment of the circle. The smaller segment is called the **minor segment** and the larger one is called the **major segment.**

Thus, the area bounded by the arc and chord of the circle is called the segment of the circle.

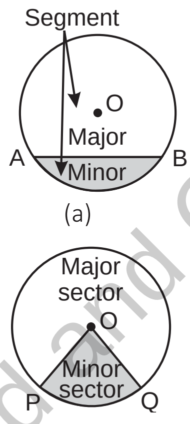


Fig. 5.51

In Fig. 5.51 (b), OP and OQ are the two radii of the circle. These radii divide the circle into two regions called the **sectors** of the circle.

The smaller region is called the **minor sector** and the larger region is called the **major sector**.

Thus, the area bounded by the radii and arc is called a **sector**.

Note

Points P and Q are common to both the arcs [see Fig. 5.51 (b)].

Region of a Circle

Like other simple closed curves, a circle also divides the plane into three regions (parts) as follows:

Interior region

The part of the plane which consists of all the points such as E, F is called the **interior region** of the circle [see Fig. 5.52 (a)].

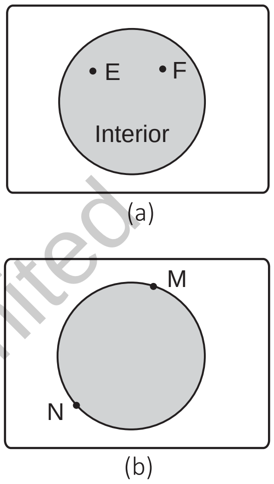


Fig. 5.52

Boundary region

The part of the plane which consists of all the points such as M, N [Fig. 5.52 (b)], lying on the circle forms the **boundary** of the circle.

The interior of the circle along with its boundary (*i.e.*, circle itself) is called the **circular region**.

Exterior region

The part of the plane which consists of all points such as R, S [Fig. 5.53] is called the **exterior region** of the circle.

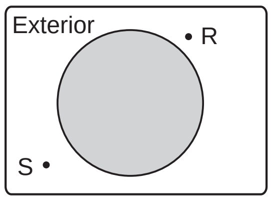


Fig. 5.53

Semicircular region

Recall that the end points of a diameter of a circle divide the circle into two equal parts called *semicircles*. Each of the regions enclosed by the diameter and the semicircle is called the **semicircular region** [Fig. 5.54].

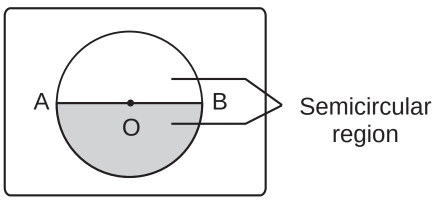


Fig. 5.54

Exercise 5.5

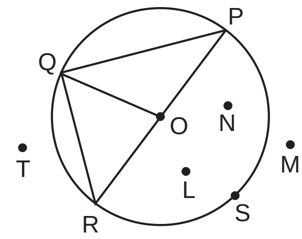
1. In the given figure, name the following:

(a) Three radii (b) Three chords

(c) A diameter (d) Points on the circle

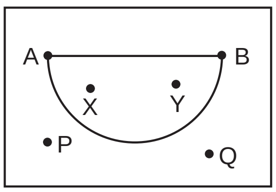
(e) Points in the interior of the circle

(f) Points in the exterior of the circle



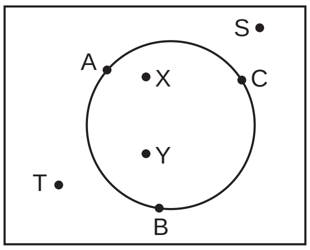
2. Look at the following figures and name the points:

(a)



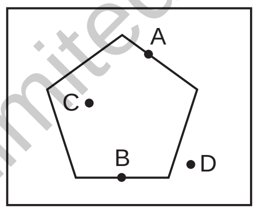
(i) in the interior region.
(ii) in the exterior region.
(iii) on the boundary region.

(b)



(i) in the interior region.
(ii) in the exterior region.
(iii) on the boundary region.

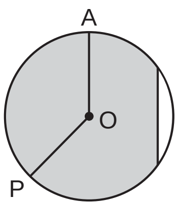
(c)



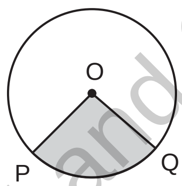
(i) in the interior region.
(ii) in the exterior region.
(iii) on the boundary region.

3. In each of the following circles, what is the shaded region called?

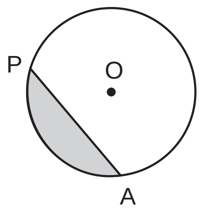
(a)



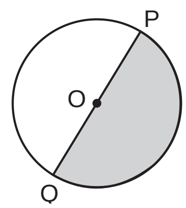
(b)



(c)

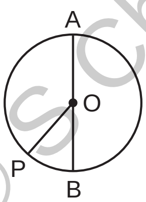


(d)

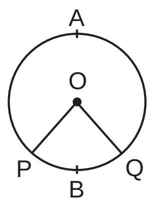


4. For each of the given circles with centre O, complete the corresponding statements.

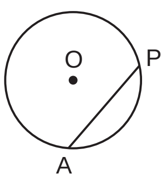
(a) AB is a ________ of the circle.



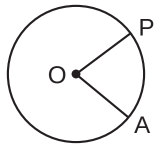
(b) PBQ is a/an ________ of the circle.



(c) AP is a ________ of the circle.



(d) OP and OA are ________ of the circle.



5. Fill in the blanks.

(a) A _________ is a chord passing through the centre of the circle.

(b) End points of the _________ of a circle divides it into two semicircles.

(c) The _________ is the total length of a circle.

(d) A _________ is a region in the interior of the circle enclosed by its arc and a chord.

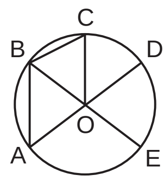
(e) A _________ is a region in the interior of the circle enclosed by an arc and a pair of radii on the other two sides.

Competency Based Exercise

21st CS

1. Tick (✓) the correct answer.

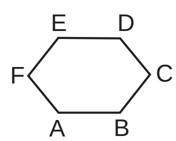
(a) In the given figure, how many line segments represent the radius of the circle?



(i) 2 (ii) 3

(iii) 4 (iv) 5

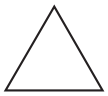
(b) The sum of the total number of vertices and total number of angles of a given polygon is:



(i) 10 (ii) 11

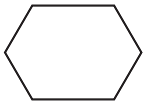
(iii) 12 (iv) 13

(c) Which of the following figures is not a polygon?

(i) 

(ii) 

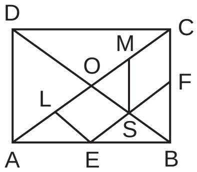
(iii) 

(iv) 

(d) The number of diagonals of a triangle is:

(i) 0 (ii) 1 (iii) 2 (iv) 3

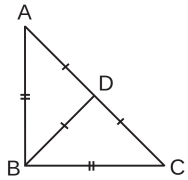
(e) The number of triangles in the given figure is:



(i) 5 (ii) 6

(iii) 7 (iv) 13

(f) In the given figure, if AB = BC and AD = BD = DC, then the number of isosceles triangles in the figure is:



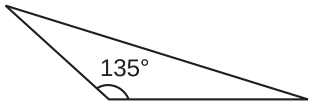
(i) 1 (ii) 2 (iii) 3 (iv) 4

(g) If all the diagonals of a regular hexagon are drawn, then the number of points of intersection not containing its vertices is:

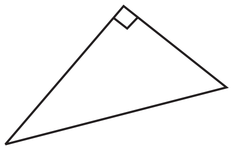
(i) 6 (ii) 7 (iii) 12 (iv) 13

2. **Identify the following triangles based on their angles.**

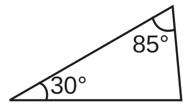
(a)



(b)

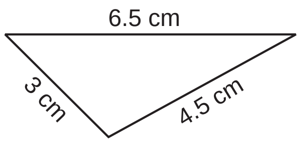


(c)

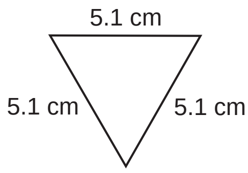


3. **Identify the following triangles based on their sides.**

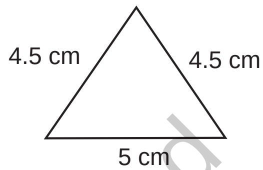
(a)



(b)



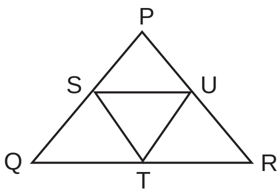
(c)



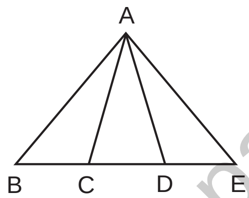
4. A polygon has prime number of sides. Its number of sides is equal to the sum of the least two consecutive primes. Find the number of diagonals of the polygon.

5. **How many triangles are there in each of the following figures? Write the name of each triangle.**

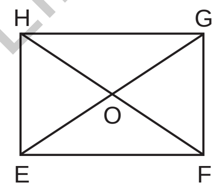
(a)



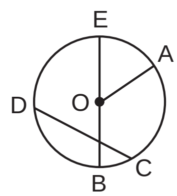
(b)



(c)



6. **In the given circle, state the term for each of the following.**

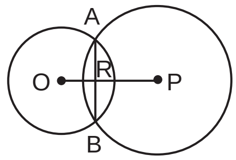


(a) OA ____________________ (b) BE ____________________

(c) DC ____________________ (d) $\overset{\frown}{DBC}$ ____________________

(e) $\overset{\frown}{DEC}$ ____________________

7. In the given figure, if OP ⊥ AB, then find the number of right angles formed.



8. Draw a rough sketch of a hexagon and draw its diagonals.

Challenge! 21st CS

1. In the given figure, in ∆ABC, AD = BD = DC. Then, ∠ABC = ∠BAC + ________.

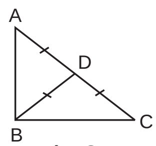


Fig. Q. 1

2. In the given figure, ABCDEF is a regular hexagon. If O is its centre, then find the measure of ∠COD.
[**Note:** A polygon in which all sides are equal and all angles are equal is called a regular polygon.]

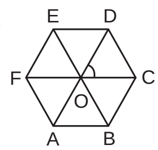


Fig. Q. 2

Let's Work in Mind

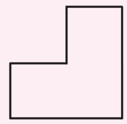
21st CS

1. What is the minimum number of sides of a polygon?
2. Name the quadrilateral whose diagonals are equal and they bisect each other at right angles.
3. What is the sum of all the angles of a triangle?
4. Name the regular polygon with 3 sides.
5. Which regular polygon has 4 sides?
6. Name the longest chord of a circle.
7. How many diagonals are there in a triangle?

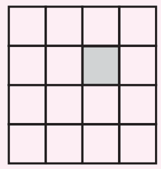
SMART TIME

LS

1. Make four copies of the following figure and cut them. Arrange them to form a larger version of the original figure.



2. A farmer has a square farm represented by 4 × 4 square grid shown alongside. The shaded part of the grid represents his house in the farm. The farmer wishes to retire and wants to divide the farm into five identical land pieces (same size and same shape) for his 5 sons. Can you help him to do so?



ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

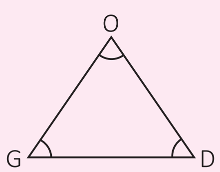
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A) :**



In ΔGOD, $\angle G = \angle O = \angle D$.

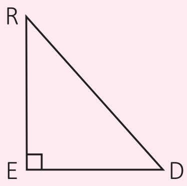
Reason (R) : ΔGOD is an equilateral triangle.

2. **Assertion (A) :** The angles of a triangle are 90°, 90° and 30°.

Reason (R) : There cannot be two right angles in a triangle.

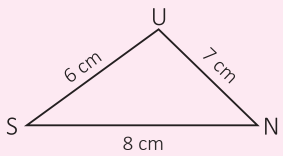
3. **Assertion (A) :** A triangle with all sides 10 cm have angles 60°, 60° and 60°.

Reason (R) : Equilateral triangles have all angles equal.

4. **Assertion (A)** : 

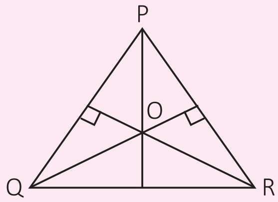
ΔRED is a right triangle.

Reason (R) : In right triangle, one angle is always 90°.

5. **Assertion (A)** : 

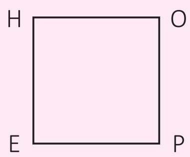
Perimeter of ΔSUN = 21 cm

Reason (R) : Perimeter of a triangle is the sum of lengths of all its sides.

6. **Assertion (A)** : 

O is the centroid of ΔPQR. It is point of concurrence.

Reason (R) : Point of concurrence is the point where more than two lines meet.

7. **Assertion (A)** : 

HOPE is a quadrilateral.

Reason (R) : A four-sided figure is called a quadrilateral.

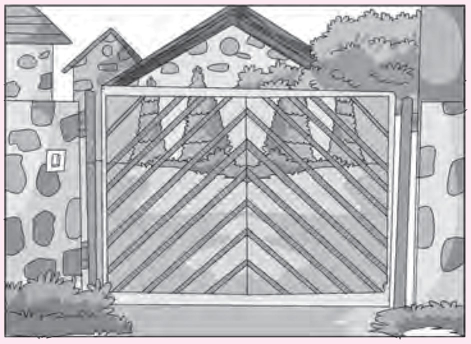
8. **Assertion (A)** : A square is a parallelogram.

Reason (R) : In parallelograms, pair of opposite sides are equal.

CASE STUDY

CC

Romila, Kanta, Arshad and Komal were walking through a street and noticed the new gate of their neighbour Colonel Uncle. They had the following arguments. Give your opinion for each argument.



1. Romila : The gate has triangles on both sides of the central line.
2. Kanta : The gate has quadrilaterals and triangles on both sides.
3. Arshad : The gate has right-angled triangles on both sides of the central line.
4. Komal : The gate has trapeziums, parallelograms and triangles on both sides.

6 Three-dimensional Shapes

What Learners Will Achieve

- identify and name the 3D shapes in their surroundings.
- understand the elements of 3D shapes (faces, edges and vertices).
- recognise the nets of solid shapes (cube, cuboid, cylinder, cones and tetrahedron)

Warm-up

LS

What we already know

Point, line, line segment, angle, etc., are some basic ideas of geometry. These ideas are the building blocks of geometry. It also includes the concept of solid shapes like cube, cuboid, cylinder, cone, sphere, etc. In fact, geometry has now become a tool for dealing with problems in other fields too. Remember, when you go with your mother to buy shoes, the shopkeeper keeps the shoes in a box. The size of the box depends on the size of the shoes. The box is made from cardboard. The manufacturer of these boxes uses geometry to decide the size of the cardboard so that there is no wastage or wastage is minimum.

Fig. 6.1

Now, try to solve the following.

1. What is the shape of the box (see Fig. 6.1)?
2. What shape will you get if you trace its one face on a sheet of paper (see Fig. 6.1)?
3. What is the difference between a square and a rectangle?
4. The two faces of the box meet in a line segment, called edge. Count how many edges the box has.
5. The three edges of the box meet at a point, called vertex of the box. How many vertices does the box have?

DID YOU KNOW?

Greek mathematician Euclid is often referred to as the 'Father of Geometry' for his revolutionary ideas and influential textbook called 'Elements' that he wrote around the year 300 BCE.

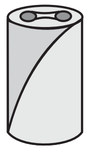
THREE-DIMENSIONAL SHAPES

We have already learnt about plane figures such as polygons, circles, etc. These figures have only two dimensions so these are called **two-dimensional** or **2D shapes**. We can draw such figures on the notebooks.

Following are some shapes (Fig. 6.2) which we see in our day-to-day life:

(a) A ball is a sphere.

(b) An ice cream cone.



(c) A can is a cylinder.

(d) A box is a cuboid.

(e) A playing die is a cube.

(f) This is the shape of a pyramid.

Fig. 6.2

Each shape is a solid shape and not flat like surface of a blackboard, top of the table, etc. These figures have three dimensions and so these are called **three-dimensional** or **3D shapes**.

If you put each of these shapes on a plane, only a part of it touches the plane.

Now, let us discuss these 3D shapes one by one.

Cuboid

A **cuboid** is a 3-dimensional figure bounded by six rectangular surfaces or **faces**.

A shoebox, a brick, an almirah, etc., are examples of the shape of a cuboid.

A shoebox

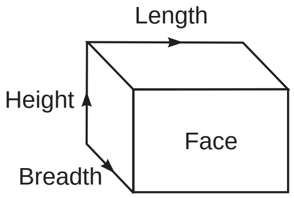
A brick

An almirah

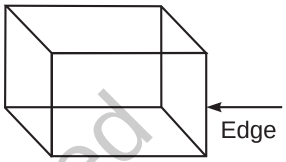
Fig. 6.3

Elements of a cuboid

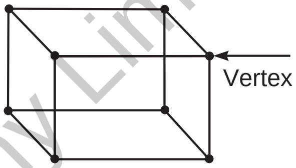
- A cuboid has 6 faces.
- Any two adjacent faces of a cuboid meet in a line segment, called the **edge** of the cuboid. There are **12 edges** in a cuboid.



6 faces
(a)



12 edges
(b)



8 vertices
(c)

Fig. 6.4

- The point of intersection of the three edges of a cuboid is called the **vertex**. There are 8 vertices in a cuboid.
- A cuboid has three distinct dimensions known as length, breadth (or width) and height.

> **Note**
>
> Each face of a cuboid has 4 edges and 4 vertices or corners.

Cube

A cuboid in which length, width and height are equal is called a **cube**.

A die, a sugar cube, an ice cube, etc., are all examples of the shape of a cube.

The playing die

The sugar cube

Fig. 6.5

Elements of a cube

A cube has 6 square faces, 12 edges and 8 vertices.

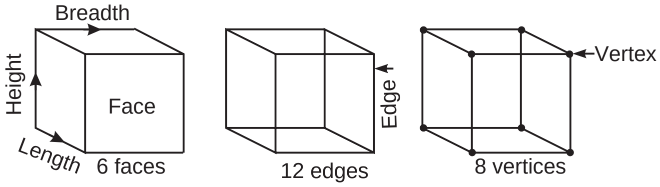


Fig. 6.6

Cylinder

A solid having a curved surface with circular ends is called a **cylinder**.

A circular pipe, gas cylinder, measuring jar, garden roller, etc., are all examples of a cylinder.

Fig. 6.7

Elements of a cylinder

- A cylinder has a curved surface and two circular faces.
- It has no vertex. It has two circular edges and no straight edge.

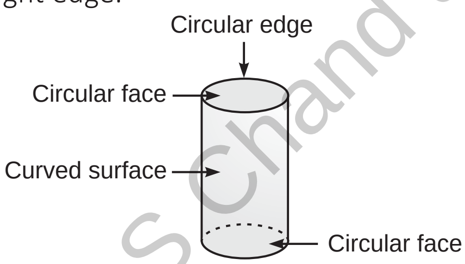


Fig. 6.8

Cone

A **cone** looks like the cap of a circus clown. It has a **curved surface** and **one circular face**.

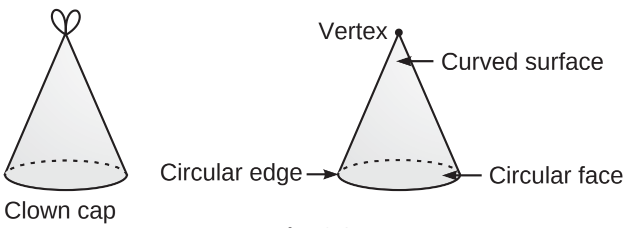


Fig. 6.9

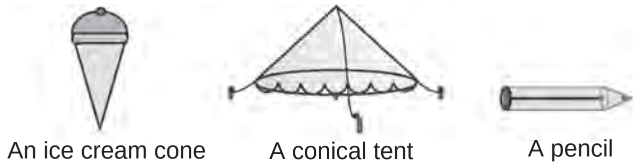


Fig. 6.10

An ice cream cone, a conical tent, the tapered end of a pencil are in the shape of a cone.

Sphere

A **sphere** is a solid shape bounded by a curved surface.

Fig. 6.11

A football, round marble, etc., are in the shape of a sphere.

Elements of a sphere

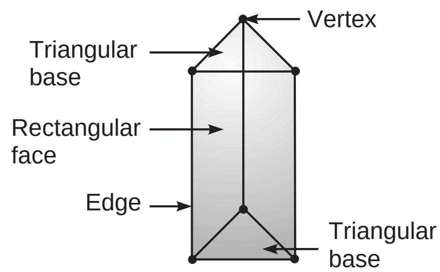
- A sphere has no edges and no vertices.
- It has only one curved surface.

Prism

A **prism** is a solid with two identical polygonal bases and rectangular lateral faces. A prism is identified on the basis of its base.

For example, a prism in which the two bases are triangles is called a **triangular prism** [Fig. 6.12].

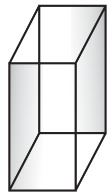
- A triangular prism has 6 vertices, 3 rectangular faces, 2 triangular faces (bases) and 9 edges.



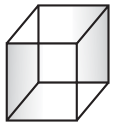
Triangular prism

Fig. 6.12

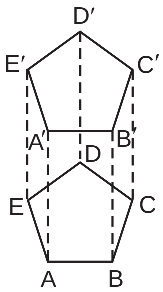
- A cuboid is also a prism with bases as rectangles. It has 8 vertices, 6 rectangular faces and 12 edges.
- A cube is also a prism with the base as a square. It has 8 vertices, 6 square faces and 12 edges.



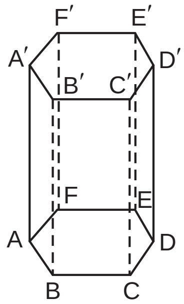
(a) Rectangular prism



(b) Square prism or cube



(c) Pentagonal prism



(d) Hexagonal prism

Fig. 6.13

Pyramid

A **pyramid** is a solid geometric figure that has a single polygonal base and whose side or lateral faces are triangles having a common vertex, called the **vertex** of the pyramid.

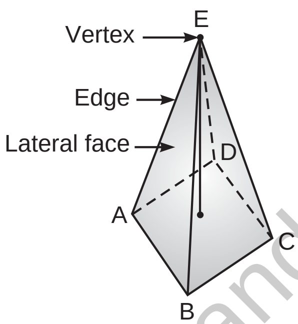
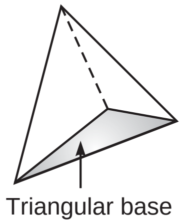


Fig. 6.14

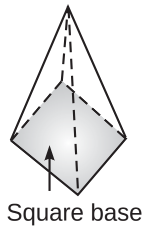
- The side faces of a pyramid are called its **lateral faces**.
- The pyramid whose base is a triangle is called a **triangular pyramid** or **tetrahedron** [Fig. 6.15 (a)].

  It has 4 vertices, 4 faces and 6 edges.
- The pyramid whose base is a square is called a **square pyramid** [Fig. 6.15 (b)].

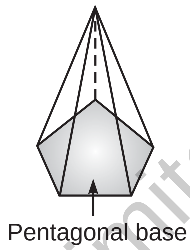
  It has 5 faces, 5 vertices and 8 edges.
- The pyramid whose base is a pentagon is called a **pentagonal pyramid** [Fig 6.15 (c)]. It has 6 faces, 6 vertices and 10 edges.



(a) Triangular pyramid
(Tetrahedron)



(b) Square pyramid



(c) Pentagonal pyramid

Fig. 6.15

Let us study some more examples.

Ex. 1. **Count the number of visible faces in the given diagram.**

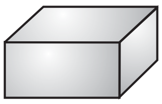


Fig. 6.16

Sol. Observe the given figure.

The given figure has three visible faces.

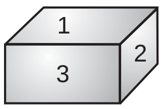


Fig. 6.17

Ex. 2. **In the given shape, find the number of faces, edges and vertices.**

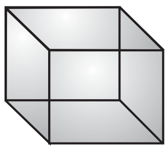


Fig. 6.18

Sol. By observing the given shape, we have

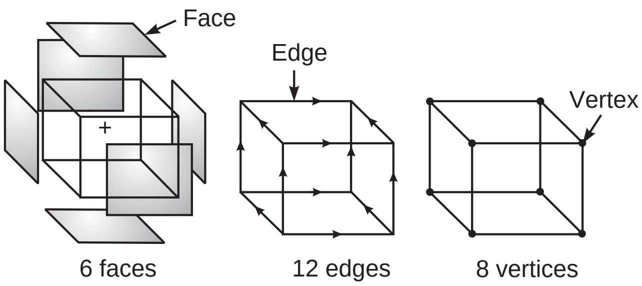


Fig. 6.19

Thus, the given shape has 6 faces, 12 edges and 8 vertices.

Exercise 6.1

1. Match the following.

Shapes	Objects
(a) Sphere	(i) A soft drink can
(b) Cylinder	(ii) A cricket ball
(c) Cuboid	(iii) A dice
(d) Cube	(iv) A chalk duster

2. Answer the following questions.

(a) What is the other name for a square prism?

(b) Which solid shape has no vertices?

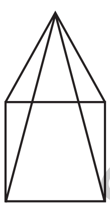
(c) A football is an example of which solid figure?

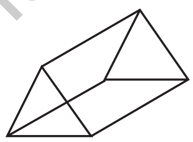
(d) Write the number of faces, edges and vertices of a tetrahedron.

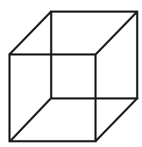
3. Complete the following table.

S.No.	Shape	Number of Edges	Number of Vertices	Number of Faces
(a)	Cuboid			
(b)	Cone			
(c)	Cylinder			
(d)	Triangular pyramid			
(e)	Pentagonal pyramid			

4. Find the number of faces, edges and vertices in each of the following figures.

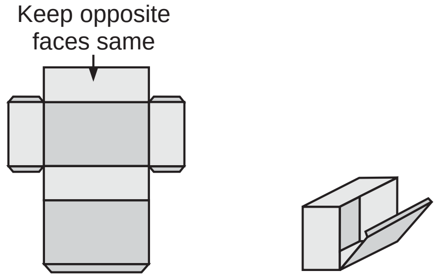
(a)


(b)


(c)


NETS OF SOLIDS

A **net** is a flat shape (2D) which can be folded to form a 3D shape. The adjoining figure shows the net of a cuboid. Most of the solids have their nets. Some solids can have more than one type of nets. Let us study to identify more than one net for a solid with the help of the following activity.



(a) Net of a cuboid (b) Folding of net

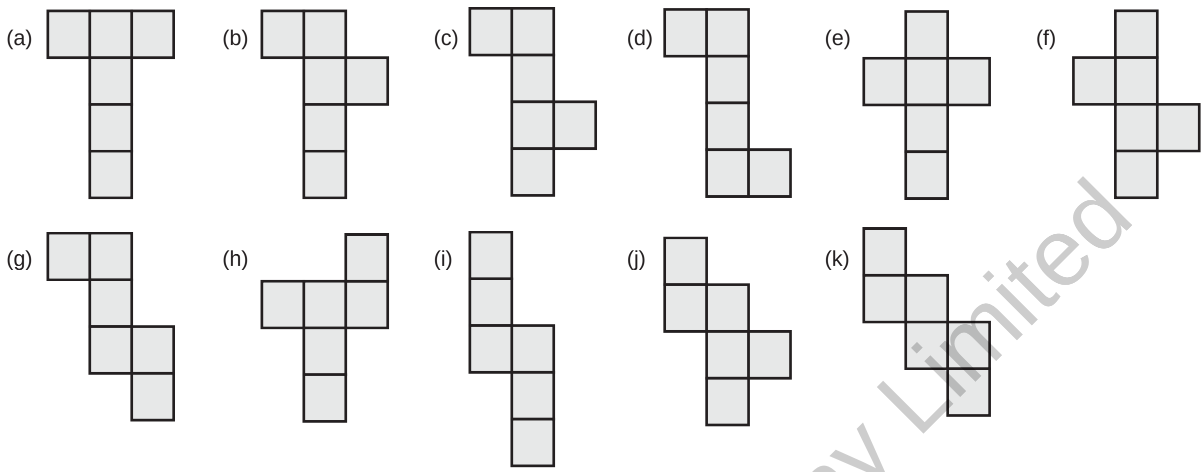
Fig. 6.20

Let Us Do

AIL

Objective: To identify the nets of a cube

Materials required: Cut-outs of all possible nets consisting of six squares of equal dimensions, glue, etc.



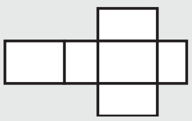
Procedure: **Step 1:** Take each cut-out of nets one by one and fold them along the edges.

Step 2: Separate the nets making cubes. Reject others.

Step 3: Use glue to join the edges to form cube.

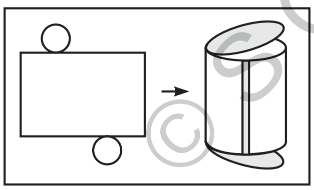
Note

To create nets of a cuboid, rectangles of equal dimensions can be used for opposite faces.
For example,

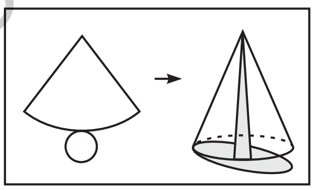


Nets of Some more Solids

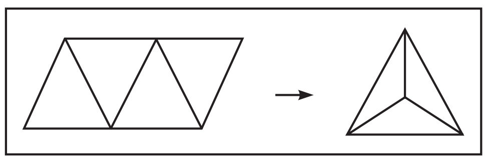
Let us study the nets of some other solids.



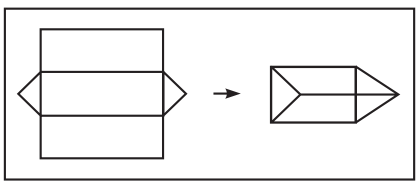
(a) Net of a cylinder



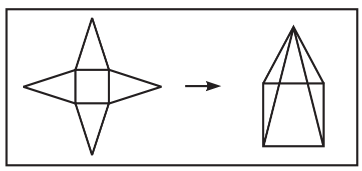
(b) Net of a cone



(d) Net of a triangular pyramid (tetrahedron)



(c) Net of a triangular prism

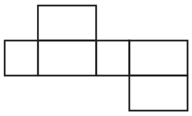


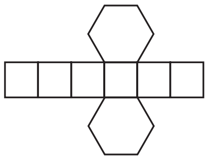
(e) Net of a square pyramid

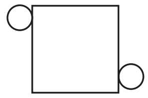
Fig. 6.21

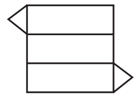
Exercise 6.2

1. Identify the 3D shape that can be formed by folding the following nets.

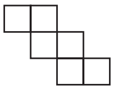
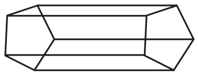
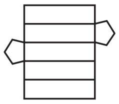
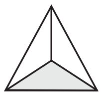
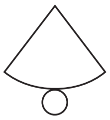
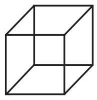
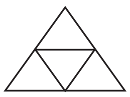
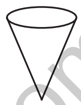
(a) 

(b) 

(c) 

(d) 

2. Match the following nets with their solids and names.

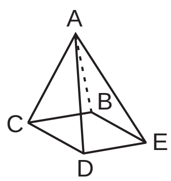
Nets		Solids		Names
(a)		(i)		(A) Tetrahedron
(b)		(ii)		(B) Cone
(c)		(iii)		(C) Cube
(d)		(iv)		(D) Pentagonal prism

Competency Based Exercise

21st CS

1. Tick (✓) the correct answer.

(a) What do the letters in the adjoining figure represent?



(i) Pyramid (ii) Faces

(iii) Vertices (iv) Edges

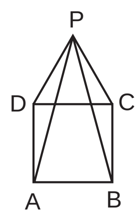
(b) The sum of the number of faces, vertices and edges of a triangular pyramid is:

(i) 14 (ii) 10 (iii) 6 (iv) 4

(c) Which of the following solids has a curved surface?

(i) Cube (ii) Cuboid (iii) Prism (iv) Cone

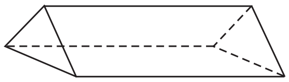
(d) In the given figure, a point P in the space (*i.e.*, not lying in the plane of ABCD) has been joined with the vertices A, B, C and D of a square ABCD. The figure so formed has:

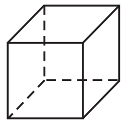


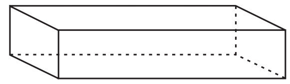
(i) 8 edges and 5 faces. (ii) 6 edges and 6 faces.

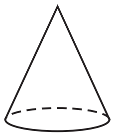
(iii) 4 edges and 8 faces. (iv) 3 edges and 9 faces.

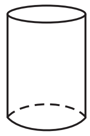
2. **Observe the following solids and count the number of faces, edges and vertices in each.**

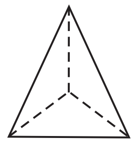
(a) 

(b) 

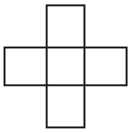
(c) 

(d) 

(e) 

(f) 

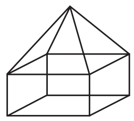
3. Which 3D shape will be formed by folding this net?



4. **Identify the solids which the following figures resemble.**
   (a) A juice pack (b) A tennis ball (c) A chalk box (d) A water bottle
5. **Write the number of triangles in the nets of a:**
   (a) triangular prism (b) square pyramid
6. Are all square prisms cubes? Give explanation in support of your answer.

Challenge! 21st CS

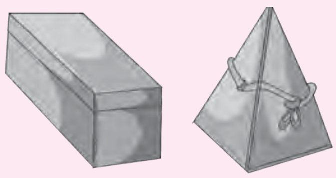
1. What shape would you get if you cut and open a birthday cap?
2. Which solid shape does not have any net?
3. Which two solid shapes are combined to form this adjoining shape? Also, find the total number of faces and edges in the given figure.



CASE STUDY

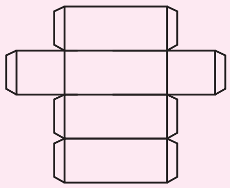
CC

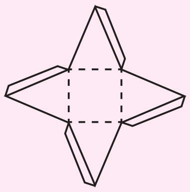
Amitabh bought some chocolates and a pair of shoes on Father's Day. The shape of the shoe box and chocolate box are shown in the adjacent picture. He wants to wrap these boxes nicely. He and his younger brother Anuj bought packing papers and cut the nets to pack the gift boxes perfectly.



Anuj saw the perfect packing and screamed with joy. Amitabh guided Anuj to packing process by making him answer the following questions:

1. What is the shape of shoe box and chocolate box?
2. How many faces, vertices and edges are there in the shoe box?
3. How many faces, vertices and edges are there in the chocolate box?
4. Which of the following nets of packing paper to be used for shoe box and chocolate box?





Let's Work in Mind

21st CS

1. Name the 3D shape(s) which do not have any vertices.
2. Can you name two shapes that have only flat faces?
3. Give two examples of a cube.
4. What shape does the lateral face of a prism represent?

ASSERTION – REASONING QUESTIONS

21st CS

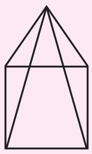
Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

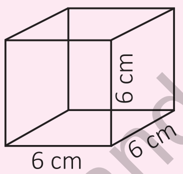
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A) :**  is a square pyramid.

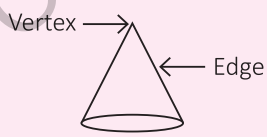
   Reason (R) : A pyramid whose base is a triangle is called a square pyramid.

2. **Assertion (A) :** 

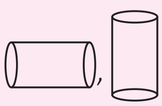
is a cube.

   Reason (R) : A cuboid in which length, breadth and height are equal is a cube.

3. **Assertion (A) :** A cone has one vertex and two edges.

   Reason (R) : 

4. **Assertion (A) :** A cylinder has two circular faces.

   Reason (R) :  has two circular edges.

5. **Assertion (A) :** The total number of vertices in a hexagon are 6.

   Reason (R) : Hexagon is a 6-sided figure.

6. **Assertion (A) :** Pentagon has five interior angles.

   Reason (R) : Pentagon has five sides.

7

Integers

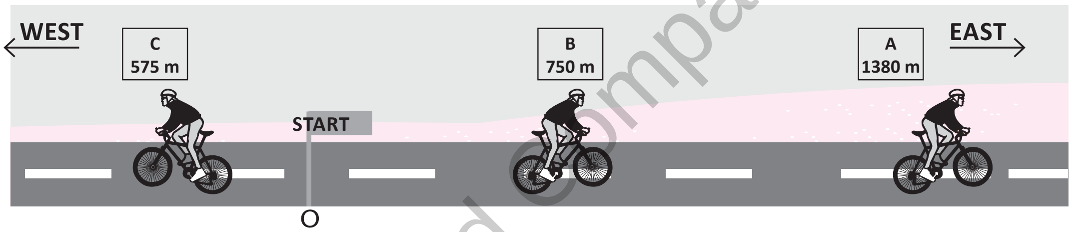
What Learners Will Achieve

- understand the need of negative numbers and integers.
- represent the integers on a number line.
- compare and order integers.
- perform addition and subtraction operations on integers.
- represent sum and difference of two integers on a number line.

EL

HOW NEGATIVE NUMBERS ARISE?

A road is in the East-West direction.



Three cyclists A, B and C start at the same point O on the road. A and B ride towards the **East** direction whereas C rides towards the **West**. Their location on the road after ten minutes is as follows:

- **Cyclist A:** At a distance of 1380 m from O.
- **Cyclist B:** At a distance of 750 m from O.
- **Cyclist C:** At a distance of 575 m from O.

How far are the three cyclists from each other?

A student of class VI-A solved the problem and gave the following answer:

1. Distance of A from B = 1380 m – 750 m = 630 m

2. Distance of A from C = 1380 m – 575 m = 805 m

3. Distance of B from C = 750 m – 575 m = 175 m

Do you agree with the above answers?

> **DID YOU KNOW?**
>
> Negative numbers were first used by the Indians in the 7th century.
>
> They used it for bookkeeping (maintaining accounts), positive quantities for assets and negative for debts. **'Brahmagupta'** famous astronomer in about 630 AD has also used negative numbers in his work.

To know whether the above solution is right or wrong, we need to define a new group or category of numbers, which can indicate the opposite situations like up and down, East and West, right and left, more and less, profit and loss, etc. We call this category of numbers as **integers**.

Integers are all natural numbers, zero and negative of all natural numbers put together. Negative numbers may represent the idea of being below a standard value.

In this chapter, we shall learn about integers, their representation on a number line, operations of addition and subtraction on integers and use of integers in solving problems related to our day-to-day life.

INTEGERS

- **Natural numbers** are 1, 2, 3, 4, ...
- **Whole numbers** are 0, 1, 2, 3, ...
- Numbers with a negative sign are less than zero (0) and are called **negative numbers**.
- The numbers –1, –2, –3, ... together with whole numbers 0, 1, 2, 3, ... form a new collection of numbers, *i.e.,* ... –3, –2, –1, 0, 1, 2, 3, ... called **integers**.
- The largest or the smallest integer cannot be determined.

Representation of Integers on the Number Line

Draw a line. Mark a point zero (0) on it. Now, mark points at equal (unit) distances to the right of 0.

Label them successively as 1, 2, 3, ... Again, mark points at the same unit distances to the left of 0 and label them successively as –1, –2, –3, ... (see Fig. 7.1). In order to mark +3 or simply 3, move 3 steps to the right of 0.

In order to mark –3, move 3 steps to the left of 0.

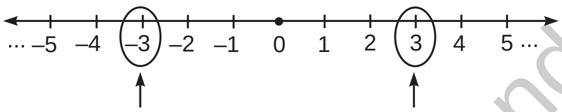


Fig. 7.1

This way, we can represent every integer on the number line.

Positive and Negative Integers

The positive integers are the integers that lie to the right of zero (0) on the number line, *i.e.*, 1, 2, 3, 4, ... are all **positive integers**.

The negative integers are the integers that lie to the left of zero (0) on the number line, *i.e.*, –1, –2, –3, –4, ... are all **negative integers**.

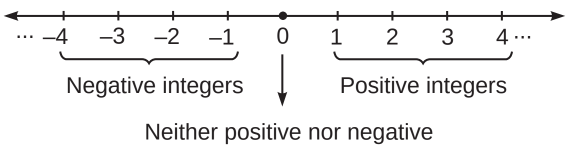


Fig. 7.2

Remember

The number zero is neither positive nor negative.

Successor and Predecessor of Integers

Successor

On a number line, if a movement of only 1 unit is made to the right of a number, we get the successor of the number.

Illustration 1: Let us find the successor of 2 and –4.

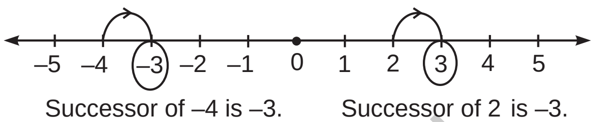


Fig. 7.3

From Fig. 7.3, it is clear that, the successor of 2 is 3 and that of –4 is –3.

Predecessor

On a number line, if a movement of only 1 unit is made to the left of the number, we get the predecessor of the number.

Illustration 2: Let us find the predecessor of 2 and –4.

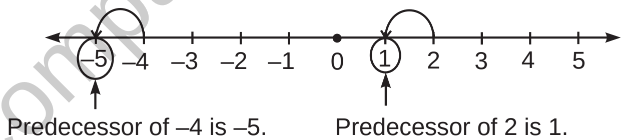


Fig. 7.4

From Fig. 7.4, it is clear that, the predecessor of –4 is –5 and that of 2 is 1.

In general, successor of any integer can be obtained by adding '1' and predecessor of any integer can be obtained by subtracting '1' from given integer.

Skill Check

- What is the successor of –10?
- What is the successor of –1?
- What is the predecessor of 0?
- What is the predecessor of –63?

Opposite of an Integer (or Negative)

In our day-to-day life, there are many situations where we talk about opposites.

The best and the easiest example to understand opposites of integers is your Savings Bank where you store coins. The number of coins added or removed from a Savings Bank are represented by an integer.

For example, if you add five coins to the Savings Bank, this would be represented by the positive integer +5. On the other hand, if you remove five coins from the Savings Bank, your action will be represented by –5.

Here, +5 and –5 are opposites of each other.

> **Note**
>
> Coins cannot be broken into parts and you can only add or remove coins, your action will always be represented by an integer.

Consider the given number line.

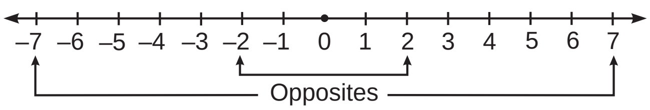


Fig. 7.5

We observe that the numbers 2 and –2 are located at the same distance from 0 on opposite sides.

The same is true for 7 and –7.

We say that 2 and –2 are opposites of each other. Also, 7 and –7 are opposites of each other.

We express these facts as follows:

$-(2) = -2$ and $-(-2) = 2$

$-(7) = -7$ and $-(-7) = 7$

Sometimes, we also say that 2 and –2 are *negatives* of each other, 7 and – 7 are *negatives* of each other, etc.

> **Note**
>
> The opposite of 0 is 0, that is, – (0) = 0.

Illustration 3: The symbol "–" before a number is used in two different ways.

1. **27 – 12 means**: Subtract 12 from 27 or from 27 subtract 12.
2. **–12 or – (12) means**: The opposite of 12 or negative of 12.

To find the 'opposite' of a number, we refer to the number line in the above figure (Fig. 7.5).

We find that:

- the opposite of a **positive** integer is a **negative** integer.
- the opposite of a **negative** integer is a **positive** integer.
- the opposite of 0 is 0 itself.

Applications of Integers

In daily life situations, *positive and negative integers,* are commonly used to represent quantities with opposite characteristics.

Negative	Zero	Positive
Left of zero (–7)	(0)	Right of zero (8 or + 8)
Loss (– ₹74)	Break even (No loss, no gain) (₹0)	Profit (₹250)
Below sea level height (–572 feet)	At sea level (0 ft)	Above sea level height (3200 feet)
Fall in price (– ₹1.25)	No change (₹0)	Rise in price (₹2.50)
Temperature below zero (–4°C)	At zero temp (0°C)	Temperature above zero (32°C)

Table 7.1

Most of the quantities can be measured with integers. Some values, however, are expressed with non-negative numbers only. Cost and measurement are two such quantities where negatives do not make any sense.

For example, the area of the lake is 4 sq km.

The cost of the table is ₹390.

Let us study some more examples.

Ex. 1. **Find the opposite of the integer 5.**

Sol. Since 5 is 5 units on the right of 0, its opposite is 5 units on the left of 0.

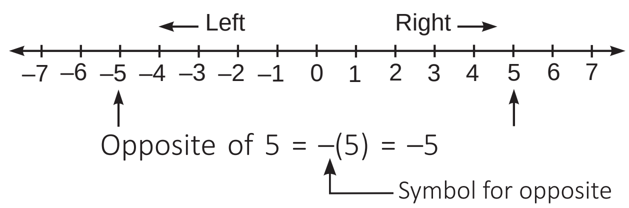


Fig. 7.6

> **Remember**
>
> To find the opposite of a non-zero integer, we simply change its sign.

Ex. 2. Write the numbers in the following situations with appropriate sign:

(a) 100 m below sea level

(b) A gain of ₹600

Sol. (a) 100 m below sea level.

Since it is below sea level, the sign will be negative.

Therefore, it can be written as –100 m.

(b) A gain of ₹600.

A gain indicates that the sign will be positive.

Therefore, it can be written as +₹600.

Ex. 3. Represent the integers –4, –1, 3, 5, –2 on the number line.

Sol. Draw a number line and show the points corresponding to the integers –4, –1, 3, 5 and –2 by the dots.

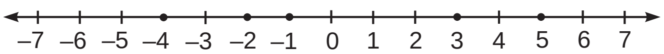


Fig. 7.7

Ex. 4. On which integer will we reach if we move 4 units to the right of –2 on the number line?

Sol. Draw a number line and mark –2 on it.

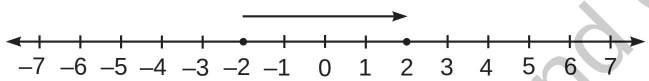


Fig. 7.8

Moving 4 units to the right of –2, we reach at 2.

Ex. 5. Write the opposite of 'increase in weight'.

Sol. The opposite of 'increase in weight' is 'decrease in weight'.

Ex. 6. Observe the given number line.

Fig. 7.9

If point D represents +9, then which point represents opposite of 9?

Sol. Given, D represents +9. So, we write the integers on the number line.

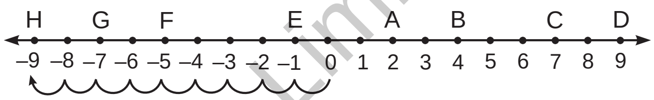


Fig. 7.10

Moving 9 units to the left side of 0, we reach H.

Therefore, the point H represents –9.

Ex. 7. In the given pair (–1, 3) of integers, which integer is on the right of the other on the number line?

Sol. First, we draw the number line and mark the given integers.

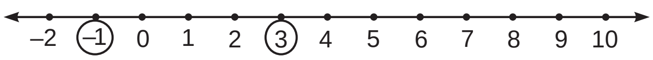


Fig. 7.11

From the number line, we observe that 3 is on the right side of –1.

Exercise 7.1

1. Tick (✓) the correct answer.

(a) The opposite of "10 km east" is:

(i) 10 km north (ii) 10 km west (iii) 10 km east (iv) 10 km south

(b) The number just to the left of –9 is:

(i) –8 (ii) –10 (iii) –11 (iv) –7

(c) Which of the following numbers will be 4 units on the right of –10?

(i) 14 (ii) 6 (iii) –6 (iv) –14

2. Write the opposite of the following integers.

(a) 5 (b) –7 (c) 15 (d) 0

3. Write the predecessor of the following integers.

(a) –7 (b) 0 (c) 7 (d) –11

4. **Write the successor of the following integers.**

(a) 5 (b) –8 (c) 0 (d) –20

5. **Write the opposite of the following.**

(a) Expenditure of ₹1000 (b) 50 km North

(c) 5°C temperature falls (d) Won by 2 seconds

6. **Express using numbers with appropriate sign.**

(a) A gain of ₹500 (b) 4 km below sea level

(c) 15°C below the freezing point (d) Gaining weight of 7 kg

7. **Represent the following integers on the number line.**

(a) –1 (b) 3 (c) –7 (d) 5 (e) –5

8. **How many integers are there between:**

(a) –7 and –4 (b) 0 and 10 (c) –6 and 7 (d) –5 and 5

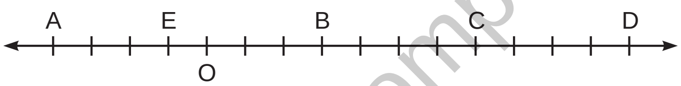
9. **Which number will we reach on the number line, if we move:**

(a) 6 units to the left of 2? (b) 5 units to the right of –4?

10. **Using a number line, write the integer which is:**

(a) 4 more than –7 (b) 3 less than –5 (c) 7 less than 4 (d) 5 more than –2

11. **Observe the given number line and answer the following questions. [Given, O represents the number zero.]**



(a) Which integer corresponds to the point B? (b) Which point corresponds to –1?

(c) Which point corresponds to –4? (d) Which integer corresponds to the point C?

12. **In the given pair of integers, which one is on the right of the other integer on the number line?**

(a) 0, –1 (b) –2, –3 (c) –8, 3

COMPARING AND ORDERING INTEGERS

Integers can be ordered (or compared) in the same way as we order whole numbers with the help of a number line. Recall that on a number line, if a whole number ***a*** lies on the left of the whole number ***b***, then we say that $a < b$. Likewise, if b lies on the right of a, we say that $b > a$. Same is true with the integers also.

To compare two integers, we mark (graph) them on a number line.

The number on the right is greater than the number on the left; equivalently, the number on the left is smaller than the number on the right. Some inequality relationships can be observed on the number line.

Illustration 1: Let us compare the integers using the symbol > or <.

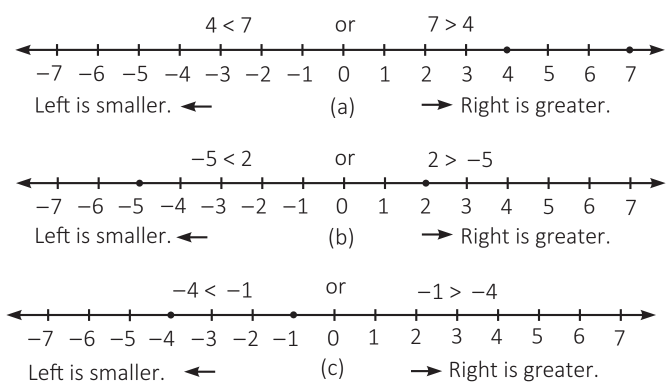


Fig. 7.12

We observe that:

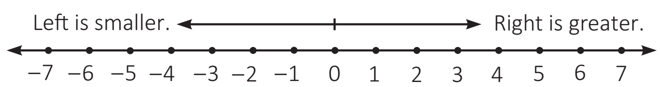


Fig. 7.13

- All positive integers are greater than 0, since they all lie to the right of zero on the number line.
- All negative integers are smaller than 0, since they all lie to the left of zero on the number line.
- Each negative integer is smaller than every positive integer or every positive integer is greater than every negative integer.
- Inequality between two integers gets reversed, if we change those two integers to their opposites.

Illustration 2: **2 < 4 and –2 > –4**

Explanation:

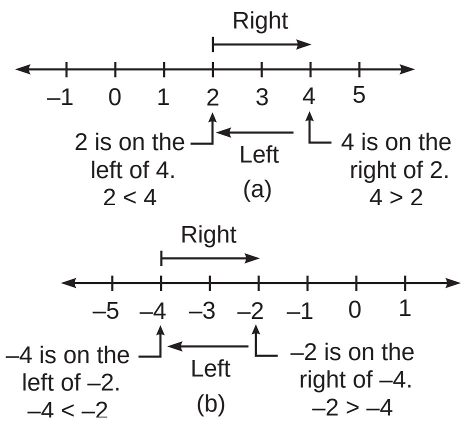


Fig. 7.14

Absolute Value of an Integer

Observe the given number line.

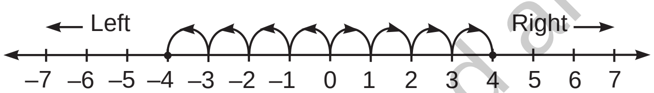


Fig. 7.15

On the number line, distance between 0 and 4 can be measured as 4 units. Distance between 0 and –4 can also be measured as 4 units.

So, we can say that the distance of any integer '*a*' on the number line from '0' is *a* units, irrespective of whether the position of '*a*' is on the left side or right side of '0'.

This distance of integer '*a*' from origin, *i.e.*, '0' is also known as 'absolute value of an integer.'

In general, it can be stated as follows:

The **absolute value** of an integer is its **numerical value** regardless of its sign. It indicates its **size** or **magnitude**. So, the absolute value is either zero or positive. It is never negative. We denote the absolute value of a number by '| |' sign.

Thus, |0| = 0; |–3| = 3; |6| = 6,

whereas –|30| = –30 or –|–30| = –30.

Note

$x > 0$; $x = 0$; $x < 0$, where x is any integer.

Think

Why absolute value of integers is always positive?

Let us study some more examples.

Ex. 8. **Determine, whether –6 < –9 is true or false?**

Sol. Draw a number line.

Fig. 7.16

By locating the numbers –6 and –9 on the number line, we find that –6 is on the right of –9. Therefore, –6 > –9.

Hence, –6 < –9 is false.

Ex. 9. **Write all the integers between –30 and –23 in increasing order.**

Sol. Mark –30 and –23 on the number line.

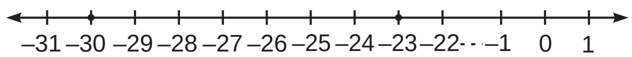


Fig. 7.17

The integers included between –30 and –23 in the increasing order are –29, –28, –27, –26, –25, –24.

Ex. 10. **Arrange the integers: 7, –8, 8, 3, 11, –7 in descending order.**

Sol. On the number line moving right to left means moving bigger to smaller. Therefore, the given integers in descending order are 11, 8, 7, 3, –7, –8.

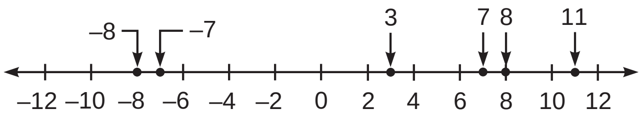


Fig. 7.18

Exercise 7.2

1. **Determine, whether the given statements are True (T) or False (F).**
   (a) $-7 < 0$ (b) $10 > -10$ (c) $-5 < -6$ (d) $-9 > -10$
2. **Compare the following integers using <, = or >.**
   (a) –10 □ 10 (b) 8 □ –8 (c) –5 □ 0 (d) 0 □ +3
   (e) +6 □ 6 (f) –7 □ –15 (g) +1 □ –1 (h) 4 □ –11
3. **Write all the integers between the given pairs in increasing order.**
   (a) –4 and 5 (b) –11 and –3 (c) –3 and 0 (d) –6 and 2
4. **Arrange the following integers in ascending order.**
   (a) –4, 0, 7, –5, 4, 10, –7 (b) 8, –6, 7, 0, –2, –4 (c) –15, 10, –8, 9, –6, –10
5. **Arrange the following integers in descending order.**
   (a) 6, –4, 0, –3, –2, 1 (b) 16, 1, 0, –14, –3, –12 (c) +11, –8, 17, 13, –2, –6
6. **Find the absolute value of the following integers.**
   (a) –5 (b) 7 (c) 0 (d) –11

OPERATIONS ON INTEGERS

We already know how to perform the operations of addition and subtraction on whole numbers. In this section, we shall learn to perform these operations on integers also.

Addition of Integers

Addition of integers on a number line

Recall that on a number line, numbers to the right of zero are positive integers and numbers to the left of zero are negative integers.

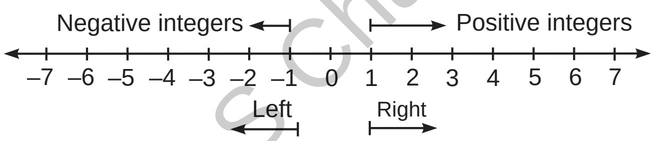


Fig. 7.19

We observe that as we move to the right on a number line, the number increases but the number decreases as we move to the left.

You may also recall that to add whole numbers 2 and 6 on the number line, we first move 2 steps to the right from 0 reaching 2 and then move 6 steps to the right of 2 reaching 8 [see Fig. 7.20].

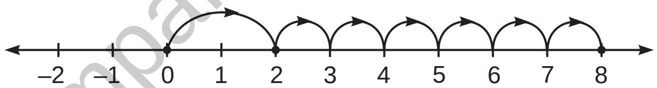


Fig. 7.20

Thus, 2 + 6 = 8.

We follow the same procedure for adding two integers on the number line, except with the difference that:

(a) if the first integer is a negative integer (say –2), then we first move 2 steps to the **left** (instead of **right** of 0) to reach –2.

(b) if the second integer is again a negative integer (say –6), then we move 6 steps to the **left** (instead of **right**) of –2.

Illustration 1: (a) To add (–2) and (–6) on the number line, we first move 2 steps to the left from 0 reaching –2, then move 6 steps to the left of (–2) reaching (–8).

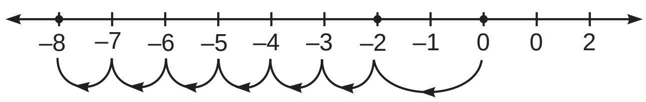


Fig. 7.21

Thus, (–2) + (–6) = –8.

(b) To add 2 and (–6) on the number line, we first move 2 steps to the **right** from 0 reaching 2, then move 6 steps to the **left** of 2 reaching (–4).

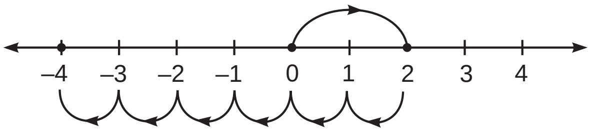


Fig. 7.22

Thus, 2 + (–6) = –4.

(c) To add (–2) and 6 on the number line, we first move 2 steps to the **left** from 0 reaching (–2), then move 6 steps to the **right** from (–2) reaching 4.

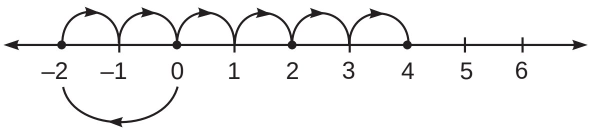


Fig. 7.23

Thus, –2 + 6 = 4.

> **Note**
>
> On a number line, adding a positive number means moving "to the right" or in the positive direction. Adding a negative number means moving "to the left" or in the negative direction.

Addition of integers with like signs without using a number line

Using a number line, we have already seen that 2 + 6 = 8 or (+2) + (+6) = (+8) and (–2) + (–6) = (–8).

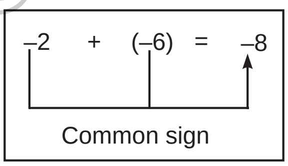
We can also find the sum of –2 and –6 without using a number line. To do that, we first recall the concept of *absolute value of an integer*.

Absolute value of –2 = |–2| = 2

Absolute value of –6 = |–6| = 6

Adding these absolute values,

|–2| + |–6| = 2 + 6 = 8.



Now, we attach the sign common to both the integers, *i.e.*, minus (–) here to the sum obtained.

Thus, (–2) + (–6) = –8.

From the above explanation, we observe that:

To add two or more integers with like signs, follow these steps:

Step 1: Add the absolute values of the integers.

Step 2: Attach the common sign to the sum obtained in Step 1.

Illustration 2: Let us find the sum (–15) + (–5).

We have, |–15| = 15, |–5| = 5

|–15| + |–5| = 15 + 5 = 20

Thus, (–15) + (–5) = –20 (Common sign of the given integers is '–').

Addition of integers with unlike signs (without using a number line)

Using a number line, we have seen that 2 + (–6) = –4 and –2 + 6 = 4.

We can also find the sum of 2 and –6 without using a number line.

- Find the absolute value of the given integers. |2| = 2, |–6| = 6
- Find the difference of the larger absolute value and the smaller absolute value. 6 – 2 = 4
- Attach the sign of the integer with the larger absolute value to the difference. Here, sign of –6 is '–'.

Thus, 2 + (–6) = –4.

Similarly, we also find the sum (–8) + 12.

We have, |–8| = 8 and |12| = 12.

|12| – |–8| = 12 – 8 = 4

Since |12| > |–8|, therefore, (–8) + 12 = 4.

From the above explanation, we observe that:

To add two integers with unlike signs, follow these steps:

Step 1: Find the absolute values of two integers.

Step 2: Subtract the smaller absolute value from the larger one.

Step 3: Prefix the sign of the integer that has the larger absolute value to the difference.

> **Note**
>
> It is obvious from the discussion that the sum of a number and its own opposite is 0, *i.e.*, 5 + (–5) = 0, (–10) + (+10) = 0, etc.

Let Us Do

EL

Understanding Operations on Integers

Objective: To explore that the sum of:

(a) two positive integers is positive.

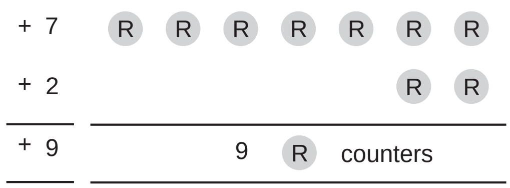
(b) a negative integer and a positive integer may be positive or negative.

Materials required: 50 counters of red colour, 50 counters of blue colour

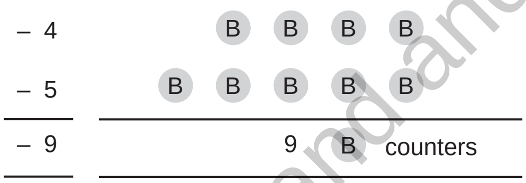
Procedure: Let us consider 'Red' colour counters as '+' integer or (R) = +1 and 'Blue' colour counters as '–' integer or (B) = –1 and make the following rule.

One red counter can cross one blue counter or vice versa, as (+1) + (–1) = 0.

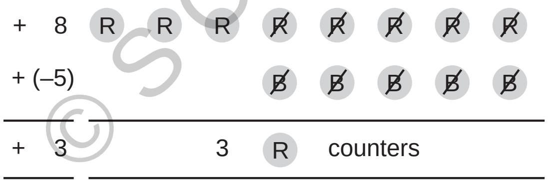
Step 1. Choose 7 red counters and 2 red counters to get 7 + 2.



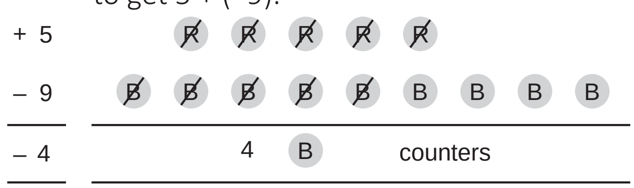
Step 2. Choose 4 blue counters and 5 blue counters to get (–4) + (–5).



Step 3. Choose randomly 8 red counters and 5 blue counters, to get 8 + (–5).



Step 4. Choose 5 red counters and 9 blue counters to get 5 + (–9).



Note

Any two different colour counters can be chosen to represent positive and negative integers.

Repeat the experiment with at least 10 different pairs of integers and complete the following table.

S. No.	A	B	Result
1.	+8	–5	(+8) + (–5) = (+3)
2.	+7	+2	(+7) + (+2) = (+9)
3.	–4	–5	(–4) + (–5) = (–9)
4.	5	–9	(5) + (–9) = (–4)
5.			
6.			
7.			
8.			
9.			
10.			

Conclusion:

(a) (+) + (+) = (_________)

(b) (+) + (–) = (+)

When the absolute value of (+) integer is _____ than the absolute value of (–) integer.

(c) (+) + (–) = (–)

When the absolute value of (+) integer is _____ than the absolute value of (–) integer.

(d) (–) + (–) = (_________)

Properties of Addition of Integers

Commutative property

Addition is commutative for integers. That means, we can add integers in any order.

Consider the following additions:

8 + (–3) = 5 and (–3) + 8 = 5

So, 8 + (–3) = (–3) + 8.

Therefore, the order of addends makes no difference. This is known as the **order** or **commutative property** of integers.

Associative property

Addition is associative for integers. That is, to find the sum of three integers, we can group them in any manner.

Illustration 1: Consider the following addition:

[(–2) + 5] + (–8) = 3 + (–8)

= –(8 – 3) [∵ |3| = 3, |–8| = 8]

= –5

and (–2) + [5 + (–8)] = (–2) + (–3) [∵ |5| = 5, |–8| = 8]

= –(2 + 3) = –5 [∵ |–2| = 2, |–3| = 3]

So, [(–2) + 5] + (–8) = (–2) + [5 + (–8)].

We can use both commutative and associative properties to see that even if we first add –2 and –8 and then add 5 to the sum obtained, we will get the same answer.

[(–2) + (–8)] + 5 = –(2 + 8) + 5

= (–10) + 5 = –5

[∵ |–10| = 10, |5| = 5]

We conclude that, to find the sum of three (or more) integers, we can group them in any manner. This is known as **grouping** or **associative property** of integers.

Illustration 2: Let us find the sum of –41 + 35 + (–60) + 50.

We have, –41 + 35 + (–60) + 50

= –41 + (–60) + 35 + 50

First, add the *negative* integers, *i.e.*,

–41 + (–60) = –(41 + 60) = –101

Next, add the *positive* integers, *i.e.*, 35 + 50 = 85.

So, –41 + 35 + (–60) + 50 = (–101) + 85

= –(101 – 85) = –16

Thus, –41 + 35 + (–60) + 50 = –16.

Identity property

The number 0 is the additive identity for integers, because the sum of an integer and 0 is the same integer.

Illustration 3: 0 + (–2) = –2; (–2) + 0 = –2

So, 0 + (–2) = (–2) + 0.

Adding a Group of Integers

To add a group of integers, follow these steps:

Step 1: Rearrange the integers so that the positive integers and negative integers are grouped together.

Step 2: Add all the positive integers and all the negative integers separately.

Step 3: Add the two sums obtained in Step 2.

Let us study some more examples.

Ex. 11. Add the following using a number line.
(a) (–2) + (–3) + (–5) (b) 8 + 4 + (–7)

Sol. (a)

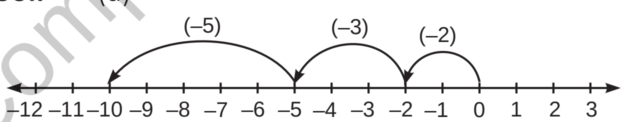


Fig. 7.24

We first move 2 steps left starting from 0 and reach –2. Next, we move 3 steps left starting from –2 and reach –5. Then we move 5 steps left starting from –5 and reach –10. Thus, (–2) + (–3) + (–5) = –10.

(b)

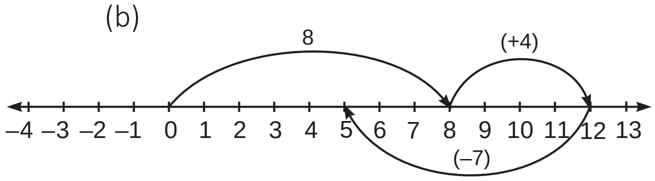


Fig. 7.25

We first move 8 steps right starting from 0 and reach 8. Next, we move 4 steps right from 8 and reach 12. Then we move 7 steps left and finally reach 5.

Thus, 8 + 4 + (–7) = 5.

Ex. 12. Add the sum of (–75) and 40 to the sum of (–100) and (–15).

Sol. We have, (–75) + 40 = –35

and (–100) + (–15) = –115

Now, add the two sums,

i.e., (–35) + (–115) = –150.

Ex. 13. Simplify 37 + (–2) + (–65) + |–8|.

Sol. We have, 37 + (–2) + (–65) + 8 (∵ |–8| = 8)

= 37 + 8 + (–2) + (–65)

(Grouping the integers with like signs)

= 45 + (–67)

= –22

(∵ |45| = 45, |–67| = 67. 45 < 67, so assign the sign of 67)

Exercise 7.3

1. **Add the following integers using a number line.**

   (a) 5 + (+3) (b) 7 + (–2) (c) (–4) + (–6) (d) (–3) + (–4) + (–1) (e) (–4) + (+6) + (–2)

2. **Add:**

   (a) 75 + (–10) (b) (–15) + 37 (c) –11 + (–5)

   (d) –15 + (3) (e) 200 + (+40) (f) (–13) + (–7) + (–50)

3. **Simplify the following.**

   (a) (–8) + (–10) + 15 + (–5) (b) 30 + (–23) + (–15) + |–7|

   (c) (–34) + (90) + |–28| + (–10) (d) 50 – (–3) + (14) + (–7)

4. Find the sum of –10 – (–4) and (–4) – (–10).
5. Find the sum of 10 + (–10) + 10 + (–10) + ..., if the number of terms is 121.
6. **Put >, < or = in the boxes to make the statements true.**

   (a) (+4) + (–2) ☐ (–2) + (+4) (b) (–3) + 0 ☐ (–3) + 1 (c) (–1) + (–2) ☐ (+2) + (–1)

   (d) (–6) + (–1) + (+5) ☐ (–3) + (–4) + (+7) (e) (+5) + (–7) ☐ 2 + (–3) + (–4)

Subtraction of Integers

Additive inverse

When a number is added to its opposite, the sum is always zero.

Since 3 + (–3) = 0 and (–3) + 3 = 0

So, 3 + (–3) = (–3) + 3 = 0.

We say that 3 is the *additive inverse* of –3 or –3 is the *additive inverse* of 3.

Since 3 and –3 are opposites of each other, we find that the additive inverse of an integer is the same as its opposite.

The opposite of an integer is called its **additive inverse**.

Recall that subtraction is the inverse process of addition.

Thus, subtracting an integer means adding the opposite (or additive inverse) of that integer.

Subtraction on the number line

Illustration 1: Let us subtract 3 from 5.

To find 5 – 3 or 5 + (–3), we first move 5 steps right from 0 and then 3 steps left from 5.

Finally, we reach at 2.

Thus, 5 – 3 = 2.

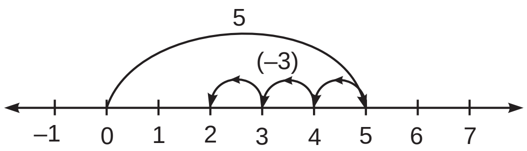


Fig. 7.26

Illustration 2: Let us subtract 4 from –3.

To find (–3) – 4 or (–3) + (–4), we first move 3 steps left from 0. Then we move 4 steps left from –3 and reach at –7.

Thus, –3 – 4 = –7.

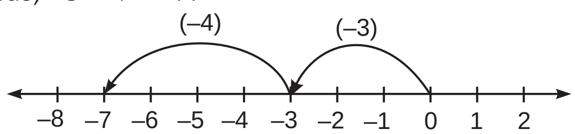


Fig. 7.27

We observe that **subtracting a positive integer from the given integer means to move left from that integer or in the negative direction**.

Illustration 3: Let us subtract (–4) from –3.

To find –3 – (–4) or –3 + (+4), we first move 3 steps left from 0 and reach at –3. Then, we move 4 steps right from –3 and reach at 1. Thus, –3 – (–4) = 1.

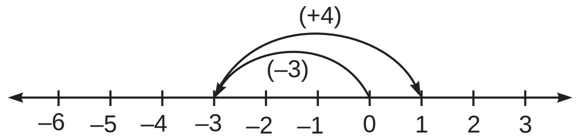


Fig. 7.28

Illustration 4: Let us subtract (–6) from 2.

To find 2 – (–6) or 2 + (+ 6), we first move 2 steps right from 0 and reach at 2.

Then we move 6 steps right from 2 and reach 8.

Thus, 2 – (–6) = 8.

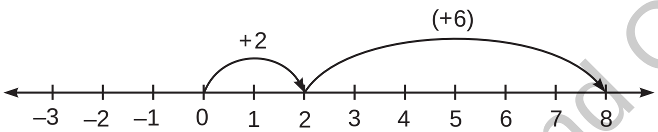


Fig. 7.29

From the above explanation, we observe that **subtracting a negative integer from the given integer, means to move right from that integer or in the positive direction**.

Skill Check

- Subtract: (a) 2 from 0 (b) –1 from 1 (c) 2 from –2.
- The predecessor of predecessor of –5 is ______.
- If '*a*' is an integer, then *a* + (–*a*) = ______.

Watch Your Step!

Subtracting –2 from 0, gives the difference 2 not –2.
As 0 – (–2) = 0 + (2) = 2.

Subtraction of two integers without using a number line

Illustration 5: Let us consider the following subtractions.

Subtract ↓ Add ↓ 14 – 5 → 14 + (–5) = 9 ↑ The opposite of 5 (–5 is the additive inverse of 5.)

Subtract ↓ Add ↓ –5 – 6 → –5 + (–6) = –11 ↑ The opposite of 6 (–6 is the additive inverse of 6.)

Subtract ↓ Add ↓ –7 – (–10) → –7 + (10) = 3 ↑ The opposite of –10 [10 is the additive inverse of (–10).]

From the above explanation, we observe that **to subtract an integer from another integer, add the opposite or the additive inverse of the integer to be subtracted**.

Subtraction of several integers

Illustration 6: Simplify: 5 – 4 – (–9) – 13

= 5 + (–4) + (+9) + (–13)
(change all subtractions to addition)

= (5 + 9) + [(–4) + (–13)]
(combining integers with the same sign)

= 14 + (–17) = –3

We observe that, to simplify numerical expressions involving more than two negative integers, follow these steps:

Step 1: Change each subtraction to addition by adding the opposite (or additive inverse) of the integers that are to be subtracted.

Step 2: Evaluate the sum.

Let us study some more examples.

Ex. 14. If the sum of two integers is –80 and one of them is –90, then find the other integer.

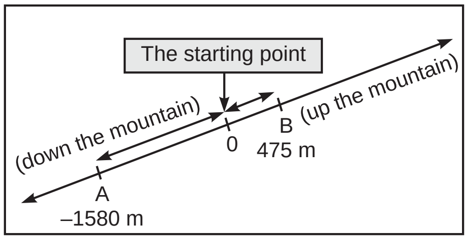
Sol. Sum of two integers = –80
One of the integers = –90
So, another integer = Sum – Given integer
= (–80) – (–90)
= –80 + (+90) = +10
Thus, the other integer is 10.

Ex. 15. Two cyclists start from the same point on a mountain along the road. One travels 1580 m down the mountain along the road in 10 minutes and the other travels 475 m up the mountain along the road in the same time. How far are they from each other after 10 minutes?

Sol. Let the starting point be represented by the number 0, the direction "up the mountain" as the positive direction, the direction "down the mountain" as the negative direction.

Distance travelled by the first cyclist in 10 minutes = –1580 m (down the mountain)

Distance travelled by the second cyclist in 10 minutes = +475 m (up the mountain).



If the positions of the cyclists after 10 minutes is indicated by the points A and B, then the distance between them is equal to the sum of distance from 0 to A and distance from 0 to B

$= |-1580 \text{ m}| + |+475 \text{ m}|$

$= 1580 \text{ m} + 475 \text{ m} = 2055 \text{ m}$

Thus, after 10 minutes, they are 2055 m apart from each other on the road.

Exercise 7.4

1. **Subtract the following, using a number line.**
   (a) 5 – 4 (b) –8 – 2 (c) 6 – (–3) (d) –3 – (–5)
2. **Find the additive inverse of the following.**
   (a) +6 (b) –8 (c) | –4 | (d) | +11 |
3. **Find the value of *x*.**
   (a) $x + 5 = 0$ (b) $5 - x = 0$ (c) $x + (-5) = 0$ (d) $-5 - x = 0$
4. **Subtract:**
   (a) 15 from 42 (b) –10 from –70 (c) –17 from 15 (d) 240 from –300
5. The sum of two integers is –14. If one of the integers is 56, find the other.
6. Two cyclists start from the same point of a mountain. One travels 1500 m downward the mountain in 15 minutes where the other travels 250 m upward the mountain in the same time. How far are they from each other after 15 minutes?
7. **Insert >, < or = in the boxes to make the statements true.**
   (a) 0 + (–10) ☐ 10 + 0 (b) 7 – 4 ☐ 4 – 7
   (c) –8 – (–2) ☐ –2 – (–8) (d) +5 – (–8) ☐ 8 – (–5)
8. **Simplify:**
   (a) 10 – 6 – (–5) – 12 + 8 (b) 100 – 3[20 + (50 – 40)] (c) 16 – [3 + {12 – 8 – (–4)}]
9. Subtract the sum of –250 and 1250 from the sum of –278 and –1222.

Competency Based Exercise

21st CS

1. Tick (✓) the correct answer.

(a) Which is the minimum temperature among the following?

(i) –200°C (ii) –273°C (iii) –270°C (iv) –250°C

(b) The additive inverse of a negative integer is:

(i) always negative (ii) always positive (iii) the same integer (iv) zero

(c) Which of the following shows the maximum rise in temperature?

(i) 0°C to 10°C (ii) –4°C to 8°C (iii) –15°C to –8°C (iv) –9°C to 0°C

(d) The greatest whole number lying between –16 and 2 is:

(i) 2 (ii) 1 (iii) 0 (iv) –17

2. Simplify the following.

(a) 15 + (–10) + (–75) + 2

(b) –70 – (–15) + (–15) – 10

(c) –(–5) + (–15) + (72) – (–10)

(d) 2 – (–18) + (–6) + 15

3. The difference between the integer *m* and (–17) is –2. Find the value of *m*.

4. Subtract the sum of –235 and 142 from the sum of –15 and –128.

5. Temperature of a place at 12:00 noon was +6°C. If the temperature is decreased by 2°C in the first hour and then increased by 1°C in the second hour, what was the temperature at 2:00 pm?

6. Find the sum, using the number line.

(a) 4 + (–5) (b) 7 + (–3) (c) (–6) + (–2) (d) (–3) + 5

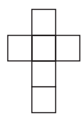
7. The sum of two integers is 2019. If one of them is –2019, then what is the other integer?

8. Given, *a* and *b* are two integers and the sum of the successor of *a* and the predecessor of *b* is subtracted from the sum of the predecessor of *a* and the successor of *b*. Find the result.

Challenge!

21st CS

1. If ∗ is an operation on integers such that for integers *m* and *n*, $m * n = m - (-n) + (-2)$, then find the value of $(-4) * (-3)$.
2. Write the numbers –6, –5, 2, 3, 4 and 7 in appropriate boxes so that the sum of all numbers across horizontal and vertical line is 0.



Let's Work in Mind

21st CS

1. Which integer is 5 less than –9?
2. What is the number that must be subtracted from –10 to get –59?
3. What is the sum of the successor and predecessor of –10?
4. Vikram gained ₹450 in one transaction and lost ₹880 in another transaction. What would be his final gain or loss?
5. How many integers are there from –5 to 5?
6. Which is the smallest 2-digit negative integer?

SMART TIME

LS

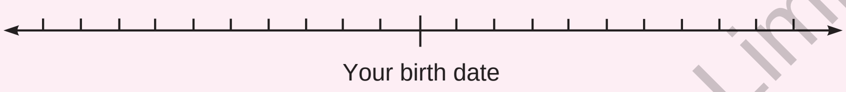
Considering that the following events are from your life. Arrange them in order of happening.

1. Birth of your elder brother or sister
2. Your birth (birth date)
3. Your father holds you for the first time
4. Your grandparents voted for the first time in general elections
5. Your entry in class VI
6. When you started walking
7. India got independence
8. Your parents were studying in school

Procedure:

Now, consider your birth (birth date) at the centre, *i.e.*, at 0, place the events before your birth on the left of zero and events after your birth right of zero in sequence of occurrence of events.

Keeping in mind that if event 2 follows event 1 then event 2 must be on the right of event 1. Similarly, if event 2 precedes, then it (event 2) must be on left of event 1. Now, show all the events on the line.



Time line

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

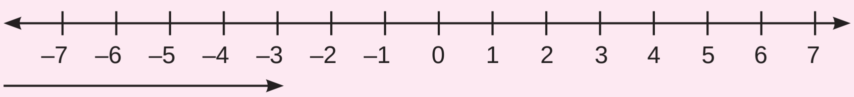
(d) Assertion (A) is false but Reason (R) is true.

1. Assertion (A) : The absolute value of –10 is 10.

Reason (R) : The absolute value of an integer is its numerical value regardless of its sign.

2. Assertion (A) : $-7 < -6$

Reason (R) : The number on the right side of a number are always greater.



3. Assertion (A) : The additive inverse of 2 is –2.

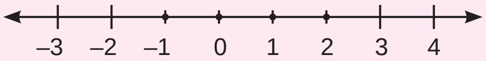
Reason (R) : The integer with opposite side is its additive inverse.

4. Assertion (A) : $(-1) + (-2) = -3$

Reason (R) : When two integers with the same sign are added, their numerical value is added and the sign remains the same.

5. **Assertion (A)** : Number of integers between –2 and 3 are 4.

   Reason (R) :



6. **Assertion (A)** : –8 + 11 = –3

   Reason (R) : When two integers with opposite sums are added, their numerical value got subtracted.

7. **Assertion (A)** : –8 + 11 = 3

   Reason (R) : When two integers with opposite signs are added, their numerical value gets subtracted and the sign of greater number is used with numerical value.

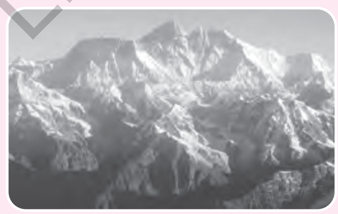
8. **Assertion (A)** : (–17) + (17) = 0

   Reason (R) : –17 is additive inverse of 17.

CASE STUDY

CC

Toolika planned Mt Everest expedition. As a first step, she collected information about the temperature variations from base camp to peak. Mt Everest is at a height peak of 8850 m. Its base is at an elevation of 5400 m. The temperature here drops at the rate of 1°C per 100 metres. She also looked at the record of the average temperature for each month.



Monthly Average Temperature Recorded on the Mount Everest

Month	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.
Temperature (in °C)	–18	–18	–21	–27	–30	–34	–36	–35	–32	–31	–25	–20

1. Arrange the temperature in ascending order, *i.e.,* from the coldest to warmest month.
2. What is the coldest month on Mount Everest?
3. If the temperature at the base camp is –5°C, then what will be the temperature at the height of 7000 m?

Self Assessment – 1

1. Match the following.

Shapes	Objects
(a) Sphere	(i) A coke can
(b) Cylinder	(ii) A cricket ball
(c) Cuboid	(iii) A dice
(d) Cube	(iv) A chalk duster

2. Fill in the blanks.

(a) Total number of vertices in an octagon is ___________.

(b) The number of integers lying between –12 and 10 is ___________.

(c) Two prime numbers whose difference is 2 are called ___________ primes.

(d) A _________ is a region in the interior of the circle enclosed by its arc and a chord.

3. Tick (✓) the correct answer.

(a) Which numeral represents five hundred five thousand five?

(i) 50,05,005 (ii) 5,05,005
(iii) 5,00,505 (iv) 55,005

(b) If a big container has 95 L 40 mL of orange juice, then the number of small containers each with capacity 60 mL which can be filled is:

(i) 159 (ii) 961
(iii) 1163 (iv) 1584

(c) Which group of numbers shows the factors of 91?

(i) 1, 13, 7, 91 (ii) 1, 3, 17, 91
(iii) 1, 7, 17, 91 (iv) 1, 13, 17, 91

(d) A figure formed by two rays in a plane having common initial point is called:

(i) a line (ii) an angle
(iii) a triangle (iv) a line segment

4. Find:

(a) the fifth common multiple of 2, 3 and 5.

(b) the estimated sum of 297 + 145 to the tens place.

5. Find 1 + (–3) – (–5) using a number line.

6. Write the following numbers in Roman numerals.

(a) 132 (b) 574 (c) 2100

7. Evaluate: 100 – [30 – 2{5 + (2 × 3 + 5)}]

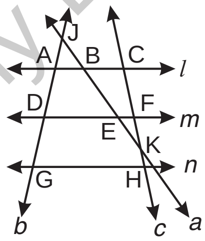
8. Find the value of the following.

(a) 8346 – 999 (b) 213 × 999
(c) 3125 × 211 + 89 × 5 × 625
(d) 36,800 × 212 – 12 × 100 × 368

9. If 11,105 is divided by a whole number, the quotient is 198 and the remainder is 17, then find the divisor.

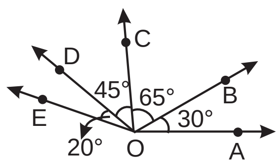
10. The length, breadth and height of a room are 7 m 68 cm, 5 m 76 cm and 8 m 32 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

11. In the given figure, name the following:

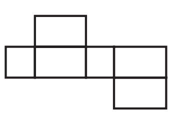


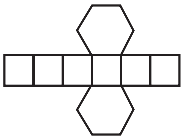
(a) Lines whose point of intersection is D.
(b) Lines whose point of intersection is H.
(c) All pairs of parallel lines.
(d) All points lying on the line *a*.
(e) Point of intersection of lines *l* and *c*.

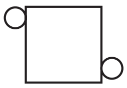
12. How many obtuse angles are there in the figure? Name them.

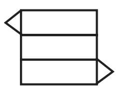


13. Identify the 3D shape that can be folded into the following nets.

(a) 

(b) 

(c) 

(d) 

8 Fractions

What Learners Will Achieve

- understand the concept of a fraction as a part of a whole and a part of a collection.
- understand the types of fractions such as proper, improper, mixed, like, unlike and equivalent.
- convert improper fractions into mixed fractions and vice versa.
- represent fractions on the number line.
- convert unlike fractions into like fractions.
- add and subtract like and unlike fractions.
- apply the concepts of fractions in real life.

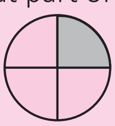
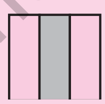
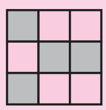
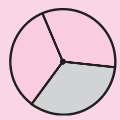
Warm-up

What we already know

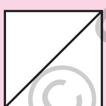
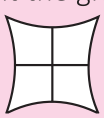
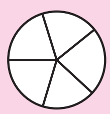
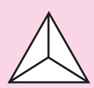
- A fraction is a number representing a part of a whole (unit). A whole can be an object or a collection of objects.
- When a part of a whole is represented by a fraction, the whole must be divided into equal parts.
- In fraction $\frac{7}{9}$, 7 is the numerator and 9 is the denominator.
- Numerator tells the number of parts to be considered and denominator tells how many parts the whole is divided into.

Now, try to solve the following.

1. What part of the figure is shaded?

(a)  (b)  (c)  (d) 

2. Shade the figures to represent the given fractions.

(a)  $\frac{1}{2}$ (b)  $\frac{3}{4}$ (c)  $\frac{2}{5}$ (d)  $\frac{3}{3}$

3. Write the numerator and denominator of the given fractions.

(a) $\frac{1}{6}$ (b) $\frac{5}{7}$ (c) $\frac{8}{11}$ (d) $\frac{6}{17}$

4. Peter painted $\frac{2}{3}$ part of a wall on one side. Harish painted $\frac{3}{4}$ part of the same wall on the opposite side. Can you say who painted the greater portion of the wall?

DID YOU KNOW?

A 'jiffy' is an actual unit of time for $\frac{1}{100}$th of a second.

INTRODUCTION TO FRACTIONS

There are two teams A and B each having two members and a mentor who guides them and caters to their needs. One day after a hard work out the mentor distributes a large pizza to the two teams as follows:

Team A: First member gets $\frac{1}{8}$ and the second gets $\frac{2}{9}$ part of the pizza.

Team B: First member gets $\frac{3}{8}$ and the second gets $\frac{1}{12}$ part of the pizza.

The mentor keeps the rest of the pizza for himself. (i) Which team got more pizza? (ii) Which individual got the largest share of the pizza? (iii) How much pizza did the mentor get?

We need to know fractions well, to answer the above questions.

In this chapter, we study about fractions, types of fractions, writing fractions to lowest terms, addition and subtraction of fractions and their use in our daily life.

Understanding Fractions

A fraction is a part of a whole (unit). A whole may be a single object or a collection of objects.

In Fig. 8.1, a unit is represented by a rectangle.

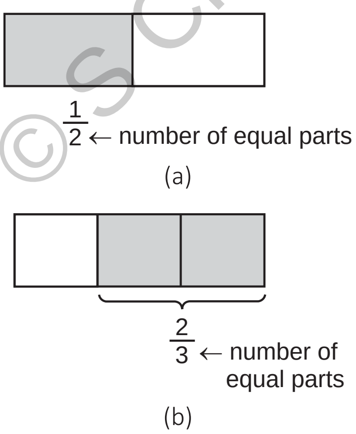


Fig. 8.1

In Fig. 8.1 (a), it is divided into 2 equal parts and one part is shaded. The shaded part is half or $\frac{1}{2}$ of the rectangle. The unshaded part is also $\frac{1}{2}$ of the rectangle.

In Fig. 8.1 (b), it is divided into 3 equal parts and two parts are shaded. The shaded part is two-thirds or $\frac{2}{3}$ of the rectangle. Unshaded part is one-third or $\frac{1}{3}$ of the rectangle.

In Fig. 8.2, a unit is represented by a collection of 12 identical balls. In Fig. 8.2 (a) to (d), balls are shaded differently, which shows as follows:

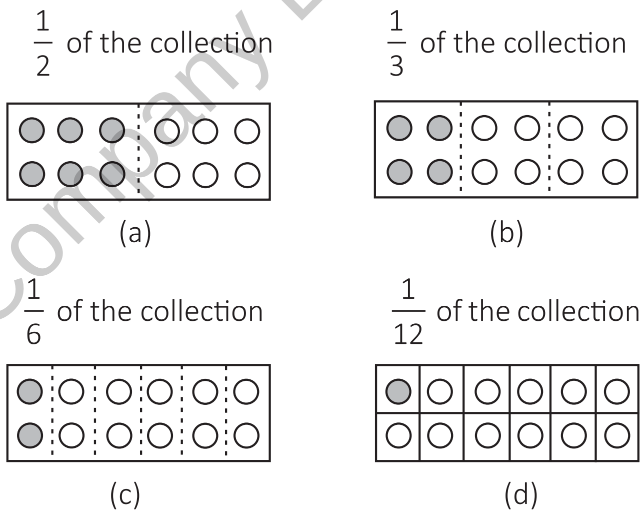


Fig. 8.2

Numbers of the type $\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12}$, etc., are called **fractions**.

Numerator and Denominator of a Fraction

If $\frac{a}{b}$ is a fraction, then a and b are respectively known as numerator and denominator of the fraction, where a and b are whole numbers but $b \neq 0$.

For example, $\frac{4}{5}$ (4 ← Numerator, 5 ← Denominator)

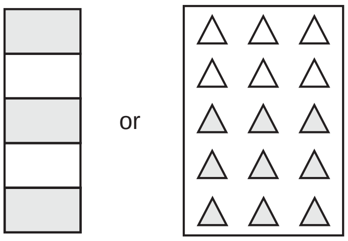
Fraction as a Division

Consider the fraction $\frac{3}{5}$. It represents '3 out of 5' or we can say a whole or collection is divided into 5 equal parts and 3 parts are taken.

Numerator is dividend.

Therefore, $\frac{3}{5}$ also can be expressed as $3 \div 5$.

Denominator is divisor.



(a) (b)

Fig. 8.3

Thus, we can say that, a fraction $\frac{a}{b}$ is the quotient of $a \div b$, where a and b are whole numbers and $b \neq 0$.

Fraction of a Collection

Let us consider the collection given in Fig. 8.3 (b).

Clearly, shaded triangles represent $\frac{3}{5}$ of 15 triangles, *i.e.*, $\frac{3}{5} \times 15$ triangles.

$\Rightarrow (3 \times 15) \div 5 = 45 \div 5 = 9$ or $3 \times 15 \div 5 = 3 \times 3 = 9$

Note

We know 'of' is an operation which means multiplication '×' and so it is solved before the process of division.

Illustration 1: Let us find $\frac{1}{5}$ of an hour.

We know that 1 hour = 60 minutes.

So, $\frac{1}{5}$ of 60 minutes $= (1 \times 60) \div 5 = 60 \div 5$

$= 12$ minutes.

Let us study some more examples.

Ex. 1 **Write the fraction represented by the shaded portion of Fig. 8.4.**

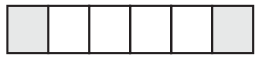


Fig. 8.4

Sol. The total number of equal parts in the figure is 6. So, the denominator is 6.

Now, the number of shaded parts is 2. So, the numerator is 2.

Thus, the shaded portion represents $\frac{2}{6}$ of the figure.

Ex. 2 **In Fig. 8.5, what fraction of the total circles are the circles having X's in them?**

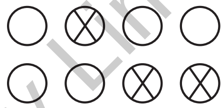


Fig. 8.5

Sol. Total number of circles = 8

Number of circles having X's in them = 3

Thus, the required fraction $= \frac{3}{8}$.

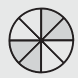
Ex. 3 **Name the following fractions:**

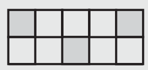
(a) $\frac{3}{4}$ **(b)** $\frac{7}{10}$ **(c)** $\frac{5}{13}$

Sol. (a) Three-fourths (b) Seven-tenths

(c) Five-thirteenths

Skill Check

- What fraction of the circle is shaded? 
- What portion of the below figure is shaded?



- The fraction for two-thirds is written as ______.
- Gunjan decided to distribute a large piece of her tenth birthday cake equally among her 6 friends. How much will each friend get?

TYPES OF FRACTIONS

Proper Fractions

A fraction in which, the numerator is less than the denominator is called a **proper fraction**. The value of a proper fraction is always less than one unit (or a whole).

For example, $\frac{1}{2}, \frac{1}{4}, \frac{4}{7}, \frac{3}{4}$, ... are all proper fractions.

Improper Fractions

A fraction in which the numerator is either greater than or equal to the denominator is called an **improper fraction**.

For example, $\frac{3}{2}, \frac{5}{3}, \frac{7}{4}, \frac{5}{5}, \frac{11}{10}, \frac{100}{61}$, ... are all improper fractions.

> **Remember**
>
> If the numerator of a fraction is equal to the denominator, then the value of the fraction is 1. If the numerator of a fraction is larger than the denominator, then the value of the fraction is greater than 1.

Some important points

1. Every natural number can be written as an improper fraction. For example, $35 = \frac{35}{1}$.

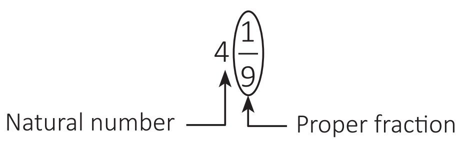
2. When a part of a unit is represented by a fraction, the unit must be divided into equal parts.

3. A fraction with numerator 1 is called a **unit fraction**. For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{9}$, etc., are all unit fractions.

> **Watch Your Step!**
>
> The shaded part cannot be represented by a fraction using 4 as the denominator because the unit in the following figure is divided into unequal parts.
>
> 
>
>
> So, it is incorrect to say that $\frac{1}{4}$ represents the shaded area.

Mixed Fractions

Improper fractions can also be written as fractions like $2\frac{1}{2}$, $4\frac{1}{9}$, $5\frac{11}{21}$, etc. Each of these fractions contains a natural number and a proper fraction. This type of fractions are called **mixed fractions**.



> **Remember**
>
> Mixed fractions are another way of writing those improper fractions where the numerator is greater than the denominator and is not its multiple.

Let us learn about mixed fractions and conversion of an improper fraction into a mixed fraction and vice versa.

Improper fraction as a natural number

Consider an improper fraction, say $\frac{12}{4}$.

The denominator 4 suggests that a unit is divided into 4 equal parts.

Also, 12 = 4 + 4 + 4 suggests that all the four parts of three units are shaded.

Thus, $\frac{12}{4}$ = 3 units.

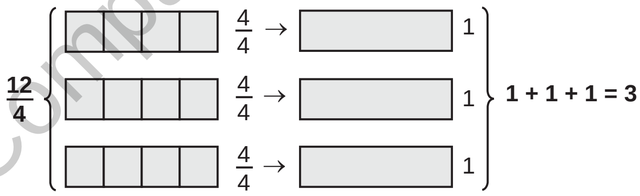


Fig. 8.6

We observe that, the denominator (4) divides evenly the numerator (12), *i.e.,* 12 ÷ 4 = 3.

In all such cases, the improper fraction is a natural number.

Improper fraction as a mixed fraction

Consider the improper fraction $\frac{13}{5}$. The denominator suggests that a unit is divided into five equal parts.

Also, 13 = 5 + 5 + 3 suggests that all the five parts of two units are shaded and three parts of the third unit are shaded.

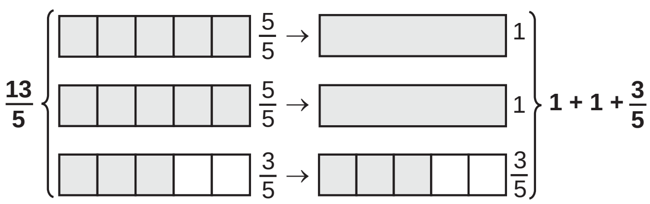


Fig. 8.7

$\frac{13}{5} = 2$ units and $\frac{3}{5}$ unit $= 2 + \frac{3}{5}$ written as $2\frac{3}{5}$. The number $2\frac{3}{5}$ is called a **mixed fraction** (or a mixed number).

Converting Improper Fractions into Mixed Fractions

Observe that when 13 is divided by 5, the quotient is 2 and the remainder is 3.

Divisor → $5\overline{)13}$ quotient 2 ← Quotient
-10
3 ← Remainder

Recall that $\frac{13}{5} = 2 + \frac{3}{5}$ or $2\frac{3}{5}$

$= \text{Quotient } (2) + \frac{\text{Remainder } (3)}{\text{Divisor } (5)} = Q\frac{R}{D}$

Recall the notation: $16 \div 2 = \frac{16}{2}$. The fraction bar is also a division symbol. Thus, it makes sense to divide when changing an improper fraction into a mixed fraction or a natural number.

Illustration 2: Consider $23 \div 5$, we have $Q = 4$, $R = 3$.

$\frac{23}{5} = 4 + \frac{3}{5} = 4\frac{3}{5}$

$\frac{23}{5}$	=	4	+	$\frac{3}{5}$	=	$4\frac{3}{5}$
↓		↓		↓		↓
Improper fraction		Natural number	+	Proper fraction	=	Mixed fraction

In general, the improper fractions that are not natural numbers can be written as:

Improper fraction $= \frac{\text{Numerator}}{\text{Denominator}}$
(Numerator > Denominator)

Mixed fraction $= \text{Quotient} + \frac{\text{Remainder}}{\text{Denominator (Divisor)}}$

Converting Mixed Fractions into Improper Fractions

A mixed fraction can be written as an improper fraction. Consider the shaded portion as shown in the Fig. 8.8. It illustrates a method for changing a mixed fraction to an improper fraction.

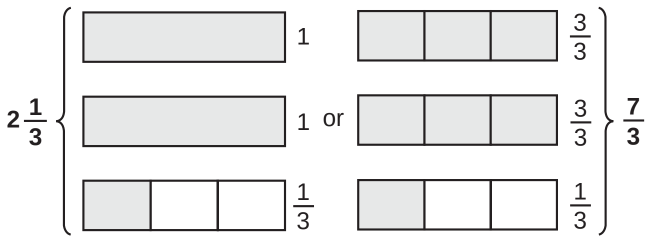


Fig. 8.8

$2\frac{1}{3} = 1 + 1 + \frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$.

So, $2\frac{1}{3} = \frac{7 \rightarrow (2 \times 3 + 1)}{3}$.

We can use the following short cut to convert a mixed fraction into an improper fraction:

Mixed Fraction $= \frac{(\text{Whole number part}) \times \text{Denominator} + \text{Numerator}}{\text{Denominator}}$

Watch Your Step!

$2\frac{1}{3} = \frac{7}{3}$, so it has 7 one-thirds. It is incorrect to say that there is 1 one-third in $2\frac{1}{3}$.

Illustration 3: $7\frac{3}{4} = \frac{7 \times 4 + 3}{4} = \frac{31}{4}$;

$5\frac{2}{3} = \frac{5 \times 3 + 2}{3} = \frac{17}{3}$

Let us study some more examples.

Ex. 4 **Identify the type of fraction: $\frac{7}{19}$ is proper or improper.**

Sol.

Numerator	Denominator	Fraction type
7	19	Proper
	(Numerator < Denominator)	

Thus, $\frac{7}{19}$ is a proper fraction.

Ex. 5 **Express $\frac{11}{5}$ as a mixed fraction.**

Sol. We have, $\frac{11}{5}$ = 2 wholes and $\frac{1}{5}$ more.

$= 2 + \frac{1}{5} = 2\frac{1}{5}$

Ex. 6 **Convert the following mixed fractions into an improper fraction.**

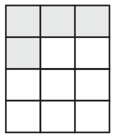
(a) $7\frac{4}{9}$ **(b)** $53\frac{3}{4}$

Sol. (a) $7\frac{4}{9} = \frac{7 \times 9 + 4}{9} = \frac{63 + 4}{9} = \frac{67}{9}$

(b) $53\frac{3}{4} = \frac{53 \times 4 + 3}{4} = \frac{212 + 3}{4} = \frac{215}{4}$

Exercise 8.1

1. What fraction of the given figure is not shaded?



2. Arushi made four cheese pizzas and three pepperoni pizzas. She cuts each into eights. How many slices did she have altogether?
3. Write a fraction with numerator 3 × 5 and denominator 19.
4. **Write the following in words.**

   (a) $\frac{2}{3}$ (b) $\frac{3}{8}$ (c) $\frac{1}{5}$ (d) $\frac{4}{9}$

5. **Determine the following.**

   (a) $\frac{1}{4}$ of 12 pens (b) $\frac{2}{3}$ of 18 balloons (c) $\frac{5}{6}$ of a day

6. **Identify the fractions as proper, improper and mixed fractions.**

   (a) $\frac{2}{7}$ (b) $1\frac{1}{2}$ (c) $\frac{5}{2}$ (d) $\frac{12}{3}$ (e) $\frac{9}{11}$ (f) $\frac{21}{17}$

7. **Fill up the boxes with >, < or =.**

   (a) $\frac{2}{3}$ ☐ 1 (b) 1 ☐ $\frac{8}{5}$ (c) $\frac{5}{5}$ ☐ 1 (d) $\frac{9}{7}$ ☐ 1

8. **Express the following as a mixed fraction.**

   (a) $\frac{13}{5}$ (b) $\frac{23}{7}$ (c) $\frac{31}{5}$ (d) $\frac{42}{19}$

9. **Convert the mixed fraction into an improper fraction.**

   (a) $3\frac{2}{3}$ (b) $5\frac{1}{3}$ (c) $7\frac{2}{5}$ (d) $13\frac{8}{9}$

10. Fill in the blanks.

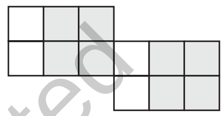
(a) $\frac{27}{5}$ as a mixed fraction equals __________.

(b) The mixed fraction $9\frac{3}{5}$ to an improper fraction is __________.

(c) The total number of halves in $4\frac{1}{2}$ are __________.

11. Sasha has 30 pencils with her. She gave one-third of the pencils to Rita. How many pencils are left with Sasha?

12. If I shade one more square in the figure as shown, what fraction would be shaded?



FRACTIONS ON THE NUMBER LINE

Whole Numbers on a Number Line

Recall how whole numbers have been represented on a number line (Fig. 8.9).

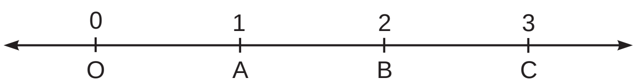


Fig. 8.9

Line segments OA, AB and BC are equal and each of these segments represents one unit.

If OA = 1 unit, then point O represents the number 0, the point A represents 1, point B represents 2, point C represents 3 and so on.

Fractions on a Number Line

Consider Fig. 8.10.

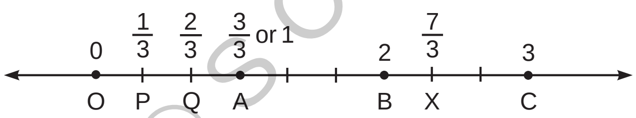


Fig. 8.10

Each unit in the given figure, *i.e.*, OA, AB or BC has been divided into **three** equal parts.

Point P represents $\frac{1}{3}$, point Q represents $\frac{2}{3}$ and point X represents $\frac{7}{3}$.

Observe that, to reach X, we have moved 7 steps, (OP, PQ, QA, ..., BX, each of length $\frac{1}{3}$ unit). What does point A represent? Of course $\frac{3}{3}$ or 1.

Illustration: Consider the Fig. 8.11.

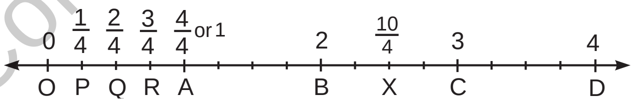


Fig. 8.11

Starting from O, each unit, *i.e.*, OA, AB, BC or CD has been divided into four equal parts.

Point P represents $\frac{1}{4}$, point Q represents $\frac{2}{4}$, point R represents $\frac{3}{4}$ and point X represents $\frac{10}{4}$. (Moving 10 steps from O, each of length $\frac{1}{4}$ unit). What does point B represent? It represents $\frac{8}{4}$ or 2.

Remember

All proper fractions are on the left of 1 as they are less than 1 on the number line.

All improper fractions are on the right of 1 (including 1 itself) on the number line.

Mixed Numbers on a Number Line

Let us represent $3\frac{7}{10}$ on a number line.

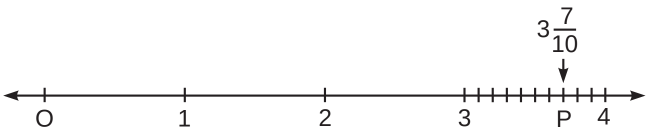


Fig. 8.12

We know that $3\frac{7}{10}$ lies between 3 and 4. So, divide the gap between 3 and 4 into 10 equal parts. Mark the 7th part as P.

Thus, point P represents $3\frac{7}{10}$.

We can use the above process to represent improper fractions also, by converting them into mixed numbers.

Let us study some more examples.

Ex. 7 **Represent the fractions $\frac{2}{5}$ and $\frac{7}{5}$ on a number line.**

Sol. Draw a number line with O representing 0, A representing 1 and B representing 2.

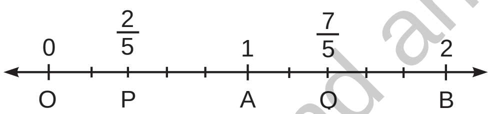


Fig. 8.13

Divide each of these units into five equal parts, because the denominator of both the fractions is 5. Next, count the appropriate number of fifths starting from 0 to represent the fractions.

The point P represents $\frac{2}{5}$ and the point Q represents $\frac{7}{5}$.

Ex. 8 **What fractions do the points X, P and M represent?**

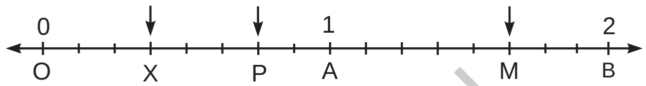


Fig. 8.14

Sol. Each unit OA or AB has been divided into 8 equal parts on the number line.
Thus, we count the markings of eighths starting from O. Hence, the point X represents $\frac{3}{8}$. P represents $\frac{6}{8}$ and M represents $\frac{13}{8}$ or $1\frac{5}{8}$.

Ex. 9 **Represent $\frac{17}{6}$ on a number line.**

Sol. We know that $\frac{17}{6} = 2\frac{5}{6}$ which lies between 2 and 3 on a number line. Since denominator is 6, divide the gap between 2 and 3 into 6 equal parts and mark the fifth part as $2\frac{5}{6}$ or $\frac{17}{6}$.

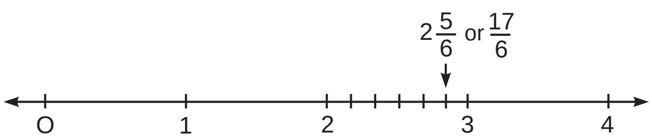


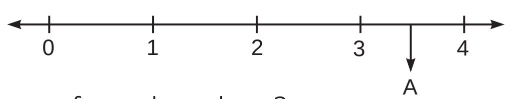
Fig. 8.15

Exercise 8.2

1. Answer the following questions.

(a) On a number line, $\frac{52}{7}$ can be represented between which two whole numbers?

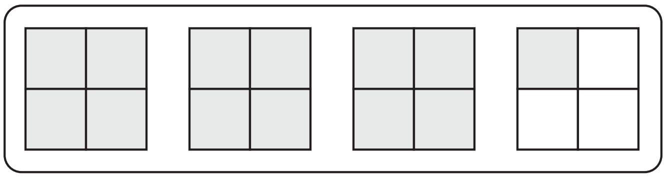
(b) On the number line, what is the probable value of A?



(c) Which of the following fractions has a value that is different from the others?

(i) $5\frac{2}{3}$ (ii) $\frac{36}{6}$ (iii) $\frac{17}{3}$ (iv) $\frac{34}{6}$

(d) Write the mixed fraction illustrated by the given figure.



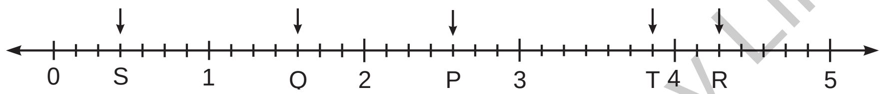
2. Represent the following fractions on a number line.

(a) $\frac{2}{4}$ and $\frac{7}{4}$ (b) $\frac{3}{7}$ and $\frac{8}{7}$ (c) $\frac{1}{3}$ and $\frac{10}{3}$

3. Represent the following improper or mixed fractions on a number line.

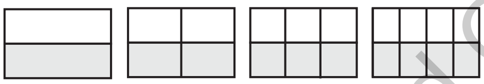
(a) $3\frac{4}{6}$ (b) $\frac{10}{3}$ (c) $\frac{15}{4}$ (d) $\frac{20}{5}$

4. What fractions do the points P, Q, R, S and T represent on the number line?



EQUIVALENT FRACTIONS

Observe the shaded portion in four units (rectangles) given below.



Fraction representing shaded portion → $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

Fig. 8.16

As the shaded portion in each of the four units is equal (same), we can verify by placing one over the other), so the fractions representing the shaded portions must be equal, that is,

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

$$\frac{2}{4} = \frac{1 \times 2}{2 \times 2}, \frac{4}{8} = \frac{1 \times 4}{2 \times 4}, \frac{3}{6} = \frac{1 \times 3}{2 \times 3}$$

Therefore, the fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ and $\frac{4}{8}$ are equivalent fractions.

Thus, we can say that two or more fractions representing the same part of a whole are called **equivalent fractions**.

Building Equivalent Fractions

Observe the following fractions:

1. $\frac{2}{3} = \frac{\mathbf{2} \times 2}{\mathbf{3} \times 2} = \frac{4}{6}, \frac{2}{3} = \frac{\mathbf{2} \times 3}{\mathbf{3} \times 3} = \frac{6}{9},$

$\frac{2}{3} = \frac{\mathbf{2} \times 5}{\mathbf{3} \times 5} = \frac{10}{15}$

Therefore, equivalent fractions of $\frac{2}{3}$ are $\frac{4}{6}, \frac{6}{9}, \frac{10}{15}$ and so on.

2. $\frac{64}{120} = \frac{64 \div 2}{120 \div 2} = \frac{32}{60}, \frac{64}{120} = \frac{64 \div 4}{120 \div 4} = \frac{16}{30}$

$\frac{64}{120} = \frac{64 \div 8}{120 \div 8} = \frac{8}{15}$

Therefore, equivalent fractions of $\frac{64}{120}$ are $\frac{32}{60}, \frac{16}{30}, \frac{8}{15}$ and so on.

Thus, we can say that to find an equivalent fraction of a given fraction, we should multiply or divide, both the numerator and denominator of the fraction by the same non-zero number.

Let us study some more examples.

Ex. 10 Determine the fraction equivalent to $\frac{7}{13}$ with 52 as denominator.

Sol. Since the required denominator is 52.

$$\therefore \quad \frac{7}{13} = \frac{7 \times 4}{13 \times 4} = \frac{28}{52}$$

Thus, the new fraction $\frac{28}{52}$ is equivalent to the given fraction $\frac{7}{13}$.

Ex. 11 Find the equivalent fraction of $\frac{36}{48}$ having numerator 9.

Sol. The required numerator is 9.
Since 36 = 4 × 9, so we divide the numerator and denominator by 4.

$$\frac{36}{48} = \frac{36 \div 4}{48 \div 4} = \frac{9}{12}$$

Thus, the new fraction $\frac{9}{12}$ is equivalent to the given fraction $\frac{36}{48}$ and has 9 as the numerator.

A Fact about Equivalent Fractions

Observe the following pair of fractions:

We know that $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent fractions.

On cross multiplying, we have

$$\frac{2}{3} \times \frac{4}{6}$$

2 × 6 = 12 ← Numerator of first × denominator of second

4 × 3 = 12 ← Numerator of second × denominator of first

We observe that 2 × 6 = 4 × 3 = 12.

Similarly, $\frac{32}{60}$ and $\frac{64}{120}$

$$\frac{32}{60} \times \frac{64}{120}$$

On cross multiplying, we have

32 × 120 = 3840, 64 × 60 = 3840

∴ 32 × 120 = 64 × 60 = 3840.

Thus, if two fractions are equivalent, then

Numerator of first × denominator of second = Numerator of second × denominator of first

In fact, the converse of this statement is also true. If the cross products of two fractions are equal, then the given fractions are equivalent.

Finding the Missing Part of Equivalent Fractions

Illustration 1: Let us find the missing numerator and denominator in the given equivalent fractions.

(a) $\frac{3}{4} = \frac{?}{20}$ (b) $\frac{7}{16} = \frac{147}{?}$

(a) $\frac{3}{4} = \frac{?}{20}$ (Divide the denominator, 20 ÷ 4 = 5)

$\frac{3}{4} = \frac{3 \times 5}{4 \times 5}$ (Multiply the numerator and denominator by 5.)

$= \frac{15}{20}$

Thus, the missing numerator is 15.

(b) $\frac{7}{16} = \frac{147}{?}$ (Divide the numerator, 147 ÷ 7 = 21)

$\frac{7}{16} = \frac{7 \times 21}{16 \times 21}$ (Multiply the numerator and denominator by 21.)

$= \frac{147}{336}$

Thus, the missing denominator is 336.

Skill Check

- Replace $\square$ by the correct number in each of the following.

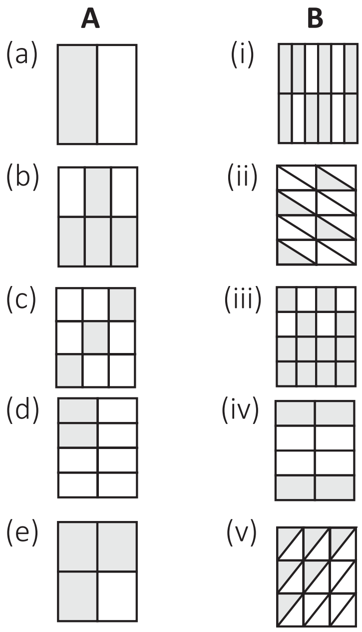
(a) $\frac{4}{11} = \frac{\square}{55}$ (b) $\frac{3}{8} = \frac{\square}{32}$ (c) $\frac{\square}{7} = \frac{20}{28}$

(d) $\frac{48}{\square} = \frac{6}{7}$ (e) $\frac{5}{7} = \frac{10}{\square}$

- Find three equivalent fractions of each of the following fractions.

(a) $\frac{3}{5}$ (b) $\frac{8}{12}$ (c) $\frac{5}{7}$ (d) $\frac{6}{11}$

Ex. 12 **Pair up the equivalent fractions represented by the shaded parts from each column.**



Sol. Pairing is as follows:

(a) ↔ (iv) since (a) represents $\frac{1}{2}$ and (iv) represents $\frac{4}{8} = \frac{1}{2}$

(b) ↔ (i) since (b) represents $\frac{4}{6} = \frac{2}{3}$ and (i) represents $\frac{8}{12} = \frac{2}{3}$

(c) ↔ (v) since (c) represents $\frac{3}{9} = \frac{1}{3}$ and (v) represents $\frac{6}{18} = \frac{1}{3}$

(d) ↔ (ii) since (d) represents $\frac{2}{8} = \frac{1}{4}$ and (ii) represents $\frac{4}{16} = \frac{1}{4}$

(e) ↔ (iii) since (e) represents $\frac{3}{4}$ and (iii) represents $\frac{12}{16} = \frac{3}{4}$

FRACTION IN ITS LOWEST TERMS

A fraction $\frac{a}{b}$ is said to be in its **lowest terms**, if its numerator and denominator have no common factor, except 1, *i.e.*, if HCF of *a* and *b* is 1. For example, the fraction $\frac{3}{4}$ is in its lowest terms (or in **simplest form**) as HCF of 3 and 4 = 1. The fraction $\frac{15}{11}$ is also in its lowest terms as HCF of 15 and 11 is 1. The fraction $\frac{16}{20}$ is not in its lowest terms as HCF of 16 and 20 is 4 (not 1).

Now, let us learn how to reduce fractions such as $\frac{16}{20}$ in their lowest terms.

Reducing Fractions into Lowest Terms

Given, the fraction $\frac{16}{20}$.

Let us try to find an equivalent fraction of it in which HCF of numerator and denominator is 1.

$\frac{16}{20} = \frac{16 \div 2}{20 \div 2} = \frac{8}{10}$, HCF of 8 and 10 is 2 ≠ 1.

So, further reducing $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$, HCF of 4 and 5 is 1.

So, fraction $\frac{16}{20}$ has an equivalent fraction $\frac{4}{5}$ in which HCF of 4 and 5 = 1.

We say that $\frac{16}{20}$ has been reduced to its lowest terms (or simplest form) as $\frac{4}{5}$.

Note

$\frac{16}{20} = \frac{16 \div 4}{20 \div 4} = \frac{4}{5}$ (∵ HCF of 16 and 20 = 4)

Alternate Method: $\frac{16}{20} = \frac{\cancel{2}\times\cancel{2}\times 2\times 2}{\cancel{2}\times\cancel{2}\times 5} = \frac{4}{5}$

(Writing the fractions as prime factorisation of the numerator and denominator.)

Illustration 2: Reduce the fraction $\frac{525}{1125}$ to its lowest terms.

Dividing the numerator and denominator by the HCF, 75, we get

$$\frac{525}{1125} = \frac{525 \div 75}{1125 \div 75} = \frac{7}{15}$$

$$\begin{array}{l} 525\overline{)1125}(2 \\ \underline{-1050} \\ \quad 75\overline{)525}(7 \\ \quad\ \ \underline{-525} \\ \qquad\ \ 0 \end{array}$$

$\therefore$ HCF = 75

Alternate Method:

$$\frac{525}{1125} = \frac{\cancel{3}\times\cancel{5}\times\cancel{5}\times 7}{\cancel{3}\times 3\times\cancel{5}\times\cancel{5}\times 5} \quad \text{(Prime factorisation)}$$

$$= \frac{7}{15} \quad \text{(To lowest terms as HCF of 7 and 15 = 1)}$$

Skill Check

Reduce the following fractions to their lowest terms.

(a) $\frac{42}{70}$ (b) $\frac{58}{174}$

Ex. 13 Which of the following fractions is equal to $\frac{1}{6}$?

$\frac{4}{20}, \frac{12}{75}, \frac{16}{96}$

Sol. $\frac{4}{20} = \frac{4\times 1}{4\times 5} = \frac{1}{5}; \frac{12}{75} = \frac{3\times 4}{3\times 25} = \frac{4}{25};$

$\frac{16}{96} = \frac{16\times 1}{16\times 6} = \frac{1}{6}$

Thus, the required equivalent fraction is $\frac{16}{96}$.

Ex. 14 What fraction of a day is 8 hours?

Sol. 1 day = 24 hours

$\frac{8 \text{ hours}}{24 \text{ hours}} = \frac{8}{24} = \frac{1}{3}$

Ex. 15 Sarthak is a tailor. He had to stitch 20 dresses. If he has stitched 15 dresses, what fraction of the dresses has he finished?

Sol. Total number of dresses = 20

Number of dresses stitched = 15

Fraction representing stitched dresses

$= \frac{15}{20} = \frac{3}{4}$

Ex. 16 Aayush had 25 pencils, Aryan had 30 pencils and Arnav had 40 pencils. After 3 months, Aayush used up 15 pencils, Aryan used up 15 pencils and Arnav used up 20 pencils. What fraction of pencils did each use up?

Sol. Number of pencils with Aayush = 25

Pencils used up in 3 months = 15

Fraction of pencils used up by Aayush

$= \frac{15}{25} = \frac{3}{5}$

Number of pencils with Arnav = 40

Pencils used up in 3 months = 20

Fraction of pencils used up by Arnav

$= \frac{20}{40} = \frac{1}{2}$

Number of pencils with Aryan = 30

Pencils used up in 3 months = 15

Fraction of pencils used up by Aryan

$= \frac{15}{30} = \frac{1}{2}$

Exercise 8.3

1. Tick (✓) the correct answer.

(a) Which of the following will give a fraction equivalent to $\frac{3}{4}$?

(i) $\frac{4+3}{4+3}$ (ii) $\frac{4-3}{4+3}$ (iii) $\frac{2\times 3}{5+3}$ (iv) $\frac{4\div 3}{4+3}$

(b) Which of the following fractions is equal to $\frac{1}{5}$?

(i) $\frac{10}{60}$ (ii) $\frac{12}{60}$ (iii) $\frac{15}{60}$ (iv) None of these

(c) Which of the following fractions is equal to $\frac{4}{5}$?

(i) $\frac{12}{15}$ (ii) $\frac{16}{25}$ (iii) $\frac{20}{30}$ (iv) None of these

2. Identify the group of fractions that are equivalent.

(a) $\frac{3}{4}, \frac{9}{10}, \frac{15}{16}, \frac{1}{10}$ (b) $\frac{2}{3}, \frac{6}{10}, \frac{14}{20}, \frac{8}{9}$ (c) $\frac{4}{5}, \frac{8}{10}, \frac{16}{20}, \frac{28}{35}$ (d) $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}$

3. Determine an equivalent fraction for the following fractions.

(a) $\frac{4}{7}$ with 20 as numerator (b) $\frac{12}{8}$ with 24 as denominator

(c) $\frac{20}{25}$ with 12 as numerator (d) $\frac{3}{5}$ with 35 as denominator

4. Find the missing denominator.

(a) $\frac{3}{7} = \frac{21}{?}$ (b) $\frac{4}{5} = \frac{36}{?}$ (c) $\frac{8}{11} = \frac{40}{?}$ (d) $\frac{2}{9} = \frac{18}{?}$ (e) $\frac{4}{15} = \frac{28}{?}$

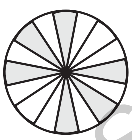
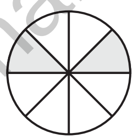
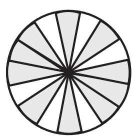
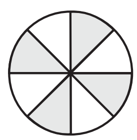
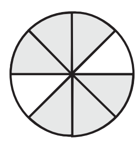
5. Find the missing numerator.

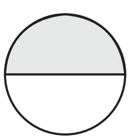
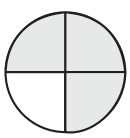
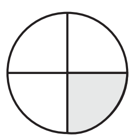
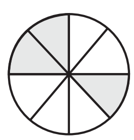
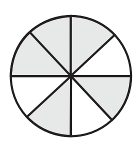
(a) $\frac{3}{7} = \frac{?}{42}$ (b) $\frac{4}{13} = \frac{?}{52}$ (c) $\frac{7}{12} = \frac{?}{60}$ (d) $\frac{8}{15} = \frac{?}{105}$ (e) $\frac{3}{8} = \frac{?}{32}$

6. Find an equivalent fraction of $\frac{56}{70}$ with:

(a) numerator 4 (b) denominator 10

7. Pair up the equivalent fractions from each row.

(a)  (b)  (c)  (d)  (e) 

(i)  (ii)  (iii)  (iv)  (v) 

8. Reduce the following fractions to their lowest terms.

(a) $\frac{17}{51}$ (b) $\frac{25}{125}$ (c) $\frac{22}{110}$ (d) $\frac{64}{144}$ (e) $\frac{126}{147}$ (f) $\frac{84}{168}$

9. What fraction of a day is:

(a) 3 hours? (b) 6 hours? (c) 9 hours? (d) 18 hours?

10. What fraction of a month of 30 days is:

(a) 6 days? (b) 12 days? (c) 16 days? (d) 21 days?

11. Pankaj coloured 5 pages of a colouring book containing 60 pages, Raju coloured $\frac{1}{6}$ of the pages of the same book. Did they colour the same fraction of pages?

12. How many natural numbers are there from 2 to 20? What fraction of them are prime numbers?

13. In each of the following, what fraction of total dresses to be stitched have been finished?

(a) Total dresses to be stitched 25, dresses finished 15.

(b) Total dresses to be stitched 45, dresses finished 25.

14. Swati wants to finish a novel as soon as possible. So far, she has read only 42 pages. What fraction of the number of pages has she read, if there are 147 pages in the novel? Simplify, the fraction to its lowest terms.

15. Lali had 30 pencils, Mili had 35 pencils and Joly had 45 pencils. After 3 months, Lali used up 20 pencils, Mili used up 15 pencils and Joly used up 25 pencils. What fraction of pencils did each use up?

LIKE AND UNLIKE FRACTIONS

Like Fractions

Fractions with the same denominator are called **like fractions**.

For example, $\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{5}{9}, \frac{8}{9}$, ... are all like fractions.

Unlike fractions

Fractions with different denominators are called **unlike fractions**.

For example, $\frac{7}{9}, \frac{15}{17}, \frac{7}{23}$, ... are all unlike fractions.

Converting Unlike Fractions into Like Fractions

In the previous section, we have learnt about equivalent fractions. Here, we use that idea to convert unlike fractions into like fractions.

Illustration 1: Consider the fractions $\frac{5}{8}$ and $\frac{7}{12}$.

Both the fractions have different denominators 8 and 12. So, first find their LCM.

2	8, 12
2	4, 6
	2, 3

The LCM of 8 and 12 = 2 × 2 × 2 × 3 = 24

Now, find the fractions equivalent to $\frac{5}{8}$ and $\frac{7}{12}$ with the common denominator as 24.

$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$ (24 ÷ 8 = 3, so multiply by 3)

$\frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$ (24 ÷ 12 = 2, so multiply by 2)

Thus, $\frac{15}{24}$ and $\frac{14}{24}$ are a pair of like fractions.

COMPARING FRACTIONS

We already know how to compare whole numbers. In this section, we shall learn how to compare fractions.

Comparing Like Fractions

Let us compare two fractions, say $\frac{2}{7}$ and $\frac{5}{7}$, having the same denominator (but unequal numerators). Observe that the shaded portion in Fig. 8.17 (a) representing the fraction $\frac{2}{7}$ is smaller than the shaded portion in Fig. 8.17 (b) representing the fraction $\frac{5}{7}$.

$\frac{2}{7}$ (a) $\frac{5}{7}$ (b)

Fig. 8.17

i.e., $\frac{2}{7}$ is less than $\frac{5}{7}$. This is written symbolically as $\frac{2}{7} < \frac{5}{7}$. (Note that 2 < 5)

Also, $\frac{5}{7}$ is larger than $\frac{2}{7}$. This is written symbolically as $\frac{5}{7} > \frac{2}{7}$. (Note that 5 > 2)

In general, it can be stated as follows:

If two fractions have the same denominator, the fraction with greater numerator is greater or with smaller numerator is smaller.

Illustration 2: Observe the following:

$\frac{5}{9} > \frac{2}{9}$ because 5 > 2; $\frac{13}{21} > \frac{8}{21}$ because 13 > 8;

$\frac{15}{91} > \frac{7}{91}$ because 15 > 7.

Comparing Unlike Fractions

Let us compare $\frac{7}{9}$ and $\frac{5}{12}$.

These are unlike fractions. To compare them, we first, convert these fractions into equivalent fractions with the LCM of the denominators as the common denominator, *i.e.*, like fractions.

$$\frac{7}{9} = \frac{7 \times 4}{9 \times 4} = \frac{28}{36} \text{ and } \frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$$

($\because$ LCM of 9 and 12 = 36)

Now, compare $\frac{28}{36}$ and $\frac{15}{36}$.

Since, 28 > 15, $\frac{28}{36} > \frac{15}{36}$, *i.e.*, $\frac{7}{9} > \frac{5}{12}$.

Alternate Method:

We have, $\frac{7}{9}$ and $\frac{5}{12}$.

By cross multiplication:

$$\frac{7}{9} \times \frac{5}{12}$$

$$7 \times 12 = 84 \text{ and } 5 \times 9 = 45$$

$$\because \quad 84 > 45 \Rightarrow \frac{7}{9} > \frac{5}{12}$$

In general, it can be stated as follows:

If the fractions to be compared do not have a common denominator, then first convert the fractions into equivalent fractions with LCM of the denominators as the common denominator. Then, compare them as we do for like fractions.

Comparing Fractions with the Same Numerator

Let us consider two fractions $\frac{5}{9}$ and $\frac{5}{12}$. These are unlike fractions. They have the **same numerator** but different **denominators**.

Now, building equivalent fractions with the same denominator, we have

$$\frac{5}{9} = \frac{5 \times 4}{9 \times 4} = \frac{20}{36} \text{ and } \frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$$

($\because$ LCM of 9 and 12 = 36)

$$\Rightarrow \frac{20}{36} > \frac{15}{36} \quad (\because 20 > 15) \quad \Rightarrow \frac{5}{9} > \frac{5}{12}.$$

Note

Here numerators are the same (*i.e.*, 5) and the fraction with smaller denominator is greater.

In general, it can be stated as follows:

If two fractions have the same numerator, the one with smaller denominator is greater.

Illustration 3: Observe the following:

$\frac{2}{5} > \frac{2}{7}$ because 5 < 7; $\frac{4}{7} > \frac{4}{11}$ because 7 < 11;

$\frac{3}{4} > \frac{3}{8}$ because 4 < 8.

Let Us Do AIL

Objective: (A) To create a fraction chart using paper folding

(B) To compare the strips of fraction chart obtained to conclude "As the denominator increases, the fraction decreases"

Materials required: 12 paper strips of the same size, a pen/pencil and colours

Procedure:

Step 1: Take first strip and colour completely with the same colour. This strip represents 1 or whole.

Step 2: Take a second strip and fold it to divide it in two equal parts. Unfold and mark the crease with a black pen. Colour half with different colour and leave half of it uncoloured.

Step 3: Take a third strip and fold it to get three equal parts. Unfold and mark the two creases with a black pen. Colour one-third of the boxes.

Step 4: Repeat the procedure till you divide the strip into 12 equal parts.

Step 5: Obtain a fraction chart as shown (see Fig. 8.18).

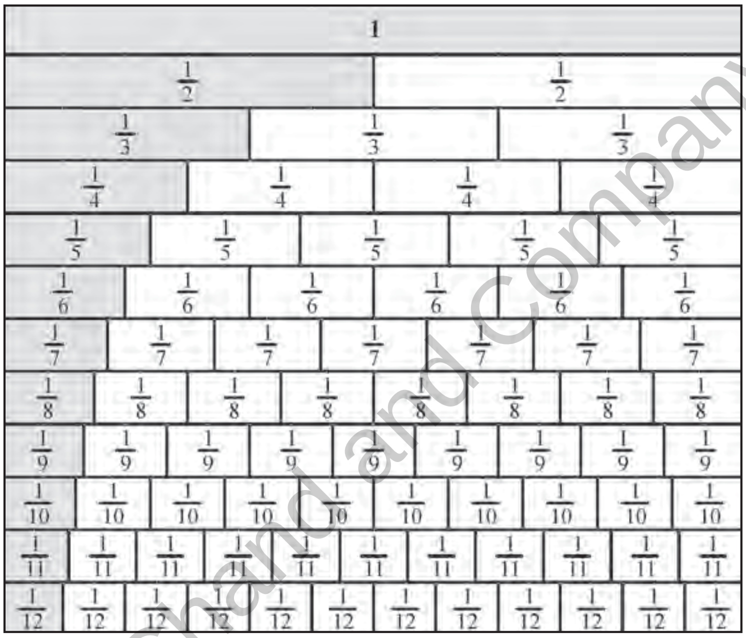


Fig. 8.18

Note for the teacher: Encourage the students to speak their mind and trigger their thinking and lead them towards the conclusion that as the denominator increases the fraction decreases. Further depict the fractional strip on the number line and explain how the fractions can be represented on a number line.

Step 6: Observe that as you are increasing the number of parts, each part is becoming smaller.

So, we can conclude:

As the denominator ______________, the fraction______________.

Comparing Mixed Fractions

When whole number parts are not equal

To compare two mixed fractions with their whole number parts not equal, compare their whole number parts. The fraction with the greater whole number part is greater.

Illustration 4: Consider $2\frac{3}{4}$ and $1\frac{4}{5}$.

We have, $2\frac{3}{4} > 1\frac{4}{5}$ [Since, whole number part 2 > 1.]

Whole number parts

When whole number parts are equal

If the whole number parts of two mixed fractions are equal, then compare their fractional parts. The mixed fraction with the greater fractional part is greater.

Illustration 5: Compare $2\frac{5}{12}$ and $2\frac{7}{16}$.

Since the whole number parts are equal, we compare their fractional parts $\frac{5}{12}$ and $\frac{7}{16}$.

First, convert these fractional parts into like fractions.

$\frac{5}{12} = \frac{20}{48}$ and $\frac{7}{16} = \frac{21}{48}$ ($\because$ LCM of 12 and 16 = 48)

Since 21 > 20, $\frac{21}{48} > \frac{20}{48}$, *i.e.*, $\frac{7}{16} > \frac{5}{12}$.

Thus, $2\frac{7}{16} > 2\frac{5}{12}$.

Skill Check

Put the appropriate sign >, < or = in the boxes.

(a) $1\frac{7}{8} \square 3\frac{7}{8}$ (b) $2\frac{3}{4} \square 2\frac{4}{5}$

Comparing Improper Fractions

Recall that an improper fraction is always greater than or equal to 1, while a proper fraction is always less than 1. Therefore, **every improper fraction, whole number or mixed fraction is larger than any proper fraction**.

Illustration 6: Observe the following:

$2\frac{3}{4} > \frac{5}{13}$; $1\frac{7}{9} > \frac{201}{417}$; $2 > \frac{387}{388}$

To compare two improper fractions, first convert them into mixed fractions and then compare.

Illustration 7: Compare $\frac{11}{4}$ and $\frac{14}{5}$.

Convert the given fractions into mixed fractions.

$\frac{11}{4} = 2\frac{3}{4}$ and $\frac{14}{5} = 2\frac{4}{5}$

To compare $2\frac{3}{4}$ and $2\frac{4}{5}$, compare the proper fraction parts as the whole number parts are the same.

$\frac{3}{4} = \frac{15}{20}$ and $\frac{4}{5} = \frac{16}{20}$

($\because$ LCM of 4 and 5 = 20)

Since 15 < 16, $\frac{15}{20} < \frac{16}{20}$.

Therefore, $2\frac{15}{20} < 2\frac{16}{20}$ or $2\frac{3}{4} < 2\frac{4}{5}$.

Thus, to compare two fractions and write the inequality statement between them:

Step 1: change the given fractions to equivalent fractions with the LCM of the denominators as the common denominator.

Step 2: compare the new fractions through their numerators.

Step 3: replace the new fractions by the original ones.

ASCENDING AND DESCENDING ORDERS OF FRACTIONS

When a group of fractions is arranged in order from the smallest to the largest, they are said to be in **ascending order** and when arranged from the largest to the smallest, they are said to be in **descending order**.

Consider the fractions $\frac{2}{5}$, $\frac{3}{4}$ and $\frac{7}{10}$. Since these fractions are unlike, we first convert them into like fractions.

LCM of the denominators 5, 4 and 10 = 20.

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}, \quad \frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20},$$

$$\frac{7}{10} = \frac{7 \times 2}{10 \times 2} = \frac{14}{20}$$

Since 8 < 14 < 15, so $\frac{8}{20} < \frac{14}{20} < \frac{15}{20}$

or $\frac{2}{5} < \frac{7}{10} < \frac{3}{4}$.

Therefore, the fractions in ascending order are $\frac{2}{5}, \frac{7}{10}, \frac{3}{4}$ and in descending order are $\frac{3}{4}, \frac{7}{10}, \frac{2}{5}$.

Thus, to list a group of fractions in ascending or descending order:

Step 1: find the LCM of the denominators.

Step 2: convert the unlike fractions into like fractions.

Step 3: arrange the new fractions in the desired order of their numerators.

Step 4: replace each of the new fractions by the original fraction.

Let us study some more examples.

Ex. 17. Identify the larger of the two fractions and write the inequality in each case.

(a) $\frac{3}{9}, \frac{5}{12}$ **(b)** $2\frac{3}{4}, \frac{17}{36}$

Sol. (a) The LCM of 9 and 12 is 36.

So, $\frac{3}{9} = \frac{3\times4}{9\times4} = \frac{12}{36}, \frac{5}{12} = \frac{5\times3}{12\times3} = \frac{15}{36}$

Now, $\frac{12}{36}$ is smaller than $\frac{15}{36}$.

(Since, 12 < 15).

Thus, $\frac{3}{9}$ is smaller than $\frac{5}{12}$.

Therefore, the inequality statement is $\frac{3}{9} < \frac{5}{12}$.

(b) The two fractions $2\frac{3}{4}$ and $\frac{17}{36}$ can be compared with the knowledge of proper and improper fractions.

$2\frac{3}{4} = 2 + \frac{3}{4}$, which is more than 2.

$\frac{17}{36} < 1$ (It is a proper fraction.)

Thus, $2\frac{3}{4} > \frac{17}{36}$.

Note

Mixed numbers (improper fractions) are always larger than proper fractions.

Skill Check

- Identify, whether the given statement is true or false.

(a) $\frac{2}{5} > \frac{3}{4}$ (b) $\frac{11}{30} < \frac{7}{18}$

- Write the following fractions in ascending and descending orders.

(a) $\frac{1}{7}, \frac{5}{7}, \frac{3}{7}, \frac{6}{7}, \frac{4}{7}$ (b) $\frac{3}{4}, \frac{7}{12}, \frac{12}{5}$

Ex. 18. List the mixed fractions $7\frac{5}{8}, 7\frac{1}{2}, 7\frac{9}{16}$ in ascending order.

Sol. Since the whole number part is the same in each mixed fraction, let us compare their proper fractional parts. Keeping the whole number part as it is, convert the fractional parts to equivalent fractions with the LCM of the denominators as the common denominator.

The LCM of 8, 2 and 16 is 16.

We have, $7\frac{5}{8} = 7\frac{10}{16}, 7\frac{1}{2} = 7\frac{8}{16}$ and $7\frac{9}{16} = 7\frac{9}{16}$.

Listing the new fractions in ascending order of the numerators, we have

$7\frac{8}{16} < 7\frac{9}{16} < 7\frac{10}{16}$ (Since 8 < 9 < 10)

or $7\frac{1}{2} < 7\frac{9}{16} < 7\frac{5}{8}$

Thus, the fractions in the desired order are $7\frac{1}{2}, 7\frac{9}{16}$ and $7\frac{5}{8}$.

Ex. 19 Sidhant solved $\frac{2}{7}$ part of an exercise while Sarthak solved $\frac{4}{5}$ of it. Who solved the greater part?

Sol. To find the greater part of the exercise, we compare $\frac{2}{7}$ and $\frac{4}{5}$.

We change them to like fractions.

LCM of 7 and 5 is 35.

Therefore, $\frac{2}{7} = \frac{2 \times 5}{7 \times 5} = \frac{10}{35}$

and $\frac{4}{5} = \frac{4 \times 7}{5 \times 7} = \frac{28}{35}$.

Since 28 > 10, so $\frac{28}{35} > \frac{10}{35}$, *i.e.*, $\frac{4}{5} > \frac{2}{7}$.

Thus, Sarthak solved the greater part of the exercise.

Ex. 20 Latika read 20 pages of a book containing 80 pages. Kirti read $\frac{1}{3}$ of the same book. Who read more?

Sol. Total number of pages in the book = 80

Number of pages read by Latika = 20

Fraction of the book read by Latika

$= \frac{20}{80} = \frac{1}{4}$

Fraction of the book read by Kirti $= \frac{1}{3}$

Since $\frac{1}{3} > \frac{1}{4}$, therefore Kirti read more.

Exercise 8.4

1. Classify the set of fractions as like or unlike.

(a) $\frac{3}{11}, \frac{5}{11}, \frac{8}{11}, \frac{2}{11}$ (b) $\frac{12}{11}, \frac{5}{12}, \frac{3}{8}, \frac{8}{15}$ (c) $\frac{2}{10}, \frac{13}{10}, \frac{7}{10}, \frac{6}{10}$ (d) $\frac{8}{11}, \frac{8}{10}, \frac{8}{9}, \frac{8}{14}$

2. Convert the following fraction pairs into like fractions.

(a) $\frac{8}{15}, \frac{3}{5}$ (b) $\frac{11}{20}, \frac{7}{10}$ (c) $\frac{8}{15}, \frac{4}{21}$ (d) $\frac{7}{15}, \frac{11}{20}$

3. Fill in each box with the proper symbols <, > or = .

(a) $\frac{8}{7}$ ☐ 1 (b) $\frac{99}{100}$ ☐ 1 (c) $\frac{5}{12}$ ☐ $\frac{5}{8}$ (d) $\frac{6}{13}$ ☐ $\frac{3}{8}$

4. Identify the larger of the two fractions.

(a) $\frac{3}{4}, \frac{5}{6}$ (b) $\frac{5}{8}, \frac{7}{12}$ (c) $\frac{7}{8}, \frac{10}{11}$ (d) $\frac{4}{5}, \frac{5}{7}$

5. Write True (T) or False (F) for the following statements.

(a) $\frac{7}{8} > \frac{7}{10}$ (b) $\frac{4}{11} < \frac{5}{9}$ (c) $\frac{5}{6} < \frac{7}{8}$ (d) $2\frac{3}{5} < 2\frac{2}{3}$

6. Which of the following fractions has the maximum value?

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$

7. Arrange the following fractions in descending order.

(a) $\frac{7}{10}, \frac{11}{15}, \frac{2}{5}$ (b) $\frac{3}{4}, \frac{11}{16}, \frac{25}{32}$ (c) $\frac{5}{12}, \frac{5}{6}, \frac{19}{24}$

8. List the following mixed fractions (numbers) in ascending order.

(a) $4\frac{2}{3}, 4\frac{1}{5}, 4\frac{3}{7}$ (b) $5\frac{1}{6}, 5\frac{2}{3}, 5\frac{3}{4}$

9. Renu solved $\frac{2}{3}$ part of an exercise while Shilpa solved $\frac{3}{4}$ of it. Who solved the greater part?

10. Rakhi read 40 pages of a book containing 90 pages while Minu read $\frac{2}{5}$ of the book. Who read less?

ADDITION OF FRACTIONS

We already know that, one half and one half of a whole together make 1 whole, *i.e.*, $\frac{1}{2} + \frac{1}{2} = 1.$

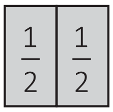


Fig. 8.19

But what about $\frac{2}{3} + \frac{1}{4}$? In this section, we shall learn addition of two or more fractions.

Addition of Like Fractions

Illustration 1: Let us find the sum of $\frac{2}{5}$ and $\frac{1}{5}$.

Notice that $\frac{2}{5}$ and $\frac{1}{5}$ are like fractions.

Observe that in the Fig. 8.20, a rectangular unit is divided into 5 equal parts.

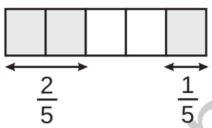


Fig. 8.20

The total number of shaded parts are 3 and together they represent the fraction $\frac{3}{5}$.

Therefore, $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$.

Illustration 2: Now, let us find the sum of $\frac{3}{7} + \frac{5}{7}$.

Three parts are shaded in the left rectangle and five parts are shaded in the right rectangle.

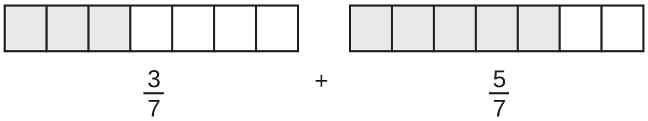


Fig. 8.21

Combining them we get eight shaded parts as shown in Fig. 8.22.

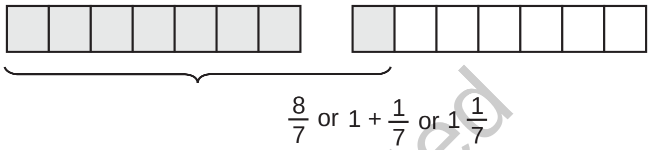


Fig. 8.22

Therefore, $\frac{3}{7} + \frac{5}{7} = \frac{8}{7} = 1\frac{1}{7}$.

(Sum of numerators or shaded parts)

(Common denominator or the number of equal parts a rectangle divided into)

Let us represent this addition on the number line.

To add $\frac{3}{7}$ and $\frac{5}{7}$ on a number line, divide each unit of the number line into 7 equal parts.

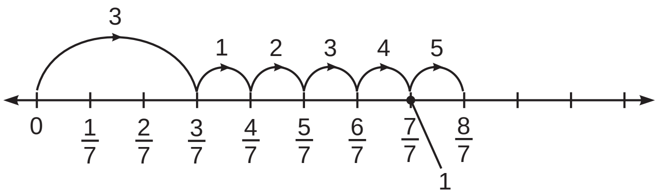


Fig. 8.23

From the third part $\left(\frac{3}{7}\right)$, move five parts $\left(\frac{5}{7}\right)$ to the right and reach at $\frac{8}{7}$.

In general, it can be stated as follows:

The sum of like fractions = $\frac{\text{Sum of the numerators}}{\text{Common denominator}}$

Illustration 3: $\frac{4}{7} + \frac{2}{7} + \frac{5}{7} = \frac{4+2+5}{7}$

$= \frac{11}{7}$ or $1\frac{4}{7}$

Skill Check

Add using a number line.

(a) $\frac{2}{5}+\frac{4}{5}$ (b) $\frac{3}{8}+\frac{7}{8}$

Addition of Unlike Fractions

Illustration 4: Let us find the sum of $\frac{1}{6}$ and $\frac{1}{3}$.

Notice that $\frac{1}{6}$ and $\frac{1}{3}$ are unlike fractions.

First, the LCM of the denominators 3 and 6 is 6.

$$\frac{1}{6}=\frac{1}{6} \text{ and } \frac{1}{3}=\frac{1\times 2}{3\times 2}=\frac{2}{6}$$

Now, $\frac{1}{6}+\frac{1}{3}=\frac{1}{6}+\frac{2}{6}=\frac{1+2}{6}$

$$=\frac{3}{6}=\frac{3\div 3}{6\div 3}=\frac{1}{2}$$

Thus, to add unlike fractions:

Step 1: convert the fractions into like fractions with the LCM of the denominators as the common denominator.

Step 2: add the converted like fractions by adding their numerators and retaining the common denominator.

Step 3: simplify the sum to the lowest terms.

Watch Your Step!

$\frac{2}{5}+\frac{2}{3}=\frac{2+2}{5+3}=\frac{4}{8}$ or $\frac{1}{2}$, which is wrong.

$\frac{2}{5}+\frac{2}{3}=\frac{2\times 3+2\times 5}{5\times 3}=\frac{6+10}{15}=\frac{16}{15}$ or $1\frac{1}{15}$ is correct.

Addition of Mixed Fractions

Here, we will explain addition of mixed fractions through illustrations.

Illustration 5: $1\frac{2}{5}+2\frac{1}{5}=\frac{7}{5}+\frac{11}{5}$

(Changing to improper fractions)

$$=\frac{7+11}{5}=\frac{18}{5}=3\frac{3}{5}$$

Illustration 6: $5\frac{1}{2}+7\frac{2}{3}=\frac{11}{2}+\frac{23}{3}$

(Changing to improper fractions)

$$=\frac{11\times 3}{2\times 3}+\frac{23\times 2}{3\times 2} \quad (\because \text{LCM of 2 and 3} = 6)$$

$$=\frac{33}{6}+\frac{46}{6}=\frac{33+46}{6}=\frac{79}{6}=13\frac{1}{6}$$

Let us study some more examples.

Ex. 21 Find the sum of $\frac{2}{3}+\frac{1}{4}$.

Sol. The LCM of 3 and 4 = 12

$$\frac{2}{3}=\frac{2\times 4}{3\times 4}=\frac{8}{12}$$

(Multiply the numerator and denominator by 4)

$$\frac{1}{4}=\frac{1\times 3}{4\times 3}=\frac{3}{12}$$

(Multiply the numerator and denominator by 3)

Thus, $\frac{2}{3}+\frac{1}{4}=\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$

Ex. 22 Find the sum of $\frac{2}{9}+\frac{5}{7}+\frac{13}{3}$.

Sol. $\frac{2}{9}+\frac{5}{7}+\frac{13}{3}=\frac{14}{63}+\frac{45}{63}+\frac{273}{63}$

($\because$ LCM of 9, 7 and 3 = 63)

$$=\frac{14+45+273}{63}=\frac{332}{63} \text{ or } 5\frac{17}{63}$$

(The sum is already in its lowest terms.)

Ex. 23 Add $2\frac{1}{2}+4\frac{2}{3}+1\frac{3}{4}$ and write the sum as a mixed fraction.

Sol. $2\frac{1}{2}+4\frac{2}{3}+1\frac{3}{4}=\frac{5}{2}+\frac{14}{3}+\frac{7}{4}$

(Writing in improper form)

$$=\frac{5\times 6}{2\times 6}+\frac{14\times 4}{3\times 4}+\frac{7\times 3}{4\times 3} \quad (\because \text{LCM of 2, 3, 4} = 12)$$

$$=\frac{30}{12}+\frac{56}{12}+\frac{21}{12}$$

$$=\frac{30+56+21}{12}=\frac{107}{12}=8\frac{11}{12}$$

Ex. 24 **Namita bought $\frac{3}{5}$ m of a lace and Kavita bought $\frac{3}{4}$ m of the same type of lace. Find the total length of the lace they bought.**

Sol. Length of the lace bought by Namita

$$= \frac{3}{5} \text{ m}$$

Length of the lace bought by Kavita

$$= \frac{3}{4} \text{ m}$$

Total length of the lace $= \left(\frac{3}{5} + \frac{3}{4}\right)$ m

$$= \left(\frac{3 \times 4}{5 \times 4} + \frac{3 \times 5}{4 \times 5}\right) \text{ m}$$

($\because$ LCM of 4 and 5 = 20, we write the given fractions as equivalent fractions with denominator as LCM.)

$$= \left(\frac{12}{20} + \frac{15}{20}\right) \text{ m} = \frac{12 + 15}{20} \text{ m}$$

$$= \frac{27}{20} \text{ m} = 1\frac{7}{20} \text{ m}$$

Therefore, they bought total $1\frac{7}{20}$ m of the lace.

Ex. 25 **Ankita purchased $2\frac{1}{2}$ kg potatoes and $1\frac{3}{4}$ kg tomatoes. What is the total weight of vegetables purchased by her?**

Sol. Total weight of vegetables = Weight of potatoes + Weight of tomatoes

$$= \left(2\frac{1}{2} + 1\frac{3}{4}\right) \text{ kg} = \left(\frac{5}{2} + \frac{7}{4}\right) \text{ kg}$$

$$= \left(\frac{5 \times 2}{2 \times 2} + \frac{7}{4}\right) \text{ kg} = \left(\frac{10}{4} + \frac{7}{4}\right) \text{ kg}$$

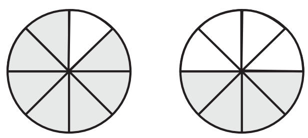
$$= \left(\frac{10 + 7}{4}\right) \text{ kg} = \frac{17}{4} \text{ kg} = 4\frac{1}{4} \text{ kg}$$

Therefore, Ankita purchased total $4\frac{1}{4}$ kg of vegetables.

Exercise 8.5

1. **Answer the following questions.**

   (a) Find the sum of the shaded portions of the given circles.



   (b) What is the value of $5\frac{2}{3} + 3\frac{2}{3}$?

   (c) Find the value of a in $\frac{3}{4} + \frac{7}{12} = \frac{a}{12}$.

2. **Add the following fractions and reduce the sum to its lowest terms.**

   (a) $\frac{1}{8} + \frac{3}{8}$ (b) $\frac{7}{9} + \frac{5}{9}$ (c) $\frac{5}{12} + \frac{4}{12}$ (d) $\frac{2}{3} + \frac{5}{6}$ (e) $\frac{5}{8} + \frac{3}{4} + \frac{1}{2}$ (f) $\frac{3}{5} + \frac{7}{15} + \frac{1}{10}$

3. **Add and write the sum as a mixed fraction.**

   (a) $5\frac{1}{5} + 2\frac{3}{5}$ (b) $4\frac{2}{7} + 1\frac{1}{2}$ (c) $7\frac{1}{3} + 8\frac{1}{6}$ (d) $4\frac{2}{3} + 1\frac{1}{5} + 6\frac{2}{3}$ (e) $6\frac{1}{2} + 2\frac{3}{4} + 3\frac{1}{8}$ (f) $3\frac{4}{5} + 5 + \frac{2}{15}$

4. The diagram given represents a cake Sujata baked. Sujata ate 2 slices and gave her friends 3 slices. What fraction of the cake did Sujata and her friends eat?

5. Rahim mixes $\frac{2}{3}$ L of yellow paint with $\frac{3}{4}$ L of blue paint to make green paint. How much green paint (in litres) does he make?

6. Sohan painted $\frac{3}{5}$ parts of the picture and her sister painted $\frac{1}{3}$ part of the same picture. What fraction of picture did they paint together?
7. Pankaj studies for $4\frac{1}{2}$ hours at home and watches TV for $\frac{3}{4}$ hour. Find the total time spent by him in studying and watching TV.
8. Veena bought a notebook for ₹$15\frac{3}{4}$ and a pen for ₹$8\frac{1}{2}$. How much money should she pay to the shopkeeper?
9. The weight of an empty gas cylinder is $16\frac{2}{3}$ kg and it contains $14\frac{2}{5}$ kg of gas. What is the weight of the cylinder filled with gas?
10. One fine morning Rakesh walked $4\frac{1}{2}$ km, Ritu walked $5\frac{1}{3}$ km and Mukherji walked $4\frac{1}{6}$ km. Find the total distance covered by them.
11. Kirti bought two pieces of ribbons measuring $5\frac{1}{6}$ m and $3\frac{3}{4}$ m. She added one more piece of length $4\frac{1}{12}$ m to it. Find the total length of the ribbon with her.

SUBTRACTION OF FRACTIONS

In the previous section, we have learnt to add two or more fractions. Now, we shall learn subtraction of fractions.

Subtraction of Like Fractions

Let us consider the Fig. 8.24 (a). The shaded part is $\frac{5}{7}$.

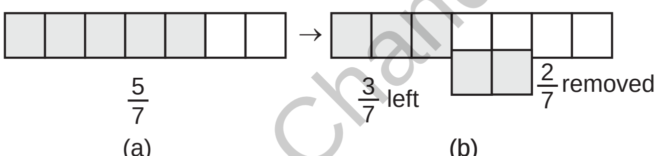


Fig. 8.24

When $\frac{2}{7}$ shaded parts are removed. It is left with $\frac{3}{7}$ shaded parts,

i.e., $\frac{5}{7}-\frac{2}{7}=\frac{3}{7} \leftarrow (5-2)$

Subtraction of like fractions can also be illustrated with the help of a number line.

To subtract $\frac{3}{5}$ from $\frac{7}{5}$ on a number line, divide each unit on the number line into 5 equal parts.

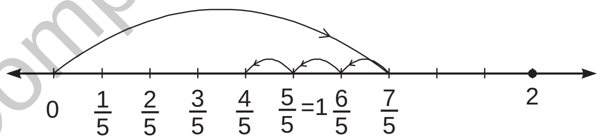


Fig. 8.25

Recall that subtracting a positive number on a number line means moving to the left.

From the seventh part $\left(\frac{7}{5}\right)$, move three parts $\left(\frac{3}{5}\right)$ to the left and reach $\frac{4}{5}$.

Therefore, $\frac{7}{5}-\frac{3}{5}=\frac{4}{5}$.

In general, it can be stated as follows:

The difference of like fractions

$$= \frac{\text{The difference of the numerators}}{\text{The common denominator}}$$

Skill Check

Subtract using a number line.

(a) $\frac{4}{7}-\frac{2}{7}$ (b) $\frac{17}{10}-\frac{7}{10}$

Subtraction of Unlike Fractions

Fractions with different denominators can be subtracted, just as in addition, by converting the fractions into equivalent fractions with the LCM of the denominators as the common denominator.

Illustration 1: Let us find $\frac{2}{3} - \frac{1}{2}$.

We have,

$$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} \quad (\because \text{ LCM of 3 and 2 is 6.})$$

$$= \frac{4-3}{6} = \frac{1}{6}$$

Thus, to subtract unlike fractions:

Step 1: Convert the fractions into like fractions.

Step 2: Subtract the numerator of the new smaller fraction from the numerator of the new greater fraction and retain the common denominator.

Step 3: Simplify the resulting fraction to its lowest terms, if needed.

Subtraction of Mixed Fractions

The process of subtracting two mixed fractions is similar to that for adding two mixed fractions.

Illustration 2: Let us find $4\frac{3}{5} - 3\frac{3}{7}$.

$$4\frac{3}{5} - 3\frac{3}{7} = \frac{23}{5} - \frac{24}{7}$$

(Converting into improper fractions)

$$= \frac{23 \times 7}{5 \times 7} - \frac{24 \times 5}{7 \times 5} \quad (\because \text{ LCM of 5 and 7 = 35})$$

$$= \frac{161}{35} - \frac{120}{35} = \frac{161-120}{35} = \frac{41}{35} = 1\frac{6}{35}$$

Let us study some more examples.

Ex. 26 Subtract $1\frac{7}{8}$ from 4.

Sol. We first express each fraction as improper fractions and then subtract.

$$4 - 1\frac{7}{8} = \frac{4}{1} - \frac{15}{8} = \frac{4 \times 8}{1 \times 8} - \frac{15 \times 1}{8 \times 1}$$

$(\because$ LCM of 1 and 8 = 8)

$$= \frac{32}{8} - \frac{15}{8} = \frac{32-15}{8} = \frac{17}{8} = 2\frac{1}{8}$$

Ex. 27 Simplify: $2\frac{4}{5} - 1\frac{5}{7}$

Sol. First convert mixed fractions into improper fractions, then proceed as usual.

$$2\frac{4}{5} - 1\frac{5}{7} = \frac{14}{5} - \frac{12}{7}$$

$$= \frac{14 \times 7}{5 \times 7} - \frac{12 \times 5}{7 \times 5}$$

$(\because$ LCM of 5 and 7 is 35.)

$$= \frac{98}{35} - \frac{60}{35} = \frac{38}{35} = 1\frac{3}{35}$$

Ex. 28 Asmita's house is $\frac{7}{10}$ km from her school. She took a bus for $\frac{1}{2}$ km and then walked some distance to reach her school. How far did she walk?

Sol. The distance of the school from Asmita's house = $\frac{7}{10}$ km

Distance covered by bus = $\frac{1}{2}$ km

Distance she walked = $\left(\frac{7}{10} - \frac{1}{2}\right)$ km

$$= \left(\frac{7}{10} - \frac{1}{2} \times \frac{5}{5}\right) \text{ km} = \left(\frac{7}{10} - \frac{5}{10}\right) \text{ km}$$

$$= \frac{2}{10} \text{ km} = \frac{1}{5} \text{ km}$$

Therefore, she walked $\frac{1}{5}$ km.

Ex. 29 Sujata and Debashish have bookshelves of the same size filled with books. Sujata's shelf is $\frac{3}{5}$th full with books and Debashish shelf is $\frac{4}{7}$th full. Whose bookshelf has more books and by how much?

Sol. Fraction of Sujata's bookshelf covered with books = $\frac{3}{5}$

Fraction of Debashish's bookshelf covered with books = $\frac{4}{7}$

To compare these two fractions, we convert them into like fractions.

LCM of 5 and 7 = 35

$\frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$ and $\frac{4}{7} = \frac{4\times5}{7\times5} = \frac{20}{35}$

Since $\frac{21}{35} > \frac{20}{35}$, therefore $\frac{3}{5} > \frac{4}{7}$.

So, Sujata's bookshelf has more books.

Also, the fraction (part of the shelf) in which Sujata has more books is

$\frac{21}{35} - \frac{20}{35} = \frac{21-20}{35} = \frac{1}{35}$.

Ex. 30 Mohit takes $3\frac{1}{3}$ minutes to take a full round of the school ground and Karan takes $\frac{13}{4}$ minutes to do the same. Who takes less time and by how much?

Sol. Time taken by Mohit to complete a round

$= 3\frac{1}{3}$ minutes $= \frac{10}{3}$ minutes

Time taken by Karan to complete a round

$= \frac{13}{4}$ minutes

First, we convert both the fractions into like fractions.

LCM of 3 and 4 = 12

$\frac{10}{3} = \frac{10\times4}{3\times4} = \frac{40}{12}$ and $\frac{13}{4} = \frac{13\times3}{4\times3} = \frac{39}{12}$

So, $\frac{40}{12} > \frac{39}{12}$ or $\frac{10}{3} > \frac{13}{4}$.

Therefore, Karan takes less time.

Also, Karan takes $\left(\frac{40}{12} - \frac{39}{12}\right)$ minute $= \frac{1}{12}$ minute less than Mohit to complete a round.

Ex. 31 Anita filled 20 litres of petrol in her car in the morning. During the day, she used $16\frac{1}{5}$ litres of petrol. How much petrol was left in the tank in the evening?

Sol. Quantity of petrol in her car in the morning = 20 litres

Quantity of petrol used during the day

$= 16\frac{1}{5}$ litres $= \frac{81}{5}$ litres

Remaining petrol in the car in the evening

$= \left(20 - \frac{81}{5}\right)$ litres $= \left(\frac{20\times5}{5} - \frac{81}{5}\right)$ litres

$= \left(\frac{100}{5} - \frac{81}{5}\right)$ litres $= \left(\frac{100-81}{5}\right)$ litres

$= \frac{19}{5}$ litres $= 3\frac{4}{5}$ litres

Therefore, $3\frac{4}{5}$ litres petrol was left in the evening.

Ex. 32 Mrs Gupta bought $7\frac{1}{2}$ litres of milk. She consumed $2\frac{3}{4}$ litres of milk in the morning and $3\frac{4}{5}$ litres of milk in the evening. How much milk was left with her?

Sol. Total quantity of milk with Mrs Gupta

$= 7\frac{1}{2}$ litres $= \frac{15}{2}$ litres

Quantity of milk consumed in the morning

$= 2\frac{3}{4}$ litres $= \frac{11}{4}$ litres

Quantity of milk consumed in the evening

$= 3\frac{4}{5}$ litres $= \frac{19}{5}$ litres

Quantity of milk left with her

$= \left(\frac{15}{2} - \frac{11}{4} - \frac{19}{5}\right)$ litres

$= \left(\frac{15\times10}{2\times10} - \frac{11\times5}{4\times5} - \frac{19\times4}{5\times4}\right)$ litres

($\because$ LCM of 2, 4 and 5 = 20)

$= \left(\frac{150}{20} - \frac{55}{20} - \frac{76}{20}\right)$ litres

$= \left(\frac{150-55-76}{20}\right)$ litres

$= \frac{19}{20}$ litre.

Exercise 8.6

1. Subtract:

(a) $\frac{9}{10} - \frac{7}{10}$ (b) $\frac{8}{15} - \frac{3}{15}$ (c) $1 - \frac{2}{3}$ (d) $\frac{2}{5} - \frac{4}{15}$

2. Subtract as directed.

(a) $\frac{4}{9}$ from $\frac{5}{6}$ (b) $5\frac{1}{3}$ from 8 (c) $4\frac{1}{7}$ from $6\frac{3}{7}$ (d) $3\frac{3}{5}$ from $7\frac{2}{3}$

3. Find the difference.

(a) $7 - 3\frac{4}{5}$ (b) $8\frac{2}{3} - 5\frac{1}{4}$ (c) $3\frac{3}{4} - 1\frac{3}{8}$

4. What should be subtracted from $17\frac{2}{3}$ to get $12\frac{3}{5}$?

5. Simplify the following.

(a) $2\frac{1}{4} + 1\frac{1}{2} - \frac{3}{8}$ (b) $\frac{9}{10} - \frac{1}{4} - \frac{2}{5}$ (c) $10 - 4\frac{2}{3} - 2\frac{3}{4}$ (d) $4\frac{3}{7} + 2\frac{4}{7} - 1\frac{3}{14}$

6. In the number sentence below, which number does Δ represent?

$3\frac{1}{4} + \frac{\Delta}{4} = 4$

7. Anisha ate $\frac{1}{3}$ of a cake, Pranshi ate $\frac{1}{5}$ and Riya ate $\frac{1}{6}$ of the cake. What part of the cake was still uneaten?

8. A wire of length $3\frac{3}{4}$ long, broke into two pieces. If one piece is $1\frac{1}{2}$ m long, what is the length of the other piece?

9. Mrs Bajaj bought $8\frac{1}{2}$ litres of milk. Out of this, $5\frac{3}{4}$ litres was consumed. How much milk is left with her?

10. Deepak takes $5\frac{1}{2}$ minutes to cross the bridge and Rahul takes $\frac{13}{4}$ minutes to do the same. Who takes more time and by how much?

11. A man brought $3\frac{1}{2}$ buckets of paint to paint a hall. He used $1\frac{3}{4}$ buckets to paint the hall. How much paint is left with him?

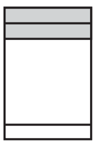
12. The perimeter of a triangle is $14\frac{2}{5}$ m. If two of its sides are $4\frac{4}{5}$ m and $5\frac{3}{5}$ m respectively, find the length of the third side.

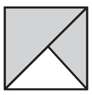
Competency Based Exercise

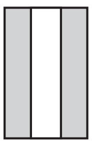
21st CS

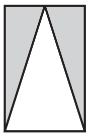
1. Tick (✓) the correct answer.

(a) Which one shows $\frac{2}{3}$ of the shaded picture?

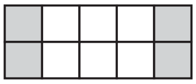
(i) 

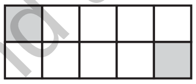
(ii) 

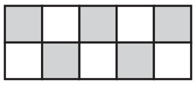
(iii) 

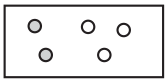
(iv) 

(b) Which picture shows that $\frac{2}{5}$ is equivalent to $\frac{4}{10}$?

(i) 

(ii) 

(iii) 

(iv) 

(c) What fraction of a straight angle is a right angle?

(i) $\frac{1}{3}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{4}$ (iv) $\frac{1}{8}$

(d) The energy content of different foods are shown in the given table.

Food	Milk	Potatoes (cooked)	Rice	Wheat
Energy content (per kg)	$3\frac{1}{2}$ joules	$3\frac{7}{10}$ joules	$5\frac{3}{10}$ joules	$3\frac{1}{5}$ joules

The food which provides minimum energy is:

(i) milk (ii) potatoes (iii) rice (iv) wheat

(e) The food which we eat remains in the stomach for a maximum of four hours. For what fraction of a day, does it remain there?

(i) $\frac{1}{4}$ (ii) $\frac{1}{6}$ (iii) $\frac{4}{12}$ (iv) $\frac{1}{24}$

(f) What fraction represents the number of prime numbers in the first twenty positive integers?

(i) $\frac{1}{2}$ (ii) $\frac{2}{5}$ (iii) $\frac{8}{19}$ (iv) $\frac{9}{20}$

(g) Garima's father got a job at the age of 20 years and he got retired from the job at the age of 60 years. What fraction of his age till retirement was he in the job?

(i) $\frac{1}{3}$ (ii) $\frac{2}{3}$ (iii) $\frac{1}{2}$ (iv) $\frac{1}{4}$

2. Simplify the following.

(a) $\frac{3}{5}+\frac{2}{7}$ (b) $8\frac{2}{7}-3$ (c) $4-3\frac{1}{4}$ (d) $5\frac{3}{4}+3\frac{1}{8}$ (e) $1\frac{7}{15}+7\frac{1}{3}-2\frac{4}{5}$ (f) $10\frac{2}{5}-\left(2\frac{3}{4}+4\frac{1}{2}\right)$

3. Arrange $\frac{7}{10}, \frac{4}{5}, \frac{7}{8}$ and $\frac{3}{4}$ in ascending order.

4. Vinod travelled $4\frac{1}{2}$ km by bus and then walked $1\frac{2}{3}$ km to reach his home. How much did he travel to reach his home?

5. Rashmi gave $3\frac{3}{4}$ litres out of the $7\frac{1}{2}$ litres of juice she purchased to her friends. How many litres of juice is left with her?

6. In a class test, Shuchi got 14 marks out of 20 in Maths and 20 marks out of 30 in Science. In which subject her performance was better?

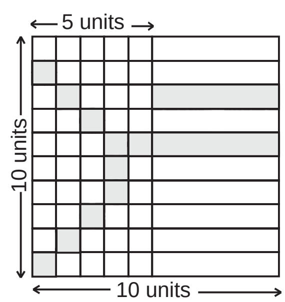
7. Subtract the sum of $3\frac{4}{5}$ and $2\frac{2}{3}$ from $9\frac{1}{5}$.

8. Alisha wants to finish a novel as soon as possible. So far, she has read only 54 pages. What fraction of the number of pages has she read, if there are 180 pages in the novel?

9. When Sunita weighed herself on Monday, she found that she had gained $1\frac{1}{4}$ kg. If her weight on Monday was $48\frac{3}{8}$ kg, what was her weight earlier?

Challenge! 21st CS

1 What fraction is represented by the unshaded portion in the given figure?



2 Subtract the sum of $4\frac{3}{4}$ and $5\frac{1}{2}$ from the sum of $2\frac{1}{4}$ and $9\frac{1}{8}$.

Let's Work in Mind

21st CS

1. What is $\frac{1}{2}+\frac{1}{4}$?
2. To find the sum $\frac{3}{4}+\frac{1}{8}+\frac{1}{2}$, which number would you use as a common denominator?
3. Raju drilled a hole that is $\frac{5}{8}$ inch wide. He has a screw that is $\frac{5}{4}$ inches wide. Is the hole wide enough to fit the screw?
4. For what value of x, $\frac{3}{4}$ is equivalent to $\frac{x}{20}$?
5. By how much is 3 less than $8\frac{2}{3}$?
6. Between which two consecutive integers will the fraction $\frac{3}{8}$ lie on a number line?
7. What is the value of $1-\frac{1}{2}-\frac{1}{3}$?

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

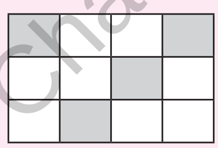
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

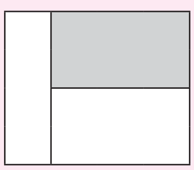
1. **Assertion (A)** :



Shaded portion represents fraction = $\frac{4}{12}$ or $\frac{1}{3}$

Reason (R) : $\frac{4}{12}$ and $\frac{1}{3}$ are equivalent fractions.

2. **Assertion (A)** :



Shaded portion represents fraction = $\frac{1}{3}$

Reason (R) : To represent fractions all parts should be equal.

3. Assertion (A) :

1	2	3	4

Fraction representing Part (2) $\neq \frac{1}{4}$

Reason (R) : All parts of given figure are equal.

4. Assertion (A) : $\frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{16}{32} = \frac{32}{64}$

Reason (R) : Improper fractions are equivalent fractions.

5. Assertion (A) : $2\frac{3}{4} < 5\frac{1}{6}$

Reason (R) : Whole number parts of fractions are compared to compare mixed fractions.

6. Assertion (A) : $2\frac{3}{4} > 5\frac{1}{4}$

Reason (R) : Fractional parts of fractions are compared to compare mixed fractions.

7. Assertion (A) : $\frac{2}{3}$ and $\frac{1}{3}$ are like fractions.

Reason (R) : If two fractions have the same denominator, the one with greater numerator is greater.

CASE STUDY

CC

On receiving the message from Jal Board about repair of water lines, Resident Welfare Association arranged water tanker for its residents. There are 40 houses in the colony.

Each of the first 25 houses collected $24\frac{1}{2}$ litres of water while the remaining 15 houses collected $20\frac{1}{2}$ litres each.

1. How much water collected by the first 25 houses in all?
2. How much water collected by the last 15 houses in all?
3. What is the total capacity of the water tank if 20 litres more water remains in the tank?

9 Decimals

What Learners Will Achieve

- express one-tenth, one-hundredth, etc., as decimal numbers.
- read and write the decimal numbers in words and figures.
- understand decimal place value chart.
- represent decimal numbers on a number line.
- convert fractions into decimals and vice versa.
- compare decimal numbers.
- express the quantities given in smaller units as decimals in larger units.
- perform addition and subtraction on decimals.

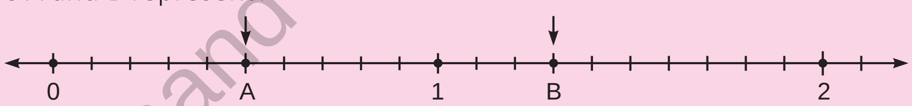
Warm-up

What we already know

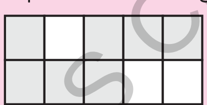
- The decimal number system, also called Hindu-Arabic number system, has base 10 and employs ten different symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to write numbers.
- The counting numbers and 0 are not sufficient to write numbers of all sizes, large or small. But using a unique symbol (**.**) called **decimal point**, it is possible.
- Decimal fractions are special types of fractions which have denominators 10, 100, 1000, etc.

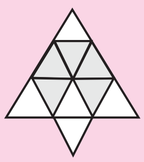
Now, try to solve the following.

1. Each unit of the number line has been divided into 10 equal parts. What decimal numbers do the letters A and B represent?

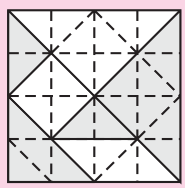


2. What parts of the figures have been shaded? Express as a decimal.

(a) 

(b) 

3. Look at the figure and answer the following questions.
   (a) Half of the figure is shaded. (True/False)
   (b) Write the fraction for the unshaded part of the figure. Write an equivalent fraction of this one with denominator 100 and then express it as a decimal.



DID YOU KNOW?

The word DECIMAL means 'based on 10'. This word is derived from the Latin word '*decima*', meaning a tenth part.

INTRODUCTION TO DECIMALS

In earlier classes, we have learnt that each place in the place value table has a value ten times the value of the next place on its right.

For example, the value of tens place is 10 times the value of ones place, the value of hundreds place is 10 times the value of tens place and so on.

Thousands	Hundreds	Tens	Ones
(1000)	(100)	(10)	(1)

Table 9.1

In other words, the value of a place is $\frac{1}{10}$ of the value of the next place on its left. Or we can say, while moving left to right in the place value table, the place value of each place becomes $\frac{1}{10}$th of the value of the place preceding it.

For example, the value of the hundreds place = 100 = $\frac{1}{10} \times 1000 = \frac{1}{10}$ of the value of thousands place, the value of tens place = 10 = $\frac{1}{10} \times 100 = \frac{1}{10}$ of the value of hundreds place, the value of ones place = 1 = $\frac{1}{10} \times 10 = \frac{1}{10}$ of the value of tens place.

What will happen, if we move one more place to the right in the place value table? The above discussion suggests that the value of that place will be $\frac{1}{10}$ of ones place, *i.e.*, $\frac{1}{10}$, read as **one-tenth**.

The value of the next place to the right of $\frac{1}{10}$ will be $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$, read as **one-hundredth**.

We observe that, the values of ones place and every place to the left of ones place is a whole number, whereas the value of each place to the right of ones place is a fraction. To distinguish between these two sets of values, we place a **dot** (**.**) between them. Places to the left of the dot (.) will be ones, tens, hundreds, ... in that order and places to the right of ones place will be tenths, hundredths, thousandths, ... and so on in that order.

We can now write $\frac{1}{10}$ as .1 or 0.1 using a dot (**.**) called a **decimal point**. 0.1 is called a **decimal number** or simply a **decimal**.

Need for Decimals

Decimals play an important role in different walks of life. We use decimals when we count money (₹24.45), place price tags on clothes or shoes (₹2549.95), display the time clocked by a competitor in a race (21.24 s) or score of a diver or a gymnast (9.12 s). Decimal system is a way to express large (3214500000000000 as 3.2145 × 10^{15}) or small (0.00000125) numbers. No wonder disciplines like science and engineering which require precise and accurate calculations use decimal system extensively.

Advantages

Decimal system has many advantages when compared with fractions. We mention a few.

1. **Easy to add:** To add $\frac{1}{2}$ and $\frac{1}{5}$, we have to first convert into like fractions and then add. It requires some effort. But adding 0.5 $\left(=\frac{1}{2}\right)$ and 0.2 $\left(=\frac{1}{5}\right)$ is very easy, $\begin{array}{r} 0.2 \\ +0.5 \\ \hline 0.7 \end{array}$ as they add up like normal numbers to give 0.7 (just line up the decimal points).
2. **Easy to know how big the number is:** It may be a little cumbersome to know how big is the number $\frac{24967}{97}$. But, if the number were 254.356 in decimal form, we can immediately say that it is slightly larger than 254.

3. **Easy to compare numbers:** If we have to compare $\frac{359}{45}$ and $\frac{256}{29}$, it will take some time. But if the numbers were written in decimal forms, the process of comparison is easy.

DECIMAL FRACTIONS AND DECIMAL NUMBERS

Consider the fractions like $\frac{1}{10}, \frac{31}{100}, \frac{9}{1000}, \frac{979}{10000}$.

These are special types of fractions in which the denominators are 10, 100, 1000 and so on. Such fractions are known as **decimal fractions.**

Numbers written using a decimal point are called **decimal numbers** or simply **decimals**.

We have, $\frac{1}{10} = .1$, $\frac{2}{10} = .2$, $\frac{3}{10} = .3$ and so on.

Here, .1, .2, .3, ... are decimal numbers.

We also write .1 as 0.1, .2 as 0.2, .3 as 0.3 and so on.

0.1 is read as zero point one. 0.2 is read as zero point two.

Similarly, $\frac{1}{100} = 0.01$, $\frac{2}{100} = 0.02$, $\frac{3}{100} = 0.03$, ...

0.01 is read as zero point zero one.

And $\frac{1}{1000} = 0.001$, $\frac{2}{1000} = 0.002$, $\frac{3}{1000} = 0.003$ and so on.

0.001 is read as zero point zero zero one.

Watch Your Step!

$\frac{15}{100}$ = .15 or 0.15. It is wrong to read as "decimal fifteen". It will be read as 'zero point one five'.

Think!

Why the digits after decimal point are not read as fifteen?

Note

If there is no whole number before the decimal point, then there is a practice to put '0' to the left of the decimal point.

Decimal Places

The number of digits to the right of the decimal point is the number of **decimal places.**

For example, the number **1.**$\underbrace{527}_{3}$ has **three** decimal places. (1.527 is read as one point five two seven.)

Parts of a Decimal Number

A decimal number consists of three parts—**a whole number part, a decimal point** and **a decimal part.**

In 57.629, the whole number part is 57 and the decimal part is 629.

Thus, 57.629 = 57 . 629

57 → Whole number; . → Decimal point; 629 → Decimal part

57.629 is read as fifty-seven point six two nine.

In the case of a whole number, the decimal point is understood to be present to the right of the digit at the ones place and '0' is written after the decimal point.

For example, 5 = 5.0, 75 = 75.0, 180 = 180.0 (↑ Decimal point in each)

Place Value Chart for Decimals

The advantage of decimal fractions (or decimals) over the other fractions lies on the fact that they are based on the same place value chart as the whole numbers. That is, decimals are written by using the standard place value table in the same way as the whole numbers.

The place value for the digits of a decimal is provided in the following chart.

Whole Number part — **Decimal part**

× 10   × 10   × 10   × 10   × 10   × 10   × 10   × 10   × 10   × 10

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
100,000	10,000	1000	100	10	1	.	$\frac{1}{10}$ = 0.1	$\frac{1}{100}$ = 0.01	$\frac{1}{1000}$ = 0.001	$\frac{1}{10000}$ = 0.0001	$\frac{1}{100000}$ = 0.00001

$\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$

Table 9.2

This chart (Table 9.2) is clearly an extension of the place value chart for the whole numbers.
We can write the decimal number 57.629 in the place value chart as follows:

Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
5	7	.	6	2	9

Table 9.3

Determining the Place Value of a Digit

To determine the place value of a digit, we do as follows:

(a) If the digit appears to the left of the decimal point, consider the entire group of the digits to the left of the decimal point as a whole number. The place value of this digit is the same as its place value in the whole number.

(b) If the digit appears to the right of the decimal point, its place value is determined by counting the number of decimal places for the digit being considered. If the digit is '*d*' and it is at the **3rd decimal place**, then its place value is $\frac{d}{1000}$.

If '*d*' is at the 2nd **decimal place**, then its place value is $\frac{d}{100}$ and so on.

Ex. 1. Determine the place values of the following digits in the decimal number 1752.3609.
(a) 6 (b) 0 (c) 7

Sol. (a) 1752.3609 (↑ 2nd decimal place)

The digit 6 appears to the right of the decimal point in the second decimal place. Thus, the place value of 6 in 1752.3609 is six-hundredths or $\frac{6}{100}$.

(b) 1752.3609 (↑ 3rd decimal place)

The digit 0 appears to the right of the decimal point in the third decimal place.
Thus, the place value of 0 in 1752.3609 is zero-thousandths or $\frac{0}{1000}$ or 0.

> **Remember**
>
> The place value of 0 is always 0 regardless of its place in the number.

(c) 1752.3609

Since 7 appears to the left of the decimal point, its place value is the same as in the whole number.

The place value of 7 in 1752.3609 is 700.

Writing Numbers in Decimals

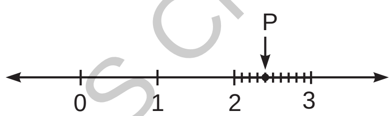
We write two ones and five-tenths as $2 + \frac{5}{10} = 2.5$, thirty-four and four-tenths as $34 + \frac{4}{10} = 34.4$, six hundred twenty-four and three-hundredths as $624 + \frac{3}{100} = 624 + 0.03 = 624.03$.

Representing Decimals on the Number Line

We have learnt the representation of whole numbers and fractions on a number line. Since decimals are also fractions, so we can represent decimals on a number line.

Illustration 1: Draw a number line and mark whole numbers 0, 1, 2, 3,...on it.

To represent 2.4 on the number line, we observe that 2.4 is more than 2 and less than 3.



2.4 = 2 + 0.4 or 2 + 4 tenths

Divide the portion between 2 and 3 into 10 equal parts and take 4 parts more from 2, which represents 4 tenths or 0.4. Mark this point as P. Thus, P represents the decimal number 2.4.

> **Skill Check**
>
> Between which two consecutive whole numbers on the number line does the number 4.8 lie?

Expanded Form of Decimals

We already know about the expanded form of a whole number.

For example, 278 = 2 × 100 + 7 × 10 + 8 × 1.

We shall now learn about expanded form of a decimal number. As in the case of whole numbers, the ***expanded form*** of a decimal number shows the place value of each digit. It is represented by the sum of the place values of each digit taken in order.

Thus, the expanded form of 0.475

= 4 tenths + 7 hundredths + 5 thousandths

$= 4 \times \frac{1}{10} + 7 \times \frac{1}{100} + 5 \times \frac{1}{1000}$ which is the same as

$\frac{4}{10} + \frac{7}{100} + \frac{5}{1000}$ or 0.4 + 0.07 + 0.005.

The chart (Table 9.4) shows some decimal numbers with their expanded form.

Decimal Number	Expanded Form
5.01	$5 + \frac{1}{100}$
742.565	$700 + 40 + 2 + \frac{5}{10} + \frac{6}{100} + \frac{5}{1000}$
0.493	$\frac{4}{10} + \frac{9}{100} + \frac{3}{1000}$
25	20 + 5

Table 9.4

If expanded form of a decimal number is given, then we can write the decimal number.

Illustration 2:

(a) $2 + \frac{3}{10} + \frac{6}{100} + \frac{1}{1000} = 2 + 0.361 = 2.361$

(b) $1 + \frac{3}{100} + \frac{7}{1000} = 1 + \frac{0}{10} + \frac{3}{100} + \frac{7}{1000}$

$= 1 + 0.037 = 1.037$

(c) 9 tenths + 6 hundredths + 5 thousandths + 3 ten thousandths

$= \frac{9}{10} + \frac{6}{100} + \frac{5}{1000} + \frac{3}{10000}$

= 0.9 + 0.06 + 0.005 + 0.0003 = 0.9653

Let us study some more examples.

Ex. 2. Identify the digit in the:

(a) hundredths place in 72.061.

(b) hundredths place in 4312.58.

(c) ten thousandths place in 0.0045.

Sol. (a) 72.061 (6 ↑ 2nd place)

The 'hundredths' place is the second place to the right of the decimal point.

The digit in the hundredths place in 72.061 is 6.

(b) 4312.58 (8 ↑ 2nd place)

The 'hundredths' place is the second place to the right of the decimal point'. The digit in the hundredths place in 4312.58 is 8.

(c) 0.0045 (5 ↑ 4th place)

The 'ten thousandths' place is the fourth place to the right of the decimal point.

The digit in the ten thousandths place in 0.0045 is 5.

Ex. 3. Write "five hundred point eight" as a decimal.

Sol. Five hundred point eight = 500.8

Ex. 4. Write the following as numerals/decimals.

(a) Nine-tenths

(b) Two tens and seven-tenths

Sol. (a) Nine-tenths = $\frac{9}{10}$ = 0.9

(b) Two tens and seven-tenths

$= 2 \times 10 + 7 \times \frac{1}{10} = 20 + \frac{7}{10} = 20.7$

Ex. 5. Write the number given in the place value table (see Table 9.5) in decimal form.

Table 9.5

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
1	0	2	0	2	5

Sol. From the place value table, we get the given number as:

$1 \times 100 + 0 \times 10 + 2 \times 1 + 0 \times \frac{1}{10} + 2 \times \frac{1}{100} + 5 \times \frac{1}{1000}$

$= 100 + 0 + 2 + \frac{0}{10} + \frac{2}{100} + \frac{5}{1000}$

= 102.025

Exercise 9.1

1. Write the decimal form for the following fractions.

(a) $\frac{7}{10}$ (b) $\frac{23}{100}$ (c) $\frac{18}{100}$ (d) $\frac{92}{1000}$ (e) $\frac{2765}{1000}$

2. Fill in the blanks.

(a) Place value of 7 in 2.0079 is _________.

(b) The number 73.0821 has digit _________ in its hundredths place.

(c) Five-hundredths can be written as _________.

(d) The decimal notation for $3 + \frac{4}{10} + \frac{5}{1000}$ is _________.

(e) The place value of a place decreases by _________ times, when moving from left to right in the place value chart.

(f) The fractions with denominators 10, 100, 1000, etc., are called __________.

3. Write the following decimals in the place value table. Also, write their number names.

(a) 32.6 (b) 102.37 (c) 479.002 (d) 97.023

4. Determine the place value for the underlined digits.

(a) $42.0\underline{9}5$ (b) $395.\underline{4}61$ (c) $123.45\underline{6}$ (d) $0.00\underline{4}5$ (e) $27.6\underline{5}02$

5. Write the following as a decimal.

(a) Three hundred six point eight (b) Two hundred four point six eight

(c) Six hundreds two tens and nine-tenths (d) Nine and four-hundredths

6. Determine, between which two consecutive whole numbers on the number line, the given numbers lie.

(a) 3.7 (b) 5.9 (c) 2.8 (d) 15.4

7. Write the number given in the place value table in decimal form.

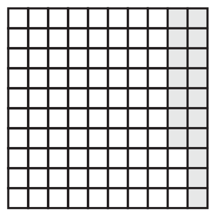
(a)

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
5	7	0	9	0	4

(b)

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
7	0	0	1	0	6

8. If 8 of the shaded squares were changed to unshaded in the figure given alongside, what decimal fraction would the remaining shaded squares represent?



9. Write the decimal number for the given expanded form.

(a) $7+\frac{4}{100}+\frac{7}{1000}$ (b) $50+3+\frac{4}{10}+\frac{7}{100}+\frac{2}{1000}$

(c) $100+30+\frac{5}{10}+\frac{7}{1000}$ (d) $300+20+5+\frac{3}{100}+\frac{4}{1000}$ (e) $200+70+\frac{6}{100}+\frac{5}{1000}$

10. Write the following decimal numbers in expanded form.

(a) 23.172 (b) 0.193 (c) 15.038 (d) 80.23 (e) 325.986

LIKE AND UNLIKE DECIMALS

- Decimals having the same number of decimal places are called **like decimals**.

  For example, 426.05 and 1.37 are like decimals (2 places) as both have two decimal places.

- Decimals having different number of decimal places are called **unlike decimals**.

  For example, 3.2, 5.03, 7.609 are unlike decimals (1 place, 2 places, 3 places) as they have different places of decimals.

Equivalent Decimal Numbers

Let us consider the following decimals:

$$0.24 = \frac{24}{100} = \frac{24\times10}{100\times10} = \frac{240}{1000} = \mathbf{0.240}$$

Thus, 0.24 = 0.240.

$$1.6 = \frac{16}{10} = \frac{16 \times 100}{10 \times 100} = \frac{1600}{1000} = \mathbf{1.600}$$

Thus, 1.6 = 1.600.

$$0.05 = \frac{5}{100} = \frac{5 \times 1000}{100 \times 1000} = \frac{5000}{100000} = \mathbf{0.05000}$$

Thus, 0.05 = 0.05000.

Observe that **appending zero(s) to the right of the last decimal place does not change the value of the decimal number**.

Thus, 28.5 = 28.50 = 28.500 = 28.5000 and so on.

By reading the above in reverse order, we notice that **dropping the zero(s) at the right end of the decimal part does not change the value of the decimal number**.

Making use of the above fact, we can convert two or more unlike decimals into like decimals by simply appending zero(s) to the right of the decimal parts in order to have the same number of decimal places in each decimal.

For example, unlike decimals 0.39 and 0.368 can be written as like decimals 0.390 and 0.368 respectively.

COMPARING DECIMALS

We already know how to compare two (or more) whole numbers. Can you tell which is greater 0.05 or 0.1? In this section, we shall learn how to compare decimals.

Comparing Two Like Decimals

Comparing like decimals (decimals having the same number of decimal places) can be easily done by ignoring the decimal points and comparing the resulting numbers.

Illustration 1: To compare 8.67 and 9.34, we compare 867 and 934. Since 867 < 934, therefore 8.67 < 9.34.

Reasoning: We know that fractional forms of like decimals have common denominators and when comparing fractions with common denominators, it is sufficient to compare only the numerators.

$$\text{Since } 8.67 = \frac{867}{100} \text{ and } 9.34 = \frac{934}{100}$$

Thus, to compare 8.67 and 9.34, it is sufficient to compare the numerators **867** and **934.**

In general, to compare like decimals, compare the whole numbers obtained by dropping the decimal point.

Skill Check

Put <, = or > in the boxes.

(a) 5.7 □ 4.9 (b) 3.15 □ 3.18 (c) 1.82 □ 1.82

Arranging Like Decimals in Ascending and Descending Orders

To arrange three or more like decimals in ascending or descending orders, proceed as given in the illustration:

Illustration 2: To arrange 13.75, 6.93 and 12.96 in ascending order, arrange the whole numbers 1375, 693 and 1296 in ascending order.

Since 693 < 1296 < 1375, therefore the desired arrangement in ascending order is 6.93, 12.96, 13.75.

Comparing Two Unlike Decimals

To compare two unlike decimals, we first convert them into like decimals and then compare as we do for like decimals.

Illustration 3: To compare 0.37 and 0.345, first convert unlike decimals into like decimals.

Since 0.345 has three decimal places, we write 0.37 as 0.370 so that it also has three decimal places.

Then compare the like decimals 0.370 and 0.345.

Here, 0.370 > 0.345 because 370 > 345.

Therefore, 0.37 > 0.345.

Arranging Unlike Decimals in Ascending and Descending Orders

To list 5.515, 5.15 and 55.51 in ascending order, first write each decimal to three decimal places, since the maximum number of decimal places in this group is three.

5.515 = 5.515

5.15 = 5.150 (Append one zero to the right.)

55.51 = 55.510 (Append one zero to the right.)

Since $5150 < 5515 < 55510$

Therefore $5.150 < 5.515 < 55.510$.

Thus, the three decimal numbers arranged in ascending order are 5.15, 5.515 and 55.51.

Ex. 6. Compare the following decimals to find which number is greater.

(a) 0.35 and 0.305

(b) 7.919 and 7.92

Sol. (a) $0.35 = 0.350$, $0.305 = 0.305$ } Write the decimals as like decimals.

Now, 350 > 305.
Thus, 0.35 > 0.305.

(b) $7.919 = 7.919$, $7.92 = 7.920$ } Write the decimals as like decimals.

Now, 7920 > 7919.

Thus, 7.92 > 7.919.

Ex. 7. Arrange the numbers 63.013, 61.3, 63.103, 63.31 in descending order.

Sol. $63.013 = 63.013$, $61.3 = 61.300$, $63.103 = 63.103$, $63.31 = 63.310$ } Convert into like decimals.

Now, 63310 > 63103 > 63013 > 61300, so 63.310 > 63.103 > 63.013 > 61.300.

(Compare the numbers.)

The desired arrangement in descending order is 63.31, 63.103, 63.013, 61.3.

(Arrange by dropping appended zeros.)

Ex. 8. Arrange the numbers 2.59, 25.9, 3.402, 0.034 in ascending order.

Sol. $2.59 = 2.590$, $25.9 = 25.900$, $3.402 = 3.402$, $0.034 = 0.034$ } Convert into like decimals.

Now, 34 < 2590 < 3402 < 25900, so 0.034 < 2.590 < 3.402 < 25.900.

(Compare the numbers.)

The desired arrangement in ascending order is 0.034, 2.59, 3.402, 25.9.

(Arrange by dropping appended zeros.)

Exercise 9.2

1. Tick (✓) the correct answer.

(a) Which of the following numbers is the largest?

(i) 0.0707 (ii) 7.07 (iii) 70.7 (iv) 0.707

(b) Identify the group in which all the decimals are arranged in descending order.

(i) 0.028, 2.008, 2.08, 2.8 (ii) 0.123, 12.3, 1.23, 0.0123

(iii) 2.4, 0.24, 24.0, 0.024 (iv) 8.0, 0.8, 0.08, 0.008

(c) The decimal not equivalent to 3.6 is:

(i) 3.60 (ii) 3.06 (iii) 3.600 (iv) 3.6000

(d) Which of the following statements is correct?

(i) 2.09 and 3.720 are like decimals. (ii) 0.723, 0.192, 0.114 are unlike decimals.

(iii) 9.001 > 9.01 (iv) 6.23 > 6.210

(e) For the numbers, 6.010, 7.001, 6.101 and 7.01, which one of the following is not correct?

(i) 7.001 and 7.01 are like decimals. (ii) 7.01 > 7.001

(iii) 6.010 < 6.101 (iv) 6.010, 7.001 and 6.101 are like decimals.

2. Convert the following decimals into like decimals.

(a) 5.7, 6.15, 7.012 (b) 0.5, 0.05, 0.0050 (c) 4.2, 42, 9.16 (d) 1.1, 1.11, 1.111

3. **Which is greater?**

(a) 0.57 or 0.4 (b) 3.72 or 3.75 (c) 41.5 or 4.9

(d) 1.234 or 12.34 (e) 3.75 or 3.705 (f) 12.07 or 12.007

4. **Fill in the boxes with appropriate sign ('<', '>' or '=').**

(a) 3.7 ☐ 3.9 (b) 72.003 ☐ 72.0035 (c) 0.415 ☐ 0.4

(d) 7.42 ☐ 7.4200 (e) 47.09 ☐ 47.0899 (f) 203.57 ☐ 203.6

5. **Arrange the following numbers in descending order.**

(a) 35.37, 40.12, 35.2, 47.03 (b) 13.9, 143.7, 15.002, 14.9, 144.2

(c) 75.012, 75.12, 75.0012, 75.0 (d) 4.1, 4.12, 4.013, 4.113

6. **Arrange the following numbers in ascending order.**

(a) 5.72, 55.02, 6.9, 0.697 (b) 15.76, 157.6, 1.576, 0.1576

(c) 35.81, 36.01, 35.8, 42.11 (d) 0.0123, 0.12, 10.13, 0.273

INTERCONVERSION OF DECIMALS AND FRACTIONS

Converting Decimals into Fractions

Earlier, we have learnt that the fractions with denominators 10, 100, 1000, etc., are decimal fractions. In this section, we will learn how to express decimals as fractions. Let us consider the following illustrations.

Illustration 1:

- 0.25 is read as "twenty-five hundredths" or "zero point two five".

$$0.25 = \frac{25}{100}$$ ← The number without the decimal point. (numerator) ← 1 followed by 2 zeros, since there are 2 decimal places. (denominator)

- 0.025 is read as "twenty-five thousandths" or "zero point zero two five".

$$0.025 = \frac{25}{1000}$$ ← The number without the decimal point. (numerator) ← 1 followed by 3 zeros, since there are 3 decimal places. (denominator)

- 23.15 is read as "twenty-three and fifteen hundredths" or "twenty-three point one five".

For decimals whose whole number part is not zero, as in the case of 23.15, it is convenient to change only the decimal part to a fraction and leave the whole number part unchanged.

For example, $23.15 = 23\frac{15}{100} = \frac{2315}{100}$

$$\left[\because \frac{23\times100+15}{100}=\frac{2300+15}{100}=\frac{2315}{100}\right]$$

Thus,

$$23.15 = \frac{2315}{100}$$ ← The number without the decimal point. (numerator) ← 1 followed by 2 zeros, since there are 2 decimal places. (denominator)

To change a decimal to a fraction, we follow these steps:

Step 1: Drop the decimal point and write the resulting number as the numerator of the fraction.

Step 2: Write the denominator as 1 followed by zeros equal to the number of decimal places or number of digits after decimal point in the decimal.

Step 3: Simplify the resulting fraction, if possible.

Ex. 9. Convert the given decimals into fractions or mixed numbers.

(a) 0.045 (b) 0.365 (c) 21.14

Sol. (a) $0.045 = \frac{45}{1000} = \frac{9\times5}{200\times5} = \frac{9}{200}$

(b) $0.365 = \frac{365}{1000} = \frac{5\times73}{5\times200} = \frac{73}{200}$

(c) $21.14 = 21\frac{14}{100}$

Now, $\frac{14}{100} = \frac{2\times7}{2\times50} = \frac{7}{50}$

Thus, $21\frac{14}{100} = 21\frac{7}{50}$.

Skill Check

- 7.018 as a mixed fraction can be expressed as ______.
- $32\frac{23}{100}$ in the decimal form is given by:

  (a) 3223 (b) 32.23 (c) 32.023 (d) 322.3
- Convert 0.075 into a fraction in the lowest terms.

Converting Fractions into Decimals

We already know how to write fractions with denominators 10, 100, 1000, etc., as decimals.

Let us recall $\frac{7}{10} = 0.7$, $\frac{15}{100} = 0.15$, $\frac{43}{1000} = 0.043$, $\frac{25}{10} = 2\frac{5}{10} = 2.5$, $5\frac{1}{100} = 5.01$.

In this section, we shall learn how to convert a fraction like $\frac{2}{5}, \frac{3}{20}, \ldots$ into decimals.

Conversion by Equivalent Fractions

Recall that fractions whose denominators are 10, 100, 1000, etc., can be written as decimals by inspection.

The number of zeros in the denominator after 1 is equal to the number of decimal places.

For example, $\frac{3}{10} = 0.3$; $\frac{215}{100} = 2.15$; $\frac{45}{1000} = 0.045$

Since $10 = 2 \times 5$, $100 = 2 \times 2 \times 5 \times 5$, etc., any fraction that has a denominator 2 or 5 or their multiples can also be written as a decimal fraction.

For example, $\frac{75}{2} = \frac{75 \times 5}{2 \times 5} = \frac{375}{10} = 37.5$;

$$\frac{12}{5} = \frac{12 \times 2}{5 \times 2} = \frac{24}{10} = 2.4$$

Ex. 10. Convert the following fractions into decimals.

(a) $\frac{3}{20}$ **(b)** $\frac{9}{125}$

Sol. We can convert the fractions into decimal fractions and then express as decimals.

(a) $\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100} = 0.15$

[Since denominator = 20 = 2 × 2 × 5]

(b) $\frac{9}{125} = \frac{9 \times 8}{125 \times 8} = \frac{72}{1000} = 0.072$

[Since denominator = 125 = 5 × 5 × 5]

Conversion by Division

For large denominators, such as 10,240 or 78,125, the above method becomes very tedious since it involves extensive work in the prime factorisation of the denominator.

Moreover, the above method cannot be used if the denominator contains any prime factor other than 2 and 5. Therefore, it is necessary to have a more general method.

Every fraction can be thought of as a division problem $\left(\frac{3}{5} = 3 \div 5\right)$.

Division is the common method of changing fractions to decimals.

Illustration 2: Let us convert $\frac{217}{8}$ into a decimal.

$$\begin{array}{r} 27.125 \\ 8\overline{)217.000} \\ -16 \\ \hline 57 \\ -56 \\ \hline 10 \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

The answer can be checked by converting $\frac{217}{8}$ into a decimal by the previous method as explained here.

We have, $\frac{217}{8} = 27.125$.

Check: $\frac{217}{8} = \frac{217 \times 125}{8 \times 125} = \frac{27125}{1000} = 27.125$.

It is interesting to note that a fraction whose denominator has only 2 or 5 as the prime factors, always has a zero remainder.

In general, to change fractions to decimals, where the only prime factors of the denominator are 2 or 5, follow these steps:

Step 1: Write the numerator (dividend) as a decimal and divide by the denominator.

Step 2: Continue dividing until the remainder is zero.

Fractions such as $\frac{1}{3}, \frac{2}{7}$, etc., can also be expressed as decimals which we will study in higher classes.

Converting Mixed Fractions into Decimals

To convert a mixed number (fraction) into a decimal, change the fractional part to a decimal and append it to the whole number part or convert the number into an improper fraction and then divide.

Illustration 3: To convert $15\frac{3}{5}$ into a decimal, we can use either of the following two methods:

Method 1

First express $\frac{3}{5}$ as a decimal: $\frac{3}{5} = \frac{3\times2}{5\times2} = \frac{6}{10} = 0.6$

Now, add this decimal to the whole number part, 15.

Thus, $15\frac{3}{5} = 15 + \frac{3}{5} = 15 + 0.6 = 15.6.$

Method 2

$$15\frac{3}{5} = \frac{78}{5} = 78 \div 5 = 15.6$$

or $$\frac{78}{5} = \frac{78\times2}{5\times2} = \frac{156}{10} = 15.6$$

$$\begin{array}{r} 15.6 \\ 5\overline{)\,78.0} \\ -5 \\ \hline 28 \\ -25 \\ \hline 3\,0 \\ -3\,0 \\ \hline 0 \end{array}$$

Skill Check

Convert the following into decimal.

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $17\frac{1}{2}$ (d) $\frac{7}{25}$

SOME USES OF DECIMAL NOTATION

In Money Values

We know that, 100 paise = ₹1

i.e., 1 paisa = $\frac{1}{100}$ rupee = ₹0.01

and 20 paise = ₹0.20;

50 paise = ₹0.50;

5 rupees 80 paise = ₹5 + ₹0.80 = ₹5.80;

100 rupees 25 paise = ₹100 + ₹0.25 = ₹100.25.

Note

We always use 2 digits to represent paise after decimal. 5 rupees 5 paise = ₹5 + ₹0.05 = ₹5.05

Watch Your Step!

5 rupees 5 paise is not written as ₹5.5 because ₹5.50 means 5 rupees 50 paise.

In Measures of Lengths

We know that,

1000 m = 1 km

i.e., 1 m = $\frac{1}{1000}$ km = 0.001 km,

25 m = $\frac{25}{1000}$ km = 0.025 km

and 500 m = $\frac{500}{1000}$ km = 0.05 km

Also, 100 cm = 1 m

i.e., 1 cm = $\frac{1}{100}$ m = 0.01 m,

15 cm = $\frac{15}{100}$ m = 0.15 m

and 7 cm = $\frac{7}{100}$ m = 0.07 m

4 m 8 cm = 4 m + 8 cm $\left(8\text{ cm} = \frac{8}{100}\text{m} = 0.08\text{ m}\right)$

= 4 m + 0.08 m = 4.08 m

Also, 10 mm = 1 cm

i.e., 1 mm = $\frac{1}{10}$ cm = 0.1 cm

and 2 mm = $\frac{2}{10}$ cm = 0.2 cm

In Measures of Mass and Capacity

We know that, 1000 g = 1 kg and 1000 mg = 1 g

i.e., 1 g = $\frac{1}{1000}$ kg = 0.001 kg and $\quad 1 \text{ mg} = \frac{1}{1000} \text{ g} = 0.001 \text{ g}$

Therefore, $75 \text{ g} = \frac{75}{1000} \text{ kg} = 0.075 \text{ kg}$

and $\quad 480 \text{ mg} = \frac{480}{1000} \text{ g} = 0.48 \text{ g}$

Also, $\quad 1000 \text{ L} = 1 \text{ kL}$ and $1000 \text{ mL} = 1 \text{ L}$

i.e., $\quad 1 \text{ L} = \frac{1}{1000} \text{ kL} = 0.001 \text{ kL}$

and $\quad 1 \text{ mL} = \frac{1}{1000} \text{ L} = 0.001 \text{ L}$

Therefore, $5 \text{ mL} = \frac{5}{1000} \text{ L} = 0.005 \text{ L}$

and $\quad 750 \text{ L} = \frac{750}{1000} \text{ kL} = 0.75 \text{ L}$

Let us study some more examples.

Ex. 11. Express 50 rupees 60 paise in rupees using decimals.

Sol. 50 rupees 60 paise = ₹50 + 60 paise

We know that, 1 paisa = ₹$\frac{1}{100}$ = ₹0.01

So, 60 paise = ₹$\frac{60}{100}$ = ₹0.60

Therefore, 50 rupees 60 paise

= ₹50 + ₹0.60 = ₹50.60

Ex. 12. Express the following using decimals.

(a) 9 m 6 cm into metres

(b) 85 m in km

(c) 456 g in kg

(d) 2 kL 9 L in kL

Sol. (a) We know that $1 \text{ cm} = \frac{1}{100} \text{ m} = 0.01 \text{ m}$

So, $6 \text{ cm} = \frac{6}{100} \text{ m} = 0.06 \text{ m}$

Therefore, 9 m 6 cm = 9 m + 6 cm

= 9 m + 0.06 m = 9.06 m

(b) We know that $1 \text{ m} = \frac{1}{1000} \text{ km} = 0.001 \text{ km}$

Therefore, $85 \text{ m} = \frac{85}{1000} \text{ km} = 0.085 \text{ km}$

(c) We know that $1 \text{ g} = \frac{1}{1000} \text{ kg} = 0.001 \text{ kg}$

Therefore, $456 \text{ g} = \frac{456}{1000} \text{ kg} = 0.456 \text{ kg}$

(d) We know that $1 \text{ L} = \frac{1}{1000} \text{ kg} = 0001 \text{ kg}$

Therefore, $2 \text{ kL } 9 \text{ L} = 2 \text{ kL} + \frac{9}{1000} \text{ kL}$

$= (2 + 0.009) \text{ kL} = 2.009 \text{ kL}$

Ex. 13. Express the following into hours.

(a) 24 minutes (b) 45 seconds

Sol. (a) We know that 60 minutes = 1 hour

So, 1 minute = $\frac{1}{60}$ hour

Therefore, 24 minutes = $\frac{24}{60}$ hour

$= \frac{24 \div 6}{60 \div 6}$ hour $= \frac{4}{10}$ hour = 0.4 hour

(b) We know that 60 seconds = 1 minute and 60 minutes = 1 hour

i.e., 60 × 60 seconds = 1 hour

So, 1 second = $\frac{1}{60 \times 60}$ hour

$= \frac{1}{3600}$ hour

Therefore, 45 seconds = $\frac{45}{3600}$ hour

$= \frac{45 \div 9}{3600 \div 9}$ hour $= \frac{5 \times 25}{400 \times 25}$ hour

$= \frac{125}{10000}$ hour = 0.0125 hour

Exercise 9.3

1. **Tick (✓) the correct answer.**

   (a) $\frac{29}{4}$ in the decimal form is given by:

   (i) 0.725 (ii) 72.5 (iii) 7.25 (iv) 7.5

(b) Which method is used to change a fraction to a decimal?

(i) Numerator ÷ denominator (ii) Denominator ÷ numerator

(iii) Numerator × denominator (iv) Denominator – numerator

(c) 3 kg 125 g in kg is:

(i) 3125 kg (ii) 31.25 kg (iii) 3.125 kg (iv) 3.125 g

2. Convert the given decimals into fractions in their lowest terms.

(a) 0.45 (b) 0.02 (c) 0.075 (d) 0.625

(e) 0.4 (f) 0.13 (g) 0.008

3. Convert the given decimals into fractions or mixed numbers.

(a) 2.005 (b) 5.025 (c) 15.04

(d) 25.06 (e) 44.25 (f) 19.750

4. Express the following fractions as decimals.

(a) $\frac{66}{100}$ (b) $1\frac{9}{1000}$ (c) $\frac{7}{1000}$ (d) $\frac{4}{50}$ (e) $\frac{3}{8}$

5. Convert the following fractions into decimals by long division method.

(a) $\frac{8}{25}$ (b) $\frac{13}{4}$ (c) $\frac{27}{6}$ (d) $3\frac{4}{5}$ (e) $\frac{93}{2}$

6. Express the following using decimals.

(a) 55 paise (in ₹) (b) 285 mm (in cm) (c) 25 kg 558 g (in kg)

(d) 3 L 45 mL (in L) (e) 87 km 450 m (in km) (f) 15 m (in km)

(g) ₹350 and 15 paise (in ₹) (h) 28 g (in kg) (i) 65 cm (in m)

(j) 5 m 4 cm (in m) (k) 40 g 40 mg (in g) (l) 15 kL 50 L (in kL)

7. Express the following using decimals in hours.

(a) 36 minutes (b) 72 seconds (c) 144 minutes (d) 198 seconds

8. Reduce $\frac{15}{24}$ to its lowest terms and then write the result as a decimal.

9. Express 3 decades as a decimal fraction of a century.

ADDING DECIMALS

We already know how to add two or more whole numbers. In this section, we shall discuss how to add two or more decimals.

Adding Like Decimals

The addition of like decimal numbers is very similar to the addition of whole numbers, extra care is needed in aligning the decimal points and digits of the same place value.

Illustration 1: Let us consider the following two problems:

Whole Numbers	Decimal Numbers
①①	①①
2 3 9 5	2 3 . 9 5
+ 9 3 2	+ 9 . 3 2
3 3 2 7	3 3 . 2 7

Thus, it can be stated that:

To add a group of like decimal numbers, write the decimal numbers in columns lining up the decimal points and the corresponding place values on either side of the decimal points: tenths under tenths, hundredths under hundredths and so on.

Adding Unlike Decimals

Illustration 2: Let us add unlike decimal numbers like 2.4, 15, 0.05 and 1.215.

We first change the numbers to like decimals and then add.

$$\begin{array}{r} 2.400 \\ 15.000 \\ 0.050 \\ +\ 1.215 \\ \hline 18.665 \end{array}$$

Change each decimal to three decimal places since the maximum number of decimal places in the group is three.

With practice, it may become unnecessary to change the decimals to like decimals and to write each digit in its proper place. It is only necessary to align the decimal points.

$$\begin{array}{r} 2.4 \\ 15 \\ 0.05 \\ +\ 1.215 \\ \hline 18.665 \end{array}$$

To add two or more decimals, we follow these steps:

Step 1: Write the decimal numbers in columns so that the decimal points and the corresponding place values on either side of the decimal points are lined up. Append zeros if required to convert the decimals into the like decimals.

Step 2: Add the decimal numbers as if they were whole numbers.

Step 3: Place the decimal point in the sum just below those in the addends.

Let us study some more examples.

Ex. 14. Add 5.21, 13.719, 0.06 and 103.15.

Sol.

$$\begin{array}{r} ①①\ ① \\ 5.210 \\ 13.719 \\ 0.060 \\ +\ 103.150 \\ \hline 122.139 \end{array}$$

(Decimal points are lined up.)

Place the decimal point.

Therefore, the sum is 122.139.

Ex. 15. Satish travelled 5 km 32 m by bus, 3 km 160 m by car and the rest 1 km 500 m on foot. How much distance did he travel in all?

Sol. Distance travelled by bus = 5 km 32 m

= 5.032 km [$\because$ 1 m = $\frac{1}{1000}$ km]

Distance travelled by car = 3 km 160 m

= 3.160 km

Distance travelled on foot = 1 km 500 m

= 1.500 km

Total distance travelled = Distance travelled by (bus + car + foot)

= (5.032 + 3.160 + 1.500) km = 9.692 km

Thus, the total distance travelled by Satish is 9.692 km.

Skill Check

Add:

(a) 2 m 10 cm and 2 m 20 cm

(b) 2 L 650 mL and 750 mL

Ex. 16. One morning, the temperature in Mumbai was recorded as 29.5°C. Afternoon, the temperature rose by 3.8°C. What was the temperature in the afternoon?

Sol. Temperature in the morning = 29.5°C

Rise in the temperature = 3.8°C

So, the temperature in the afternoon in Mumbai = 29.5°C + 3.8°C

= 33.3°C

Ex. 17. Divya spent ₹60.75 for a Maths book and ₹32.50 for a Science book. Find the total amount spent by Divya.

Sol. Amount spent for Maths book = ₹60.75

Amount spent for Science book = ₹32.50

Total amount spent by Divya

= ₹60.75 + ₹32.50 = ₹93.25

Ex. 18. Shreya bought 15 kg 258 g rice, 7 kg 50 g flour, 2 kg pulses and 500 g sugar for her family. Find the total weight of these items.

Sol. Weight of rice bought = 15 kg 258 g

= 15.258 kg

Weight of flour bought = 7 kg 50 g

= 7.050 kg

Weight of pulses bought = 2 kg

= 2.000 kg

Weight of sugar bought = 500 g

= 0.500 kg

Total weight = 15.258 kg + 7.050 kg + 2.000 kg + 0.500 kg = 24.808 kg

Thus, the total weight of items Shreya bought is 24.808 kg.

Exercise 9.4

1. **Solve the following.**
   (a) 4.56 + 7.92 (b) 12.45 + 9.3 (c) 100 + 7.8 (d) 88.9 + 18.582 (e) 1.45 + 0.56 + 0.008 + 0.7
2. **Add the following.**
   (a) 0.033, 0.049, 0.051, 0.32 (b) 1.96, 72.8, 33.827
   (c) 11.57, 230.01, 71.23, 6 (d) 485.2, 52, 112.97, 0.03
3. Ria bought a sandwich for ₹42.25 and a coffee sachet for ₹21.50. Find the total money spent by her.
4. Arjun travelled 8 km 75 m by bus, 5 km 200 m by car and rest 1 km 275 m on foot. How much distance did he travel in all?
5. Raju purchased a book, a pen and a notebook for ₹178.45, ₹55 and ₹23.75 respectively. How much money will he have to pay to the shopkeeper for these items?
6. Dinesh walked 2 km 45 m in the morning and 1 km 5 m in the evening. How much distance did he walk in all?
7. Rajni bought 5 m 40 cm cloth for her shirt and 7 m 15 cm cloth for her trousers. Find the total length of the cloth bought by her.
8. Bhanu bought 7 kg 120 g of apples, 5 kg 850 g of grapes and 4 kg 280 g of mangoes. Find the total weight of all the fruits he bought.
9. The normal temperature of water is 21.8°C. It is heated for sometime and the temperature increases by 17.6°C. What is the temperature of this lukewarm water?

SUBTRACTING DECIMALS

In the previous section, we have learnt addition of decimals. Now, we shall learn subtraction of two decimals.

Subtracting Like Decimals

The subtraction of like decimal numbers is very similar to the subtraction of whole numbers, extra care is needed in aligning the decimal points and digits of the same place value.

Illustration 1: Let us consider the following two problems.

Whole Numbers

$$\begin{array}{r} 2987 \\ -\ 352 \\ \hline 2635 \\ \hline \end{array}$$

Decimal Numbers

$$\begin{array}{r} 29.87 \\ -\ 3.52 \\ \hline 26.35 \\ \hline \end{array}$$

(7 hundredths – 2 hundredths = 5 hundredths)
(8 tenths – 5 tenths = 3 tenths)

Subtracting Like Decimals with Regrouping

If the subtraction in any column cannot be performed, simply regroup as we do in the case of whole numbers.

Illustration 2: Let us find the difference of 6.762 and 0.543.

$$\begin{array}{r} \scriptsize{5\ 12} \\ 6.7\not{6}\not{2} \\ -\ 0.543 \\ \hline 6.219 \\ \hline \end{array}$$

(3 thousandths cannot be subtracted from 2 thousandths. Regroup 6 hundredths and 2 thousandths as 5 hundredths and 12 thousandths.)

Subtracting Unlike Decimals

To subtract unlike decimals, first change the numbers to like decimals and then subtract.

Illustration 3: Let us subtract 19.249 from 75.46.

Write 75.46 as 75.460.

$$\begin{array}{r} \scriptsize{6\ 15\quad 5\ 10} \\ \not{7}\not{5}.4\not{6}\not{0} \\ -\ 19.249 \\ \hline 56.211 \\ \hline \end{array}$$

(Change to like decimals)
(Subtract)

Check the answer by addition.

```
 ①   ①
  1 9 . 2 4 9
+ 5 6 . 2 1 1
-------------
  7 5 . 4 6 0  = 75.46
```

To subtract decimal numbers, we follow these steps:

Step 1: Write the decimal numbers in columns so that the decimal points and the corresponding place values on either side of the decimal points are lined up.
Append zeros, if required to convert the decimals to like decimals.

Step 2: Subtract the decimals as if they were whole numbers.

Step 3: Place the decimal point in the difference just below those in the two decimals.

Let us study some more examples.

Ex. 19. Subtract 5.049 from 22.17.

```
 ① ⑫     ⑥ ⑩
  2  2 . 1  7  0   (2, 2, 7, 0 struck out)
-    5 . 0  4  9
----------------
  1  7 . 1  2  1
       ↑
Place the decimal point.
```

```
          ①     ①
Check:      5 . 0 4 9
        + 1 7 . 1 2 1
        -------------
          2 2 . 1 7 0  = 22.17
```

The difference 17.121 is correct.

Ex. 20. Mahima bought mangoes weighing 5 kg 200 g. Out of this, she gave 3 kg 500 g to her friend. What is the weight of mangoes left with Mahima?

Sol. Total weight of mangoes = 5 kg 200 g
= 5.200 kg

Mangoes given to her friend= 3 kg 500 g
= 3.500 kg

Weight of remaining mangoes
= (5.200 – 3.500) kg = 1.700 kg

Thus, 1.700 kg of mangoes are left with Mahima.

Ex. 21. Harsha had ₹24.50 in her mobile wallet. She bought some toffees and paid ₹11.75. Find the balance amount in her mobile wallet.

Sol. Total amount of money in mobile wallet
= ₹24.50

Amount paid for toffees = ₹11.75

Balance amount in wallet
= ₹(24.50 – 11.75)
= ₹12.75

Thus, the balance amount in her wallet is ₹12.75.

Ex. 22. Neelam travels 20 km 50 m everyday to reach her office. She travels 10 km 200 m by Metro train and the remaining distance by auto-rickshaw. How much distance does she travel by auto-rickshaw?

Sol. Total distance travelled by Neelam to reach her office = 20 km 50 m = 20.050 km

Distance travelled by Neelam by Metro train = 10 km 200 m = 10.200 km

Therefore, distance travelled by Neelam by auto-rickshaw = 20.050 km – 10.200 km = 9.850 km

Addition and Subtraction Together

Ex. 23. Simplify: 4.750 L – 6.080 L + 2.850 L

Sol. We first add 4.750 L and 2.850 L, *i.e.*,

```
  4.750 L
+ 2.850 L
---------
  7.600 L
```

Then, we subtract 6.080 L from 7.600 L, *i.e.*,

```
  7.600 L
– 6.080 L
---------
  1.520 L
```

Ex. 24. Sarika had 10 m 20 cm long cloth. She cut 2 m 50 cm and 3 m 35 cm of the cloth from it to make two dresses. How much cloth is left with her?

Sol. Length of cloth Sarika had = 10 m 20 cm
= 10.20 m

Length of cloth cut to make two dresses

= 2 m 50 cm + 3 m 35 cm

= 2.50 m + 3.35 m = 5.85 m

Thus, the length of cloth left with Sarika

= 10.20 m – 5.85 m = 4.35 m

Therefore, 4.35 cm of the cloth is left with Sarika.

Exercise 9.5

1. **Subtract the following.**
   (a) 0.546 – 0.235 (b) 90.05 – 17.65 (c) 213.65 – 123.32
   (d) 0.0123 – 0.005 (e) 327.82 – 112.5
2. **Subtract:**
   (a) 1.425 kg from 5.120 kg (b) ₹110.75 from ₹575.50
   (c) 12.27 km from 35.725 km (d) 380 mL from 2 L
3. **Simplify the following.**
   (a) 75.161 + 23.121 – 52.63 (b) 0.005 – 1.362 + 2.978 (c) 66.666 – 123.452 + 200
4. **Answer the following questions.**
   (a) What is to be added to 74.5 to get 93.85?
   (b) By how much should 27.87 be increased to get 70?
   (c) What should be added to 87.73 to get the smallest 3-digit number?
5. How many grams of sugar must be added to 7.45 kg to make it 22.5 kg?
6. Garima purchased a book worth ₹47.75 from a bookseller and gave him a 100-rupee note. How much balance did she get back?
7. Rakhi had ₹22.50 in her mobile wallet. She bought some candies and paid ₹14.80. What is the amount left in her mobile wallet?
8. The normal body temperature of a human being is 98.6°F. Due to fever, the body temperature rose to 103.2°F. How much more is the body temperature from the normal level?
9. Vijay bought 12 kg of fruits. Out of this 3 kg 50 g were apples, 2 kg 750 g were mangoes and rest were watermelons. Find the quantity of watermelons purchased by him.
10. Neelima had 17 m 75 cm long cloth. She cut 4 m 95 cm and 2 m 50 cm of the cloth from this to make two dresses. How much cloth is left with her?

Let's Work in Mind

21st CS

1. Which is greater: 0.9 or 0.09?
2. Katherine spends ₹12.50 on biscuits. How much is left with her if she had ₹25.60?
3. From ₹200 take ₹178.09. How much is left?
4. How many hundredths are there in 2 ones?
5. Between which two whole numbers does 7.06 lie?
6. How many metres of cloth is required to make 2 shirts if one shirt is prepared using 1 m 75 cm of the cloth?

Competency Based Exercise

21st CS

1. Tick (✓) the correct answer.

(a) If the grip size of a tennis racquet is $11\frac{9}{80}$ cm, then it can be written in decimals as:

(i) 111.25 cm (ii) 11.125 cm (iii) 11.0125 cm (iv) 11.1125 cm

(b) The weight of an empty LPG cylinder is 18 kg 35 g. If the LPG contained in a full cylinder is 16 kg 350 g, then the total weight of the full cylinder is:

(i) 34.85 kg (ii) 34.35 kg (iii) 34.085 kg (iv) 34.385 kg

(c) The sum of $8\frac{3}{5} + 0.03 + 1\frac{3}{4}$ is equal to:

(i) 26.40 (ii) 10.65 (iii) 10.38 (iv) 10.11

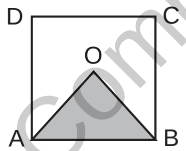
(d) Ahmed caught 232.3 kg fish and Juhi caught 157.325 kg fish. How many kg of fish should Juhi catch to equalise the fish caught by Ahmed?

(i) 74.955 kg (ii) 74.975 kg (iii) 74.795 kg (iv) 74.579 kg

(e) In the given figure, O is the centre of the square ABCD. What part of the square ABCD is represented by the shaded portion?

(i) 0.1 (ii) 0.2

(iii) 0.25 (iv) 0.5



2. For each of the following statements, write True (T) or False (F).

(a) 0.06 is the same as 0.0600.

(b) $3 + \frac{3}{100} + \frac{4}{1000}$ is equal to the decimal number 3.304.

(c) Place value of the place immediately after the decimal point in a decimal number is $\frac{1}{10}$.

(d) The place value of a digit at tenths place is 10 times the same digit at the ones place.

(e) The place value of a digit at the hundredths place is $\frac{1}{10}$ times the same digit at the tenths place.

3. Match the following.

Column A	Column B
(a) The sum of 6 tenths and 62 hundredths	(i) 4.19
(b) The sum of $2\frac{2}{5}$, 0.04 and $1\frac{3}{4}$	(ii) 30.763
(c) The sum of 4, 3.05, 10.7 and 13.013	(iii) 1.22

4. Determine the place value of the digits given against the numbers.

(a) 1.372; 7 (b) 77.0126; 2 (c) 87.209; 2

5. **Identify the digit in the following numerals.**
   (a) Hundredths place in 3.3297 (b) Thousandths place in 0.1236
6. **Express the following using decimals.**
   (a) 7 m in km (b) 32 mL in L (c) Five thousand twenty-three and seven-thousandths
   (d) Three hundred and five-thousandths
7. **Write the given numbers in expanded form.**
   (a) 0.798 (b) 57.087 (c) 30.072 (d) 327.82
8. **Express the following fractions as decimals.**
   (a) $\frac{9}{10}+\frac{3}{1000}$ (b) $300+72+\frac{5}{100}+\frac{7}{1000}$ (c) $\frac{9}{1000}$ (d) $8\frac{3}{10}$
9. **Arrange the following in ascending order.**
   (a) 0.1, 0.112, 1.1, 1.01 (b) 3.14, 3.141, 3.2, 3.157
10. **Arrange the following in descending order.**
    (a) 83.7, 83.71, 8.37, 0.8371 (b) 11.98, 11.9, 11.907, 12.011
11. **Convert the following into decimals.**
    (a) $\frac{7}{25}$ (b) $\frac{37}{2}$ (c) $\frac{3}{24}$ (d) $\frac{47}{5}$ (e) $17\frac{3}{8}$
12. **Convert the following decimals into fractions.**
    (a) 27.48 (b) 12.64 (c) 4.02 (d) 63.31
13. **Simplify the following.**
    (a) 423.687 + 238.155 (b) 0.0152 + 1.1823 + 2.8645
    (c) 17.67 – 1.99 (d) 444.869 – 333.397
    (e) 52.08 + 11.73 – 18.75 (f) 20 – 3.27 – 7.88
14. The sum of two numbers is 93. If one of the numbers is 42.872, find the other number.
15. Radha bought 27.235 kg vegetables and Sweety bought 11.720 kg vegetables. How many kg of vegetables should Sweety buy to equalise the vegetable bought by Radha?
16. Add ₹3.12, ₹1.72 and ₹9.49. By how much is the sum less than ₹20.00?
17. From the sum of 123.89 and 37.325, subtract the sum of 29.95 and 45.326.
18. The perimeter of a pentagon is 30 cm. If four of its sides measures 55 mm, 11 cm 4 mm, 4 cm 6 mm and 45 mm, find the length of the fifth side.
19. The digit at the hundreds place of a number 439.125 is interchanged by the digit at its hundredths place. What is the changed number? What is the difference in these two numbers?

Challenge! 21st CS

1. If a and b are two decimal numbers such that $a * b = a + b - 2$, find the value of 2.875 ∗ 8.23.
2. Find the value of the following expression:
   1.987 – 2.987 + 3.987 – 4.987 + ... – 48.987 + 49.987 – 50.987 + 51.987
3. If on an average, one-tenth of food eaten turns into organisms own body and is available for the next level of consumer in a food chain, then find the part of the food eaten not available for the next level of consumer.

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. Assertion (A) : The digit in the hundredth place in 72.061 is 6.

Reason (R) : The hundredth place is the second place to the right of the decimal point.

2. Assertion (A) : In 4312.58, 4312 is the whole number part and 58 is the decimal part.

Reason (R) : The fraction with denominators 10, 100, 1000, etc., are called decimal fractions.

3. Assertion (A) : $0.34 = \frac{34}{1000}$

Reason (R) : Every decimal can be written as a fraction.

4. Assertion (A) : 2.37 + 5.02 = 7.39

Reason (R) : Decimals consists of three parts— a whole number part, a decimal point and a decimal part.

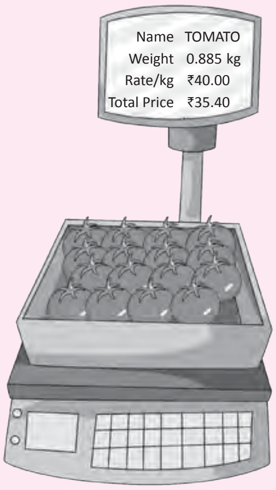
5. Assertion (A) : The decimal form of $30 + \frac{7}{100} + \frac{5}{1000}$ is 30.075.

Reason (R) : $\frac{7}{100} = 0.07$ and $\frac{5}{1000} = 0.005$ and decimal numbers can be added.

CASE STUDY

CC

Vani went to the market with her mother and purchased tomatoes. Her mother was not able to understand the numbers written on the digital weighing scale. Vani learnt decimals in her class and was able to understand the reading on the screen.



Now, answer the following questions.

1. What is the total weight of tomatoes?
2. What is the rate per kg of tomatoes?
3. What is the rate of 885 g of tomatoes shown on the screen? Is it correct?

10 Data Handling

What Learners Will Achieve

- appreciate the need of recording and organising data.
- arranging and organising data using tally marks.
- represent the given data in the form of pictograph and bar graph.
- interpret the given pictograph and bar graph.

Warm-up

What we already know LS

- Data is a collection of numbers gathered to give some information.
- To get the information from the given data quickly, the data has to be organised first in a table and then depicted through graphs pictorially.

Now, try to solve the following.

1. Collect some information about your friends and complete the table.

Name	Favourite Subjects	Favourite TV Shows	Games	Hobbies

2. Mrs Batra is blessed with a baby girl. She wants to have a unique name for her daughter, starting with the least popular letter. Can you help her by listing at least 10 names for her young angel?

3. Check the tiffin of your classmate for a week and find the most popular dish mother preferred to give for lunch.

4. Note down the temperature from the newspaper or otherwise for 15 days and find the hottest day.

DID YOU KNOW?

Statistics is a branch of Mathematics dealing with data collection, organisation, analysis, interpretation and presentation. P.C. Mahalanobis (1893-1972) is known as the father of Indian statistics.

WHAT IS DATA?

A survey was made to find out the favourite sports of 400 students. The following table gives the information based on the above survey.

Sports	Cricket	Tennis	Football	Hockey
Number of Students	150	80	100	70

Such type of information given with the help of numbers is called **numerical data** or simply **data**. Any information in raw or unorganised form such as letters, symbols or numbers that may refer to or may represent objects or ideas is called data.

For example, the collection of first letters of the name of all the students in your school is a data represented by letters. If we are looking for the number of students who have scored marks between 40–50, 50–60, 60–70, 70–80, etc., in each subject in the annual examination, we shall have data represented by numbers.

Data, as a noun, is used both with singular verb form or plural verb form.

For example, 'Additional data is available, if required'. (Singular verb form). These data are described in detailed form in the central library. (Plural verb form).

Need of Data

Information, facts, numbers are collected, examined, considered and are used in decision making.

For example,

1. Insurance companies use data like age, gender, health and a few other factors to decide the amount of premium individuals have to pay to cover their lives.
2. Data collected by Mangalyaan will be analysed so that it can be used in meaningful exploration of the planet Mars.
3. Data obtained by studying the case histories of thousands of patients suffering from a particular disease is analysed to determine its causes and to do research to find its cures.

Types of Data

There are two types of data: Primary data and Secondary data.

- **Primary data:** The data collected directly from the source is known as primary data.

  For example, class teacher records the test marks of her students.
- **Secondary data:** The data collected from any external source is called secondary data.

  For example, weather report of different cities given by newspaper, television, internet, etc.

In this chapter, we shall learn about data, recording and organisation of data in the form of a table, pictorial representation of data like pictographs, bar graphs, etc.

RECORDING AND ORGANISATION OF DATA

Recording Data

Information collected with some definite purpose, in the form of numbers is called **data**.

Raw data: Data obtained in the original form is called *raw data*.

Illustration 1: Consider the marks obtained by 10 students in a test as given below:

15, 12, 17, 18, 20, 08, 19, 13, 17, 05

The data in this form is raw data.

Illustration 2: In an organisation, an accountant recorded the employee's salary as follows:

Employee Name	Salary (in ₹)	Employee Name	Salary (in ₹)	Employee Name	Salary (in ₹)
Aman	16,000	Radha	17,000	Akhtar	19,000
Ram	17,000	Suman	17,000	Kavita	16,000
Anuradha	16,000	Kamal	18,000	Saroj	19,000
Ritu	18,000	Rajesh	18,000	Manoj	18,000

Table 10.1

Later, he noticed that he had to read the list again and again for answering questions like how many employees get salary of ₹16,000? or how many get salary of ₹18,000? So he thought of a better way to record this data.

He used a dot (•) to represent one employee and recorded the data as shown:

Salary (in ₹)	Number of Employees
16,000	• • •
17,000	• • •
18,000	• • • •
19,000	• •

Table 10.2

Now, he could easily answer the above questions without reading the list again and again.

Organisation of Data

In a class, the class teacher collected data for sizes of shoes of students in the following format:

5	4	6	8	7	4	5	8	5
7	4	7	5	8	5	6	7	4
5	7	7	6	4	5	4	5	7

She then asked the students to organise this data.

Ram's table: He used **(×)** to represent one student.

Shoes Size	Marks	Number of Students
4	× × × × × ×	6
5	× × × × × × × ×	8
6	× × ×	3
7	× × × × × × ×	7
8	× × ×	3

Table 10.3

Anuradha's table: She used (|) to represent one student.

Shoes Size	Marks	Number of Students
4	| | | | | |	6
5	| | | | | | | |	8
6	| | |	3
7	| | | | | | |	7
8	| | |	3

Table 10.4

Geeta's table: She also used (|) to represent one student but made a group of five as [|||||].

Shoes Size	Marks	Number of Students
4	| [|||||]	6
5	| | [|||||] |	8
6	| | |	3
7	| [|||||] |	7
8	| | |	3

Table 10.5

Teacher's table: The teacher also used (|) to represent one student, called them as *tally marks* and made a group of five as 卌. Observe that we can read teacher's table much clearer and faster.

Shoes Size	Tally Marks	Number of Students
4	卌 |	6
5	卌 |||	8
6	|||	3
7	卌 ||	7
8	|||	3

Table 10.6

From the above, we can see that **6** students have the size of their shoes as 4, **8** students have the size of their shoes as 5 and so on. 6 is *frequency* of shoes size 4 and 8 is the frequency of shoes size 5 and so on.

The number of times a particular observation occurs is called **frequency** of that observation.

Illustration 3: The blood groups of 40 students of Class VI are recorded as follows:

A, A, AB, O, A, A, O, O, AB, B, A, O, AB, B, A, B, O, O, B, B, A, O, O, A, B, AB, O, A, B, O, A, O, AB, O, O, A, B, O, B, AB.

Let us represent this data in the form of a table, using tally marks.

Blood Group	Tally Marks	Number of Students (frequency)
A	𝍸 𝍸 |	11
B	𝍸 ||||	9
O	𝍸 𝍸 ||||	14
AB	𝍸 |	6

Table 10.7

From the above frequency table or frequency distribution table, we observe that most common blood group is O and the least common blood group is AB.

Ex. 1. A die was thrown 30 times and the number that appeared on the top in each throw was recorded as follows:

1	4	6	2	3	5	6	2	1	1
6	3	5	6	5	6	1	4	4	6
2	1	5	4	6	5	3	6	3	5

(a) Make a table and enter the data using tally marks.

(b) Which number appeared the maximum number of times?

(c) Which number appeared the minimum number of times?

Sol. (a)

Number	Tally Marks	Number of Times
1	𝍸	5
2	|||	3
3	||||	4
4	||||	4
5	𝍸 |	6
6	𝍸 |||	8

Table 10.8

(b) The number 6 appeared the maximum number of times.

(c) The number 2 appeared the minimum number of times.

Skill Check

1. Below are the marks obtained by students in a class test in Mathematics:
   70, 75, 75, 90, 75, 80, 80, 85, 90, 80, 70, 80, 75, 90, 85, 90, 75, 85, 90
   (a) What is the frequency of 80?
   (b) Find out the marks obtained by the least number of students.
2. The choices of ice cream flavours of 20 children are as follow:
   Vanilla, Chocolate, Vanilla, Strawberry, Mango, Chocolate, Mango, Vanilla, Chocolate, Chocolate, Strawberry, Mango, Chocolate, Vanilla, Mango, Strawberry, Chocolate, Vanilla, Mango, Chocolate
   (a) Arrange the flavours in a table using tally marks.
   (b) Which ice cream flavour is preferred by the most number of children?

Let Us Do

EL 21st CS

Objective: To collect, organise and interpret the data

Materials required: Notebooks, Pen, Pencil, etc.

Procedure:

Step 1: Write down the first letter of the name of each and every student of your class. If you have 40 students, you should have 40 letters.

Step 2: Organise the data so obtained by using tally marks.

Step 3: Analyse the data by answering the following questions.

(a) Which letter is the most popular as the first letter of a name in your class?

(b) Of all the letters appearing in your data which is the least used as first letter of a name in your class?

(c) What fraction of all the letters has been used as first letter in the names of students of your class?

(d) What is the fraction of the most popular letter with respect to all the letters used as first letter in the names of students of your class?

Step 4: Do this activity by combining the raw data of all the sections.

Step 5: Compare the answers of question in step 3 (c) given (above) sectionwise and in totality (as part of step 4).

Step 6: Observe and answer the following questions.

(a) Is your section representative of the entire class 6 of your school?

(b) In which category the first letter of your name fall?

(i) most used (ii) least used (iii) in between

Exercise 10.1

1. A die was thrown 30 times and the following outcomes were noted:

2, 3, 1, 5, 5, 1, 2, 4, 6, 2, 3, 3, 1, 5, 6, 1, 4, 3, 5, 6, 1, 4, 5, 5, 3, 1, 2, 5, 5, 6

Represent the above data in tabular form using tally marks.

2. Marks obtained by 25 students in Mathematics examination out of 100 are given below:

63, 41, 87, 63, 33, 49, 78, 91, 80, 37, 52, 55, 68, 71, 43, 37, 51, 50, 67, 75, 76, 39, 52, 71, 81

(a) Arrange the data in ascending order.

(b) Find out the lowest marks obtained.

(c) How many students have scored more than 75 marks?

(d) How many students have scored less than 50 marks?

3. Complete the following table which represents the shirt size of 40 students of a school.

Shirt Size	Tally Marks	Number of Students
32	|||	3
34	𝍸 |	
36		7
38	𝍸 ||	
40		9
42		8

4. The following data gives the number of children in 20 families.

2, 3, 3, 1, 2, 2, 2, 4, 1, 4, 5, 1, 5, 4, 1, 1, 2, 3, 2, 2

Make a table and enter the data using tally marks. Find:

(a) how many families have 3 children?

(b) how many families have less than 3 children?

5. Marks obtained by 42 students of class VI in a Mathematics test are as follows:

18	16	15	20	25	24	20	25	16	15	18
16	18	15	24	20	28	27	30	16	24	25
20	28	18	27	24	24	25	18	18	25	16
20	15	20	15	20	27	29	28	16		

(a) Make a table and enter the data using tally marks.
(b) How many students have obtained the highest marks?
(c) How many students have obtained the lowest marks?

6. Fill in the blanks.

(a) Data obtained in the original form is called ____________.
(b) Information collected in the form of ____________ with some definite purpose is called data.
(c) An observation occurring five times in a data is recorded as ____________, using tally marks.
(d) Data can be arranged in a ____________ form using tally marks.
(e) Number of times, a particular observation occurs in a data is called the ____________ of the observation.

PICTORIAL REPRESENTATION OF DATA

In the previous section, we have learnt about data, collection and organisation of data in the form of tables. To study the details and characteristics of numerical data, pictures are quite useful. The study of numerical data through pictures and figures is known as **pictorical representation** or **graph** of the data.

In this chapter, we shall learn about two types of graphs: (i) Pictographs (ii) Bar graphs.

Pictographs

When numerical data is represented by picture symbols, objects or part of the objects, such a representation is called a *pictograph*.

A pictograph uses picture symbols or objects to represent a number. Each symbol represents a certain number, which is known as key. Let us observe the pictograph which gives information regarding the number of students enrolled in different courses from a certain class.

Course	Students Enrolled
Music	☺ ☺ ☺ ☺
Dance	☺ ☺ ☺ ☺ ☺ ☺
Painting	☺ ☺ ☺
Clay modelling	☺ ☺

Key 1☺ = 1 Student

Fig. 10.1

We observe that, 4 students are enrolled in music, 6 students in dance, 3 students in painting and 2 students in clay modelling, so it was easy to take a ☺ for each student.

But is it possible to take a picture or symbol for 1 thing everywhere?

Suppose, we want to show all students enrolled in the above courses from all classes of the school, data will be large and then we need to choose a scale.

Reading and Interpreting Data From a Pictograph

A pictograph helps us answer the questions on the data at a glance.

Illustration: The pictograph (Fig. 10.2) shows the number of students enrolled in different courses at a school.

Course	Students Enrolled
Music	☺ ☺ ☺ ☺ ☺
Dance	☺ ☺ ☺ ☺ ☺ ☺
Painting	☺ ☺ ☺ ☺
Clay modelling	☺ ☺ ☺

Scale: 1☺ = 1 Student

Fig. 10.2

(a) What is the number of students enrolled in dance?
(b) Which is the least popular course?
(c) How many more students are enrolled in dance as compared to painting?

Let us try to answer the above questions using this pictograph.

(a) The row for dance has six symbols of 5 students each. So, we can say that the number of students enrolled in dance is 6 × 5 = 30.

(b) Since the clay modelling row has only three symbols, we can say that clay modelling is the least popular course.

(c) Students enrolled in dance = 30 [From (a)]

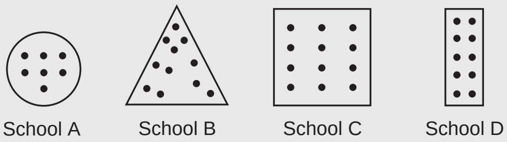
Students enrolled in painting = 4 × 5 = 20

Difference = 30 – 20 = 10

So, dance course has 10 more students enrolled as compared to the number of students enrolled for painting.

Skill Check

In the given pictograph, if one dot represents 100 students of some schools of a village of Haryana state of India.



Find out which school has the maximum number of students.

Drawing a Pictograph

To represent data in the form of a pictograph, we need a suitable symbol. Each complete symbol represents a fixed number and half the symbol represents half that number.

For example, if ☺ = 100, then ☾ = 50, etc.

Year	Number of cars produced by a company
2019	400
2020	450
2021	550
2022	600

Table 10.9

Suppose, we want to represent 100 cars and we use a symbol 🚙. What symbol could be used if we want to represent 50 cars?

We can solve such a situation by making an assumption that if 🚙 represents 100 cars, then 🚙 represents 50 cars.

We construct a pictograph for Table 10.9 using the above strategy .

Pictograph

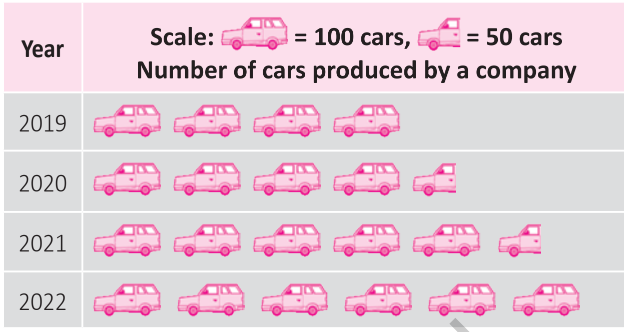


Fig. 10.3

Advantages and Disadvantages of a Pictograph

Advantages

- Like all pictorial representations, a pictograph is easy to read.
- Symbols or pictures help in understanding the data at a glance.
- If the data is spread over a period of time or among classes, *trend* in *data* is clearly visible.
- Makes data reading and its interpretation fun.

Disadvantages

- In some cases, finding a picture or a symbol to represent the data may not be easy. For example, if the data represents marks obtained by students in a particular subject or a course, a common acceptable symbol or picture may be hard to find.
- The key, unless all figures are clearly stated, may be hard to read.
- It may be difficult to draw the pictograph because of the symbols or pictures involved.

Let us study some more examples.

Ex. 2. **The following pictograph depicts the number of motorcycles produced by a company during the years 2019 to 2023.**

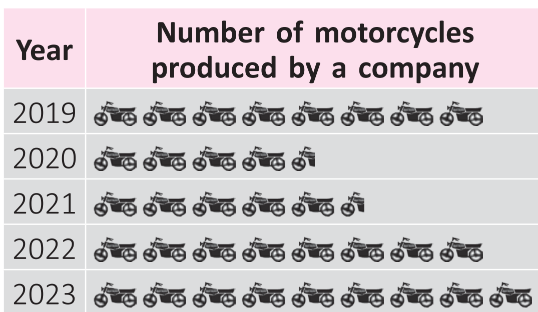


Fig. 10.4

Scale: = 1000 motorcycles, = 500 motorcycles

(a) How many motorcycles were produced during the years 2019–2021?

(b) In which year was production the highest?

(c) How many more motorcycles were produced in the year 2022 as compared to the year 2020?

Sol. (a) The number of symbols in the row of

2019: 8, 2020: $4\frac{1}{2}$, 2021 : $5\frac{1}{2}$

So, number of motorcycles produced during 2019–2021

$= \left(8+4\frac{1}{2}+5\frac{1}{2}\right) \times 1000$

$= 18 \times 1000$ (Each symbol represents 1000 motorcycles.)

= 18,000 motorcycles. (Total symbols = 18)

(b) In the year 2023, production was the highest.

(The number of symbols in the row of 2023 is more than any other row.)

(c) Motorcycles produced in 2022

$= 8 \times 1000 = 8000$

Motorcycles produced in 2020

$= 4\frac{1}{2} \times 1000 = 4500$

(or $4 \times 1000 + 1 \times 500$)

Difference in production

$= 8000 - 4500 = 3500$

Ex. 3. Following pictograph shows the number of wristwatches produced by a factory in a certain week.

Day	Number of Wristwatches
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

Fig. 10.5

Scale:  = 50 wristwatches  = 25 wristwatches

Answer the following questions.

(a) On which day were the least number of wristwatches produced and how many?

(b) On which day were the maximum number of wristwatches produced and how many?

(c) How many wristwatches were produced in the week?

Sol. (a) Least number of wristwatches were produced on Saturday.

Number of wristwatches $= 4 \times 50$

$= 200$

(b) Maximum number of wristwatches were produced on Thursday.

Number of wristwatches $= 8 \times 50$

$= 400$

(c) Total number of wristwatches produced in the week

$= (6 + 6\frac{1}{2} + 5\frac{1}{2} + 8 + 5 + 4) \times 50$

$= 35 \times 50 = 1750$

Ex. 4. Following table gives the data regarding popular school subjects among class VI students:

Subject	Number of Students
Hindi	**12**
English	**8**
Social Science	**10**
Mathematics	**14**
Science	**12**

Table 10.10

Draw a pictograph to represent the above data using a suitable key.

Sol. Let us take  = 4 students and  = 2 students. Then,

12 ÷ 4 = 3 symbols; 8 ÷ 4 = 2 symbols

$10 \div 4 = 2\frac{1}{2}$ symbols; $14 \div 4 = 3\frac{1}{2}$ symbols

12 ÷ 4 = 3 symbols

Thus, the pictograph is as shown:

Subject	Number of Students
Hindi	
English	
Social Science	
Mathematics	
Science	

Fig. 10.6

Exercise 10.2

1. Following is the pictograph of the number of cycles manufactured in a factory in a particular week.

Day	Number of Cycles Produced
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

Scale:  = 2000 cycles,  = 1000 cycles

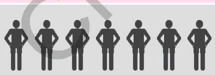
Observe the given pictograph and answer the following questions.

(a) On which day were the minimum number of cycles manufactured?

(b) On which two days were an equal number of cycles manufactured?

(c) Find the total number of cycles manufactured in the week.

2. The colours of refrigerators preferred by people living in a colony are shown by the following pictograph.

Colours	Number of People
Red	
Grey	
White	
Black	
Blue	

Scale:  = 50 people

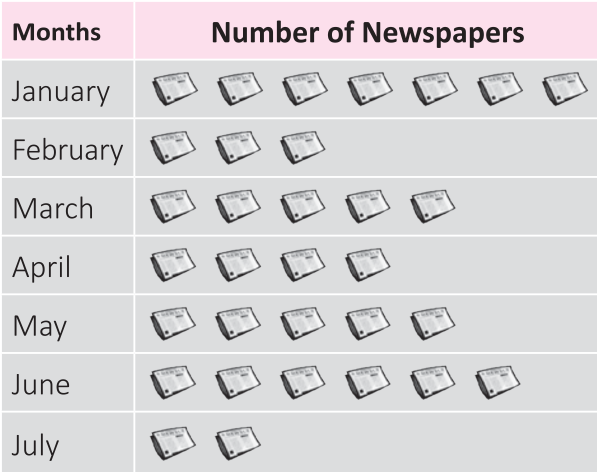
Observe the given pictograph and answer the following questions.

(a) Find the number of people preferring grey colour.

(b) Which colour is liked by most of the people? How many?

(c) How many more people prefer red colour than blue colour?

3. **The given pictograph shows the number of newspapers sold over a period of 7 months.**



Use the given information to answer the following questions.

(a) In which months the same number of newspapers were sold?

(b) How many newspapers does the symbol represent, if 3,50,000 newspapers were sold in the month of January?

(c) In which month were the least number of newspapers sold?

(d) How many newspapers were sold over a 7 month period, if each symbol equals 7000 newspapers?

4. **The table below gives the number of electric bulbs sold in a particular week by a shop.**

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of Bulbs Sold	12	14	8	10	16	6

Draw a pictograph to depict the given information.

Take scale as 1 bulb symbol = 2 bulbs.

5. **Total number of animals in six villages are given in the following table:**

Village A	**Village B**	**Village C**	**Village D**	**Village E**	**Village F**
120	90	70	80	40	60

Prepare a pictograph of these animals (use one symbol = 10 animals) and answer the following questions.

(a) How many symbols represent animals of village C?

(b) Which village has more animals: Village B or Village C?

6. **Details of the total number of students in five classes are given in the table.**

Class	I	II	III	IV	V
Students	50	35	60	30	45

Draw a pictograph for this information using one symbol ☺ to represent 10 students and ◖ to represent 5 students.

BAR GRAPHS

In the previous section, we have learnt representing data by pictographs. We now look at another way of representing data which uses bars (rectangles) in place of picture symbols. This representation of data is called a **bar graph**.

What is a Bar Graph?

A bar graph is a pictorial representation of numerical data in the form of rectangles (or bars) of equal width and of different heights.

Properties of Bar Graph

- In a bar graph, bars of equal width are drawn with different heights with equal spacing between them, using a suitable scale depending upon the data.

- If the bars are drawn on the horizontal line (as base line), then the scale of heights of bars is shown along the vertical line. If the bars are drawn on the vertical line (as base line), then the scale of heights of bars is shown along the horizontal line.
- Each bar represents only one value of the numerical data. So, there are as many bars as the values in the numerical data.
- The height (or length) of each bar indicates the corresponding value of the numerical data.

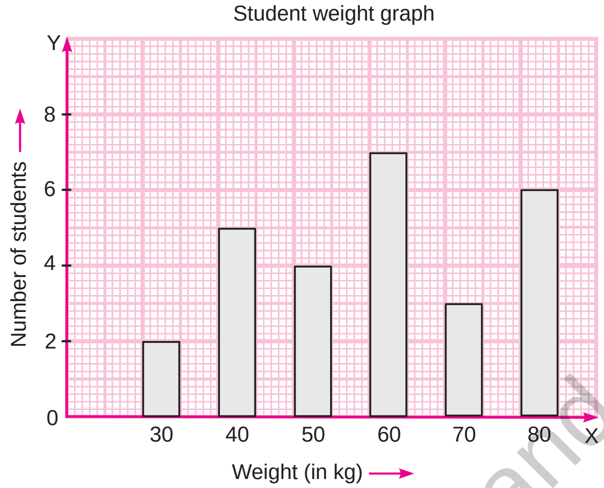


Fig. 10.7 (a) Vertical bar graph, bars are arranged vertically with equal spacing between them.

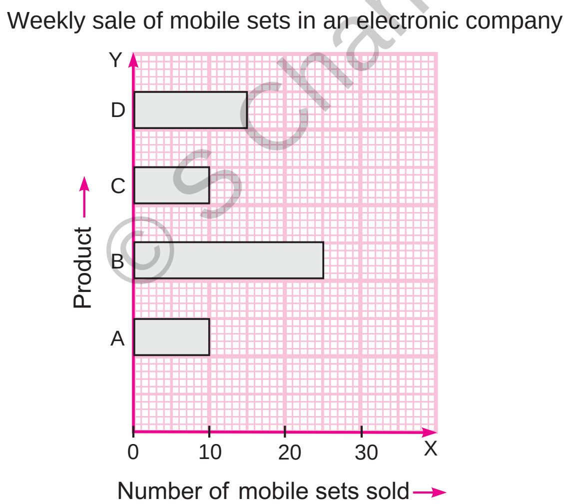


Fig. 10.7 (b) Horizontal bar graph, bars are arranged horizontally with equal spacing between them.

Reading and Interpreting Data from a Bar Graph

Illustration: The total sales of goods in a general store between 10 a.m. to 2 p.m. is given in the following bar graph.

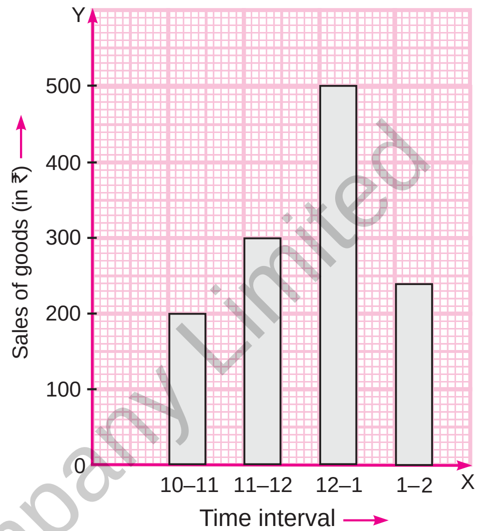


Fig. 10.8

By looking at the bar graph, we can conclude that:

(a) Total sales during the first hour, *i.e.*, between 10 a.m. to 11 a.m. = ₹200

(b) Total sales during the second hour, *i.e.*, between 11 a.m. to 12 noon = ₹300

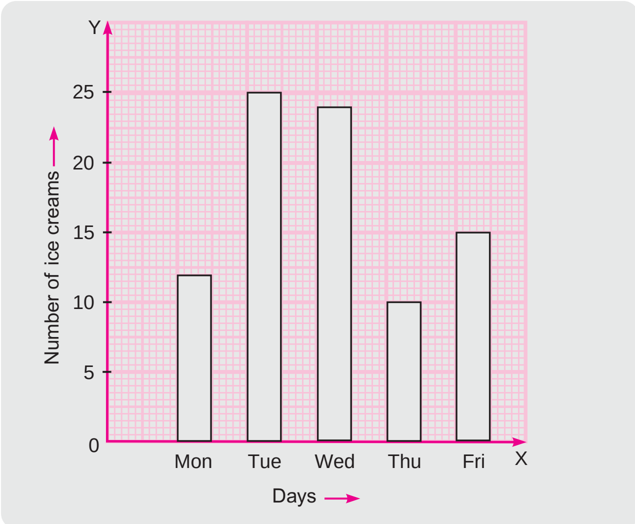
Total sales during the third (12–1) hour = ₹500

The increase in sales from the second to the third hour = ₹500 – ₹300 = ₹200

We can derive similar other information from this bar graph as shown in Fig. 10.8.

Skill Check

1. What is not correct about a bar graph?
   (a) The width of the bars should be uniform.
   (b) There should not be any gap between the bars.
   (c) The gap between any two consecutive bars should be uniform.
   (d) Bars drawn may be either horizontal or vertical.
2. The given bar graph shows the number of ice creams eaten by children in one particular week. How many ice creams did the children eat that week?



Let us study some more examples.

Ex. 5. **Using the bar graph (see Fig. 10.9), answer the following questions.**

(a) How much amount goes towards rent?

(b) What is the monthly income of Mrs Anjali, if she saves ₹10,000 per month?

Sol. (a) The height of the bar representing the item 'Rent' is 25.

So, ₹25 × 1000 = ₹25,000 goes towards rent.

Distribution of Mrs Anjali's monthly income

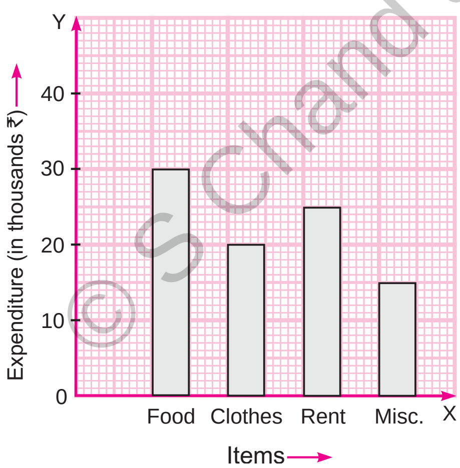


Fig. 10.9

(b) The total monthly income

= The total expenditure + Savings

= (the sum of the heights of all the bars) × ₹1000 + ₹10,000

= (30 + 20 + 25 + 15) × ₹1000 + ₹10,000

= ₹(90 × 1,000) + ₹10,000

= ₹1,00,000

Thus, the monthly income of Mrs Anjali is ₹1,00,000.

Ex. 6. **Use the bar graph (see Fig. 10.10), answer the following questions.**

(a) How many big screen TV sets are sold during the week?

(b) Which type of TV is the most popular?

Weekly sale of TV sets in an electronic shop

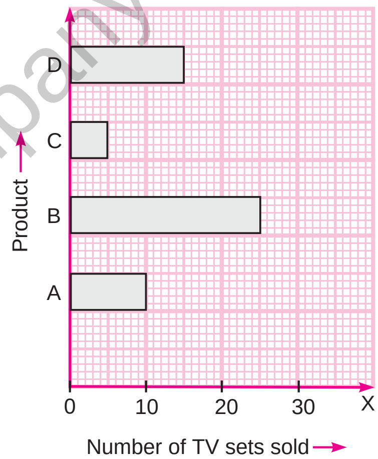


Fig. 10.10

A : Flat screen TV　　B : Miniature TV

C : Big screen TV　　D : Colour TV

Sol. (a) 5 big screen TV sets are sold during the week.

(The length of the bar for C, representing the big screen TV sets is 5.)

(b) Miniature TV is the most popular type.

(The length of the bar for B, miniature TV sets is longest.)

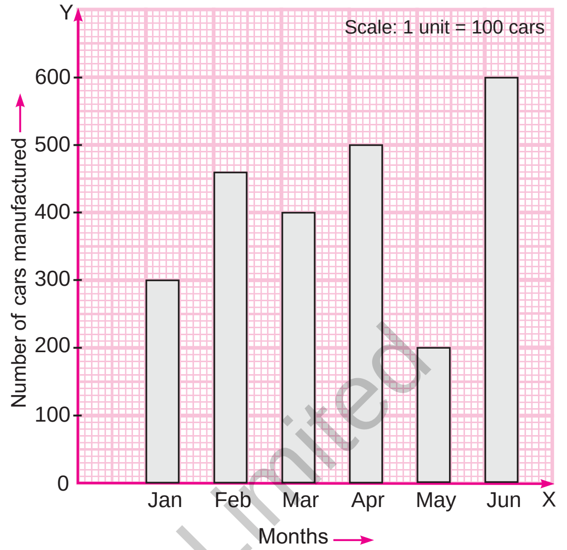
Exercise 10.3

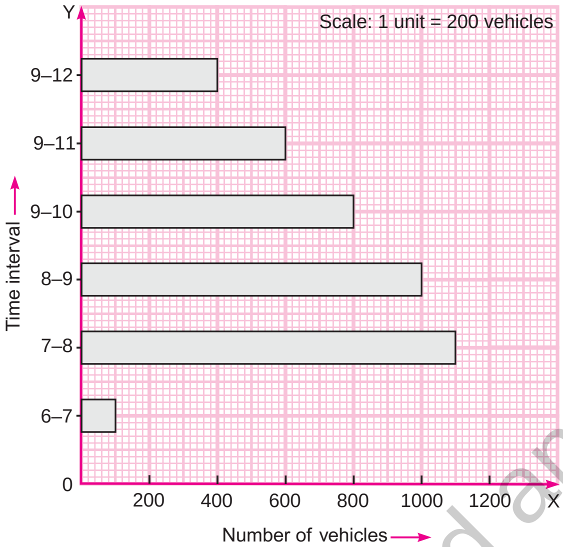
1. Read the bar graph given alongside and answer the questions that follows.

(a) Name the month when the least cars were manufactured.

(b) How many less cars were manufactured in the month of March than in the month of June?

(c) What is the ratio of cars manufactured in the month of January to that of April?





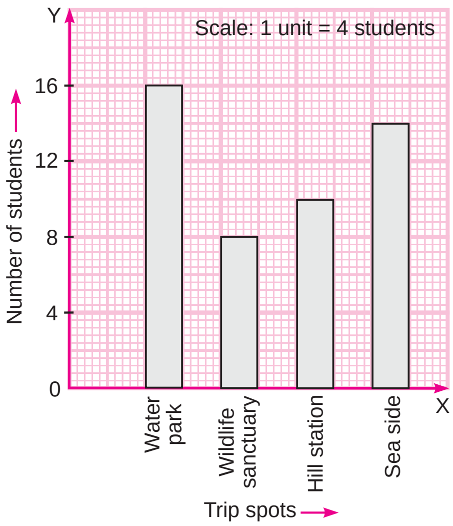
2. Observe the given bar graph, showing vehicular traffic at a busy road crossing in Mumbai. The number of vehicles passing through the crossing every hour from 6:00 a.m. to 12:00 noon is shown in the bar graph.

Now, answer the following questions.

(a) In which time interval, the traffic is minimum?

(b) In which time interval, the traffic is maximum?

(c) What is the total traffic during two peak hours?

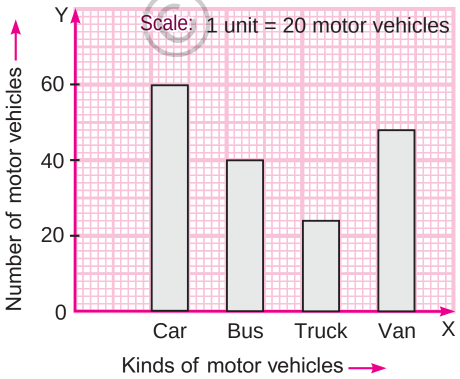


3. A class took a vote to decide where to go for a trip. Use the given bar graph to answer the following questions.

(a) How many students are there in the class?

(b) Where did the majority of students want to go?

(c) How many students voted for Hill station?

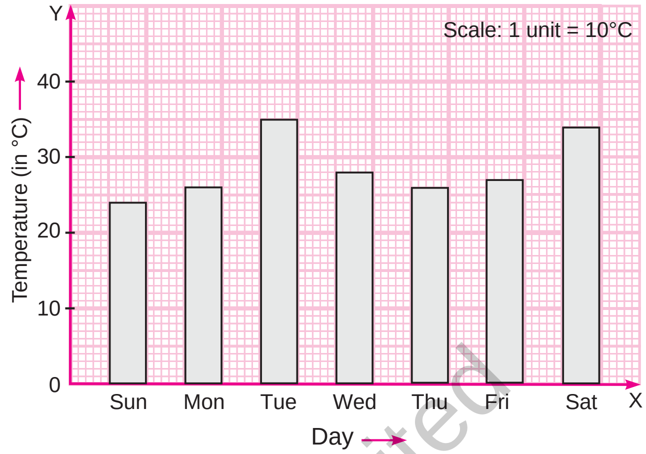


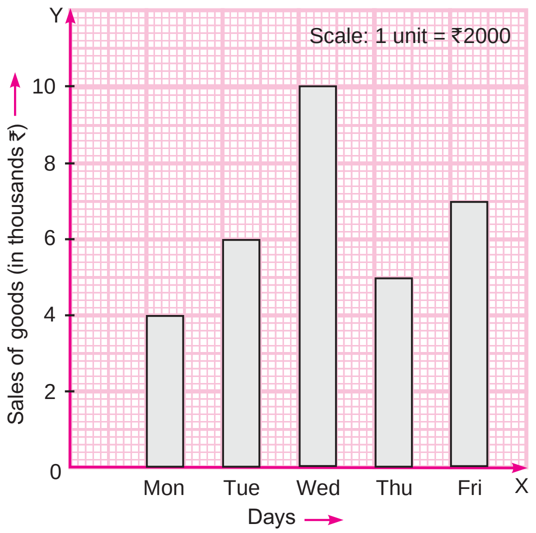
4. The given bar graph shows the number of different kinds of vehicles which were counted on the road within one hour. Use this graph to answer the following questions.

(a) Find the number of cars that exceeded the number of vans on the road.

(b) Which vehicle was the least frequent on the road?

5. **The given bar graph shows the temperature in Bridgetown, Barbados for a week in July. Use the graph to answer the following questions.**
   (a) Which day was the hottest?
   (b) Which day was the coolest?
   (c) Find the difference between the temperature on Tuesday and the temperature on Thursday.





6. **The given bar graph shows the total sales of goods in a shop from Monday to Friday. Read the graph and answer the following questions.**
   (a) Find the total sales on Monday.
   (b) On which day the sales was maximum?
   (c) On which day the sales was minimum?
   (d) Find the combined total sales on Tuesday and Friday.

CONSTRUCTING BAR GRAPHS

In the previous section, we have learnt to read and interpret a bar graph. Now, let us learn to draw or construct bar graphs.

Bar graphs are constructed on a set of two perpendicular axes, one horizontal and the other vertical.

In bar graphs, recall that there are bars or rectangles of uniform width with equal spacing in between them and are constructed on one of the axes used as a **base line**.

On the other axis, the scale for the heights (or lengths) of the rectangles is shown.

The uniformity of width is maintained in the graph for easy comparison.

The length of each bar represents the exact value of the data in the category represented by that bar.

Construction of a Vertical Bar Graph

To construct a vertical bar graph, we follow these steps:

Step 1: Draw a vertical line (axis) and a horizontal line (axis).

Step 2: Mark an appropriate scale on the vertical axis to represent the data value of each category.

Step 3: Mark the categories of data along the horizontal axis.

Step 4: Draw a vertical rectangle (bar) for each category, so that the height of the rectangle (bar) reaches the value of the data in that category.

Step 5: Give a suitable title to the graph and see that all the items are labelled clearly.

Illustration: Consider the following data.

Sales figures of a shop over a period of five years

Year	Sales (in lakhs ₹)
2019	50
2020	62
2021	45
2022	58
2023	65

Table 10.10

In order to draw a bar graph for the given data, we proceed as follows:

(i) Draw two perpendicular lines, one horizontal and other vertical.

(ii) Along the horizontal line (axis), mark the 'year' and along the vertical line (axis) the 'sales (in lakhs ₹)'.

(iii) Choose a suitable scale along the vertical axis. Let one unit length = ₹10 lakh.

(iv) Calculate heights of the bars as:

Year 2019 : 50 ÷ 10 = 5 units

Year 2020 : 62 ÷ 10 = 6.2 units

Year 2021 : 45 ÷ 10 = 4.5 units

Year 2022 : 58 ÷ 10 = 5.8 units

Year 2023 : 65 ÷ 10 = 6.5 units

(v) With these heights, draw the bars of the same width keeping uniform gap between them as shown below. It is the required bar graph.

Fig. 10.11

Skill Check

Construct a bar graph that represents the following data.

Sales figures of goods over a period of five years

Year	Sales (in lakhs ₹)
2017	50
2018	55
2019	48
2020	60
2021	65

Let us study some more examples.

Ex. 7. **The heights (in cm) of 40 students of Class VI of a particular school are given in Table 10.11.**

Heights (in cm)	144	150	155	157	164
Number of students	7	8	12	8	5

Table 10.11

Construct a bar graph to represent the information.

Sol. **Step 1:** Draw a vertical axis and a horizontal axis.

Step 2: Mark a scale on the vertical axis that will include number of students from 0 to 12.

(On this graph, we have chosen to mark the numbers from 0 to 12, using a scale of 1 unit = 2 students.)

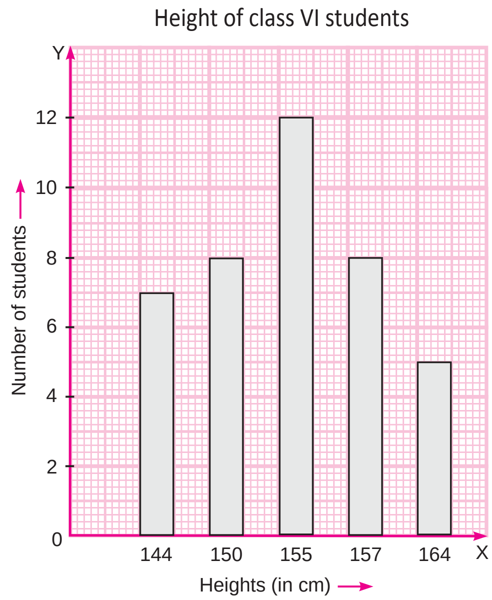


Fig. 10.12

Step 3: On the horizontal axis, mark any five points to represent the five categories of the data at equal distances. Label them with heights 144 cm to 164 cm.

Step 4: Draw bars (rectangles) of equal width for each height, keeping the marked point in the centre of the base for each rectangle. The height of each rectangle should correspond to the number of students having that particular height (in cm).

Step 5: (a) Label the horizontal axis with "Heights (in cm)" to indicate that the points marked on it represent heights.

(b) Label the vertical axis with "Number of students" to indicate that any point on this axis represents that many students having the particular height.

(c) Give a suitable title to the graph to indicate what kind of information this graph exhibits. In this case, the title "Heights of class VI students" has been used.

Ex. 8. The following table shows the approximate length of National Highways (NH) of India. Illustrate this data using a bar graph.

National Highways	Length (in km)
NH 30	2000
NH 6	1900
NH 19	1400
NH 66	1600
NH 16	1700

Table 10.12

Sol. The bar graph is as shown:

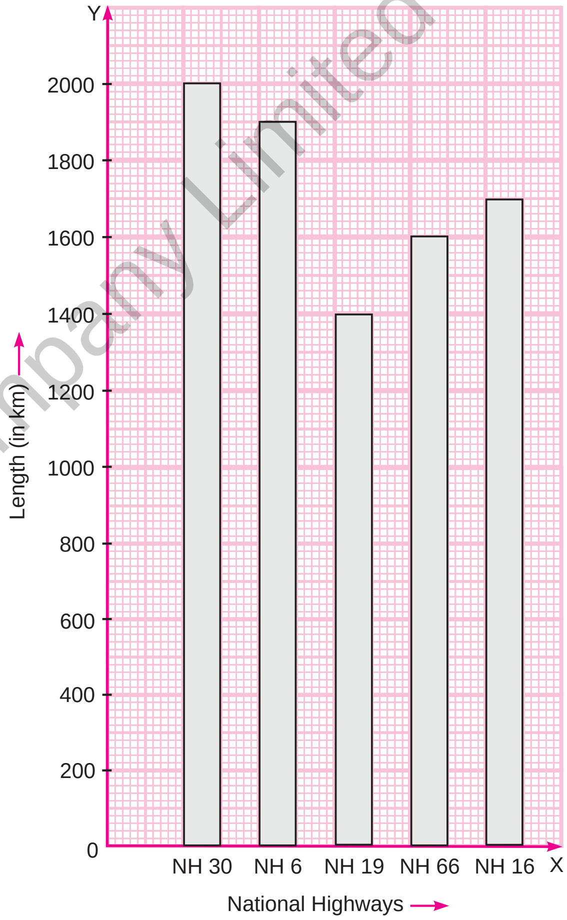


Fig. 10.13

Exercise 10.4

1. Represent the following data using a bar graph.

(a) The number of Mathematics books sold by a shopkeeper on six consecutive days is given below.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of books sold	50	35	40	70	60	65

(b) In a particular day of March 2023 at Delhi, the temperature is recorded as under.

Time	12:00 noon	2:00 p.m.	4:00 p.m.	6:00 p.m.	8:00 p.m.	10:00 p.m.
Temperature (°C)	28	32	30	28	26	24

(c) Various modes of transport used by 2000 students of a school are as follows.

Modes of transport	Car	Cycle	Private Van	School Bus	Walking
Number of students	250	380	220	750	400

2. The height of 50 students of class VI in a school are given as follows.

Height (in cm)	145	150	155	160	165
Number of students	6	10	15	12	7

Draw the bar graph to represent the data given above and answer the following questions.

(a) How many students have their height more than 160 cm?

(b) What fraction of students of total strength have height more than 155 cm?

3. The table given alongside shows the lengths of some National Highways (NH) in India. Round off the lengths to the nearest hundred kilometres and draw a bar graph for the given data.

CC

National Highways	**Length (in km)**
NH 44	3745
NH 27	3507
NH 52	2317
NH 53	1781
NH 48	2807

Competency Based Exercise

21st CS

1. Tick (✓) the correct answer.

(a) Using tally marks, which one of the following represents number seven.

(i) (||||) || (ii) 0000000 (iii) ~~||||~~ || (iv) ~~|||~~ ||

(b) The pictograph shown alongside represents some surnames of people listed in the telephone directory of a city.

The surname which appears the least number of times in the directory is:

(i) Khan (ii) Bhatia

(iii) Roy (iv) Anand

Surname	**Number of people**
Bhatia	🖐 🖐 🖐 🖐
Anand	🖐 🖐 🖐 🖐 🖐
Rao	🖐 🖐 🖐 🖐 🖐 🖐 🖐
Roy	🖐 🖐 🖐 🖐
Khan	🖐 🖐
Singh	🖐 🖐 🖐 🖐 🖐 🖐

Scale: 🖐 = 1000 people

Study the bar graph and answer questions (c) and (d).

(c) What is the length of National Highway 24?

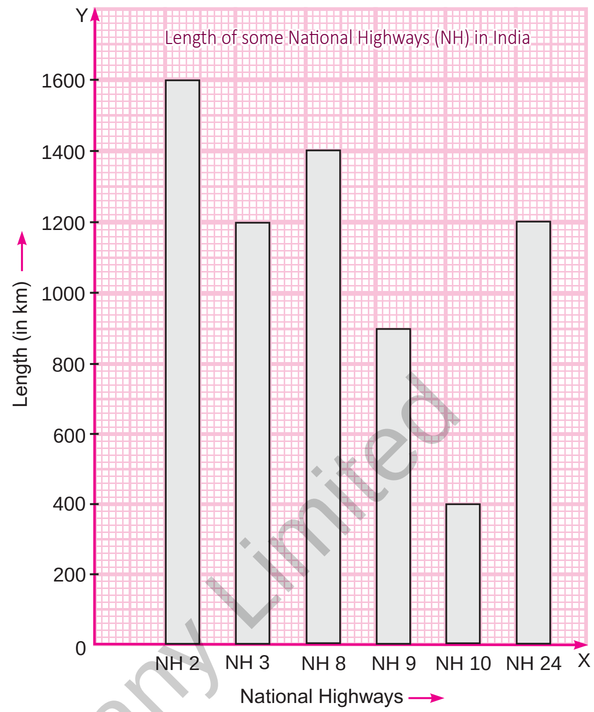
(i) 500 km (ii) 900 km

(iii) 1200 km (iv) 1400 km

(d) The length of which National Highway is about four times the National Highway 10?

(i) NH 9 (ii) NH 8

(iii) NH 3 (iv) NH 2



2. The blood groups of 30 students are recorded as under:

B, A, O, A, AB, A, B, A, A, AB, O, O, AB, B, A, B, AB, O, B, A, A, O, AB, B, A, A, O, AB, A, B

Arrange the information in a table using tally marks.

3. The given table represents the marks obtained (out of 20) by 50 students in a Mathematics test.

Complete the table and answer the following questions.

Marks	Tally Marks	Number of Students
10	|||	3
11		6
13	𝍸 𝍸	
14		10
16	𝍸 |||	
19		9
20	||||	

(a) How many students obtained marks less than 13?

(b) How many students obtained marks more than or equal to 19?

(c) How many students obtained marks more than 13 but less than 19?

4. The given pictograph represents the number of people who visited a museum during a week.

Now, answer the following questions.

(a) On which day did maximum number of people visit the museum?

(b) How many more people visited the museum on Saturday than on Thursday?

(c) If the profit of ₹20 was made on the sale of each entry ticket, find the total profit made on Sunday.

(d) Can you represent the same information in a bar graph? If yes, try it.

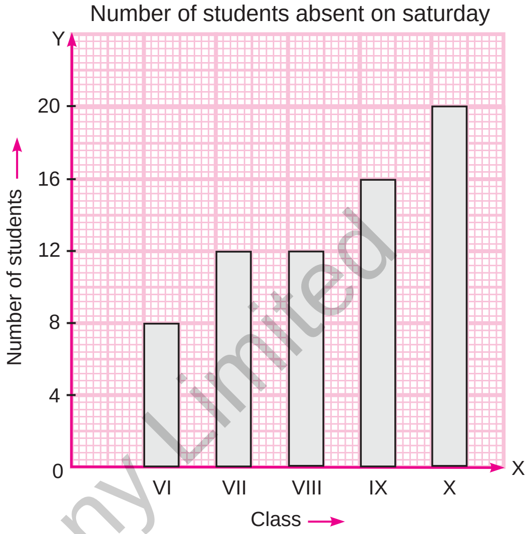
Day	Number of Visitors
Monday	☺ ☺ ☺ ☺
Tuesday	☺ ☺ ☺ ☺
Wednesday	☺ ☺ ☺ ◐
Thursday	☺ ☺ ☺ ☺ ☺ ☺
Friday	☺ ☺ ☺ ◐
Saturday	☺ ☺ ☺ ☺ ☺ ☺ ☺
Sunday	☺ ☺ ☺ ☺ ☺ ◐

Scale: ☺ = 100 visitors

5. **Prepare a pictograph for the data given for the number of students who like to play five different games.**

Game	Number of Students
Football	140
Cricket	200
Hockey	85
Badminton	125
Tennis	110

6. **The principal drew a bar graph of the students of the classes VI to X who were absent on Saturday. Study the bar graph and answer the following questions.**
   (a) How many total number of students were absent on Saturday?
   (b) For which class, the maximum number of students were absent.
   (c) For which class, 16 students were absent.



Challenge!

21st CS

Following are the favourite games of 40 students.

Cricket, football, kho-kho, hockey, cricket, hockey, kho-kho, tennis, tennis, cricket, football, football, hockey, kho-kho, football, cricket, tennis, football, hockey, kho-kho, football, cricket, cricket, football, hockey, kho-kho, tennis, football, hockey, cricket, football, hockey, cricket, football, kho-kho, football, cricket, hockey, football, football

Represent the above information through a pictograph or bar graph.

SMART TIME

CC

Write the names of 3-4 former Prime Ministers/Presidents of India and their office tenure.

S.No.	Names of ________	Tenure	Number of years
1.			
2.			
3.			
4.			

Display your records through a bar graph.

Let's Work in Mind

21st CS

1. What number do the tally marks 𝍸 𝍸 ||| represent?
2. What is the frequency of vowels in the word 'MATHEMATICS'?
3. Which marks are used in tabulation of data?
4. In a pictograph, if a symbol  represents 50 persons, then what does the symbol  represent?
5. In a pictograph, if a symbol  represents 20 flowers, then what does  stand for?

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. Assertion (A) : In the following table, the frequency of getting head by 5 students in 6 tosses of coin is shown.

Students	1	2	3	4	5
Number of Heads	2	3	4	1	5

5th student has the highest frequency of heads.

Reason (R) : Frequency is the number of time an event occurs.

2. Assertion (A) : In the given pictograph, if one dot represents 100 students of some schools in a village of Punjab.

(a) (b) (c) (d)

(a) has maximum students, *i.e.*, 600 and (b) has minimum students, *i.e.*, 300.

Reason (R) : One dot = 100

So, number of students = number of dots × 100

3. Assertion (A) : Given the following table of shoes size of students in a class.

Shoes size	4	5	6	7	8	9
Number of students	6	8	3	7	3	0

7 is frequency of shoe size 7.

Reason (R) : The number of times a particular observation occurs is called its frequency.

4. Assertion (A) : Following is the pictograph of number of students enrolled in different courses in a school.

Courses	**Students enrolled**
Music	☺ ☺ ☺ ☺ ☺
Dance	☺ ☺ ☺
Painting	☺ ☺ ☺ ☺ ☺ ☺
Clay Modelling	☺ ☺

Scale: 1 ☺ = 10 students

Least popular course is clay modelling.

Reason (R) : Frequency cannot be calculated using picture symbol.

11 Mensuration

What Learners Will Achieve

- define the perimeter of plane figures, regular as well as irregular.
- derive the formulae to find the perimeter of a rectangle, square, equilateral triangle and other regular shapes.
- define the area of plane figures, regular as well as irregular, understand area as amount of surface enclosed by a figure.
- find the area of a rectangle and a square.
- find the area of regular or irregular shapes by dividing them into small rectangles and squares.
- apply the concept of perimeter and area in real life.

Warm-up

What we already know

- The distance around the boundary of a plane shape is known as its perimeter.
- The region enclosed by a plane shape is known as its area.

Now, try to solve the following.

1. Which term (perimeter or area) is suitable to determine the following?

(a) Length of wooden photoframe ___________

(b) Length of glitter tape showing the margin of a chart ___________

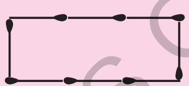
(c) Space of floor for carpeting ___________

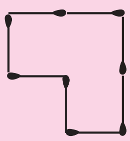
(d) Wrapping paper to cover a gift ___________

(e) Length of rope for fencing a school garden ___________

(f) Space on a wall for advertisement ___________

2. The length of a matchstick is 3 cm. Find the total length of the boundary in the following figures.

(a) 

(b) 

3. Tick **(✓)** the note or coin that covers less space.

(a) 

(b)  

DID YOU KNOW?

Babylonians had measured the circumference of the circle as 3 times the diameter and it is fairly close to today's calculation of the circumference.

PERIMETER AND AREA OF POLYGONS

In geometry, we have learnt about plane figures. When we talk about a plane figure, we immediately think of its boundary and the region enclosed by it. The first one gives rise to the concept of *perimeter* and the second one to the concept of *area*. In this chapter, we shall learn about these two concepts that are of great practical utility. For example, if a farmer wants to fence his field he would need to know the length of the boundary (called perimeter) of his field to find out how much fencing he needs. Similarly, if we want to fix a glass in a window frame, we must know the measure of the surface of the window to know the quantity of the glass required. The branch of Mathematics which deals with these concepts (*i.e.,* finding perimeter, area, volume) is called **mensuration**. We shall develop some formulae for perimeters and areas of some plane figures like square, rectangle, etc., and then use them in solving various problems.

PERIMETER

We have learnt that a simple closed curve divides the plane (on which it is drawn) into three distinct parts:

(i) The curve itself.

(ii) The portion (or part) of the plane inside the curve or bounded by the curve is called the *interior region*.

(iii) The portion of the plane outside the curve is called the *exterior region*.

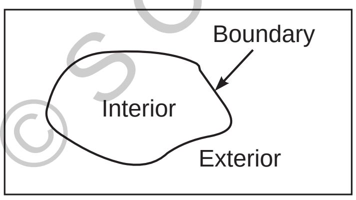


Fig. 11.1

The curve itself is the **boundary** of the interior region. We are generally interested in the figures consisting of a curve along with its interior region for practical purposes. We shall use the term **region** to mean **interior region along with the boundary**.

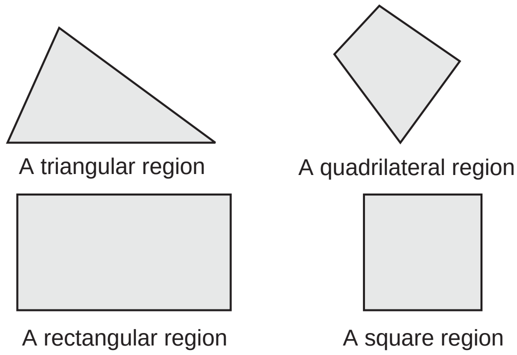


Fig. 11.2

The **perimeter of a closed figure (simple closed figure) is the distance moved along its boundary once.**

In Fig. 11.3, if we start moving along the curve starting from a point S on it and come back at S in one round, the distance so covered is the **perimeter of the curve**.

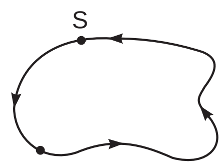


Fig. 11.3

Perimeter of a Polygon

The perimeter of a polygon is the sum of the lengths of all its sides and is generally denoted by the letter P.

Perimeter of a Triangle

The perimeter of a triangle is the sum of the lengths of all its three sides.

Illustration 1: The perimeter (P) of the triangle (see Fig. 11.4) whose sides measure 4 cm, 5.2 cm and 3.5 cm is given by:

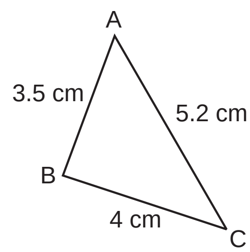


Fig. 11.4

P = BC + CA + AB

i.e., P = 4 cm + 5.2 cm + 3.5 cm

= 12.7 cm

Illustration 2: Two sides of a triangle are 12 cm and 15 cm. The perimeter of the triangle is 40 cm. To find the third side of the triangle, we proceed as follows.

Let the length of the third side of the triangle be x cm.

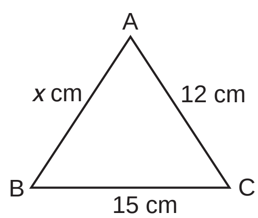


Fig. 11.5

Since we know that

Perimeter of a triangle = Sum of the lengths of its sides

$\Rightarrow$ $40 = 12 + 15 + x$

$\Rightarrow$ $40 = 27 + x$ $\Rightarrow$ $x = (40 - 27)$ $\Rightarrow$ $x = 13$

Thus, the third side of the triangle is 13 cm.

Perimeter of an equilateral triangle

We know that a triangle having all sides equal is known as an ***equilateral triangle***. So, **perimeter of an equilateral triangle = 3 × length of a side**.

Illustration 3: Let us find the perimeter of the equilateral triangle whose side is 7 cm.

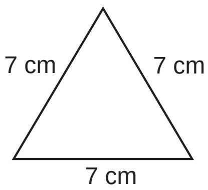


Fig. 11.6

Perimeter of an equilateral triangle = 3 × Length of a side.

Therefore, perimeter

= 3 × 7 cm = 21 cm.

Thus, the perimeter of the given equilateral triangle is 21 cm.

Perimeter of a Quadrilateral

The perimeter of a quadrilateral is the sum of the length of its four sides.

Illustration 4: The perimeter of the quadrilateral PQRS (see Fig. 11.7) is given by:

Perimeter = PQ + QR + RS + SP

i.e., Perimeter = 4 m + 2.5 m + 3 m + 5 m = 14.5 m

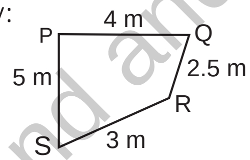


Fig. 11.7

Perimeter of a rectangle

The sides of a rectangle are generally called its length (l) and breadth (b).

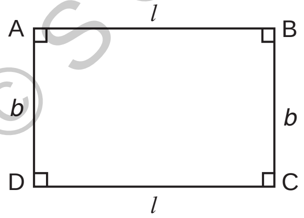


Fig. 11.8

Thus, the perimeter (P) of the rectangle [see Fig. 11.8] = Sum of the lengths of its four sides,

i.e., P = AB + BC + CD + DA

$= l + b + l + b = 2l + 2b = 2(l + b)$

$\therefore$ **Perimeter of a rectangle = $2(l + b)$**

Ex. 1. Find the perimeter of the rectangle with length 4 cm and breadth 3 cm.

Sol. Perimeter = 4 cm + 4 cm + 3 cm + 3 cm

= 2(4 cm) + 2(3 cm)

= 8 cm + 6 cm = 14 cm

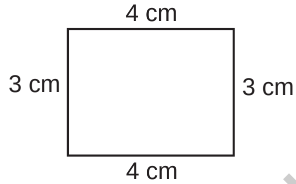


Fig. 11.9

or perimeter = $2(l + b) = 2(4 \text{ cm} + 3 \text{ cm})$

$= 2 \times 7 \text{ cm} = 14 \text{ cm}$

Ex. 2. The perimeter of a rectangle is 180 cm and its breadth is 36 cm. Find the length of the rectangle.

Sol. Perimeter of the rectangle (P) = 180 cm,

Breadth of the rectangle (b) = 36 cm

Let the length of the rectangle be l cm.

Then, $P = 2(l + b)$ (Formula)

$\Rightarrow$ $180 = 2(l + 36)$

$\Rightarrow$ $2(l + 36) = 180$

$\Rightarrow$ $l + 36 = \dfrac{180}{2} = 90$

$\Rightarrow$ $l = (90 - 36) = 54$

Therefore, the length of the rectangle is 54 cm.

Perimeter of a square

We know that a square is a rectangle whose all four sides are equal. If the length of each side of the square is a (see Fig. 11.10), then

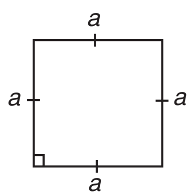


Fig. 11.10

Perimeter of a square = $a + a + a + a = 4a$

Ex. 3. Find the perimeter of a square with side 6 cm.

Sol. Perimeter of a square

$= 4a$

$= 4 \times 6$ cm

$= 24$ cm

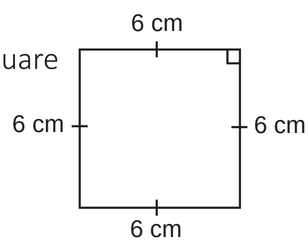


Fig. 11.11

Perimeter of a regular polygon

We know that a regular polygon has its all sides and angles of equal measure. Square and equilateral triangle are examples of regular polygons.

We already know that perimeter of a square = 4 × side (Here, number of sides = 4) and perimeter of an equilateral triangle = 3 × side (Here, number of sides = 3).

> Thus, we can say that the perimeter of an *n*-sided regular polygon = ***n* × side**.

Ex. 4. **Find the perimeter of a regular hexagon with each side measuring 7 m.**

Sol. One side of a regular hexagon = 7 m

Number of sides of a regular hexagon = 6

So, perimeter of the regular hexagon

= 6 × length of one side

= 6 × 7 m = 42 m

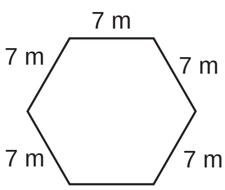


Fig. 11.12

Perimeter of an irregular polygon

The perimeter of an irregular polygon is determined by simply adding the lengths of all sides.

Ex. 5. **Find the perimeter of the polygon given in the Fig. 11.13.**

Sol. Perimeter = Sum of the lengths of all its sides

Perimeter = 9 m + 9 m + 9 m + 8.2 m + 8.2 m

= (9 + 9 + 9 + 8.2 + 8.2) m

= 43.4 m

Thus, the perimeter of the polygon is 43.4 m.

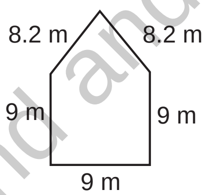


Fig. 11.13

Ex. 6. **Find the perimeter of the Fig. 11.14.**

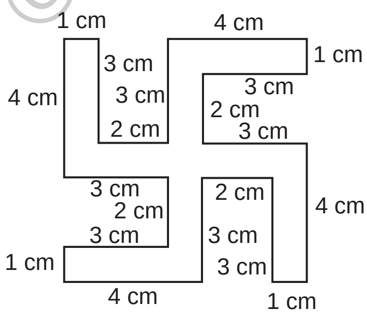


Fig. 11.14

Sol. From the figure, we have

Perimeter = 4 cm + 1 cm + 3 cm + 2 cm + 3 cm + 4 cm + 1 cm + 3 cm + 2 cm + 3 cm + 4 cm + 1 cm + 3 cm + 2 cm + 3 cm + 4 cm + 1 cm + 3 cm + 2 cm + 3 cm

= 52 cm

Applications of Perimeter in Real Life

Ex. 7. **Find the distance travelled by Priya, if she takes 3 rounds of a square park of side 50 m.**

Sol. Perimeter of the square park

= 4 × length of a side

= 4 × 50 m = 200 m

Distance covered in one round

= Perimeter of the square park

= 200 m

Therefore, distance covered in 3 rounds = 3 × 200 m = 600 m.

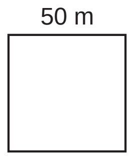


Fig. 11.15

Ex. 8. **A rectangular field is 130 m long and 80 m wide. Find the length of a wire needed to fence the field. Also, find the cost of the fence, if fencing costs ₹3.05 per metre.**

Sol. Length of wire needed to fence the field

= Perimeter of the field

= 2(130 m) + 2(80 m) = 420 m

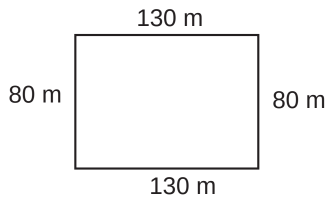


Fig. 11.16

Since the cost of fencing 1 m of the field is ₹3.05, therefore the cost of fencing 420 m of the field = 420 × ₹3.05 = ₹1281.

Skill Check

- Find the length of the wooden strip required to frame a photograph of length and breadth 30 cm and 20 cm respectively.
- Find the cost of fencing a square park of side 215 m at the rate of ₹20 per metre.

Ex. 9. Find the perimeter of the top of a rectangular table, if the table top measures 2 m 25 cm by 1 m 50 cm.

Sol. Length of the table top (l) = 2 m 25 cm = 2.25 m (∵ 1 m = 100 cm)

Breadth of the table top (b) = 1 m 50 cm = 1.50 m

Perimeter of the table top = $2(l + b)$

= 2(2.25 + 1.50) m

= 2 × 3.75 m = 7.50 m

Ex. 10. The cost of putting a fence around a square field at the rate of ₹25 per metre is ₹1600. Find the length of each side of the field.

Sol. Cost of putting a fence around the field = ₹1600

Cost of fencing one metre = ₹25

As the fencing is done along the boundary of the square, *i.e.*, along its perimeter, so

$$\text{perimeter} = \frac{\text{Total cost}}{\text{Cost per metre}}$$

$$= \frac{1600}{25} \text{ m} = 64 \text{ m}$$

Let each side of the square field = x metres

Then, the perimeter of the square field = $4x$ metres

Therefore, $64 = 4x$ or $x = \frac{64}{4} = 16$ m

Thus, the length of each side of the square field is 16 metres.

Ex. 11. A wire in the shape of a square of side 64 cm is rebent to form a rectangle of length 80 cm. Find the breadth of the rectangle so formed.

Sol. Given, side of the square = 64 cm

So, length of the wire = Perimeter of the square, *i.e.*, 4 × side = 4 × 64 cm = 256 cm

Since the wire is bent into the form a rectangle, so length of the wire will remain the same.

Now, length of the rectangle = 80 cm

Perimeter of the rectangle = 256 cm

Since $P = 2(l + b)$

⇒ 256 cm = 2(80 + b) cm

⇒ b = 128 cm − 80 cm = 48 cm

Thus, the breadth of the rectangle is 48 cm.

Ex. 12. Sushant runs around a square park of side 60 m. Bilal runs around a rectangular park with length 40 m and breadth 30 m. Who covers less distance?

Sol. Sushant runs around a square park.

Side of the square park = 60 m

Perimeter of the square park = 4 × side

= 4 × 60 m = 240 m

Therefore, distance covered by Sushant = 240 m

Bilal runs around a rectangular park.

Length of the rectangular park = 40 m

Breadth of the rectangular park = 30 m

Perimeter of the rectangular park = $2(l + b)$

= 2(40 + 30) m = 2 × 70 m = 140 m

Therefore, distance covered by Bilal = 140 m

Since 240 m > 140 m, therefore Bilal covers less distance.

Skill Check

Ritesh runs 12 times around a square field of side 70 m and Rakesh runs 10 times around a rectangular field of length 90 m and breadth 80 m. Ritesh claimed that he ran more and Rakesh said that he covered more distance. Whose claim is right?

Ex. 13. Ankur buys 9 square paving slabs, each of side $\frac{1}{2}$ m. He lays them in the form of a square as shown in Fig. 11.17. Find the perimeter of his arrangement.

Sol. Ankur arranges the square slabs in the form of a square.

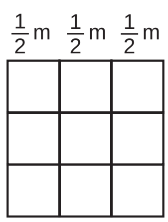


Fig. 11.17

Each side of the square

$= \frac{1}{2}\text{ m} + \frac{1}{2}\text{ m} + \frac{1}{2}\text{ m} = \frac{3}{2}\text{ m}$

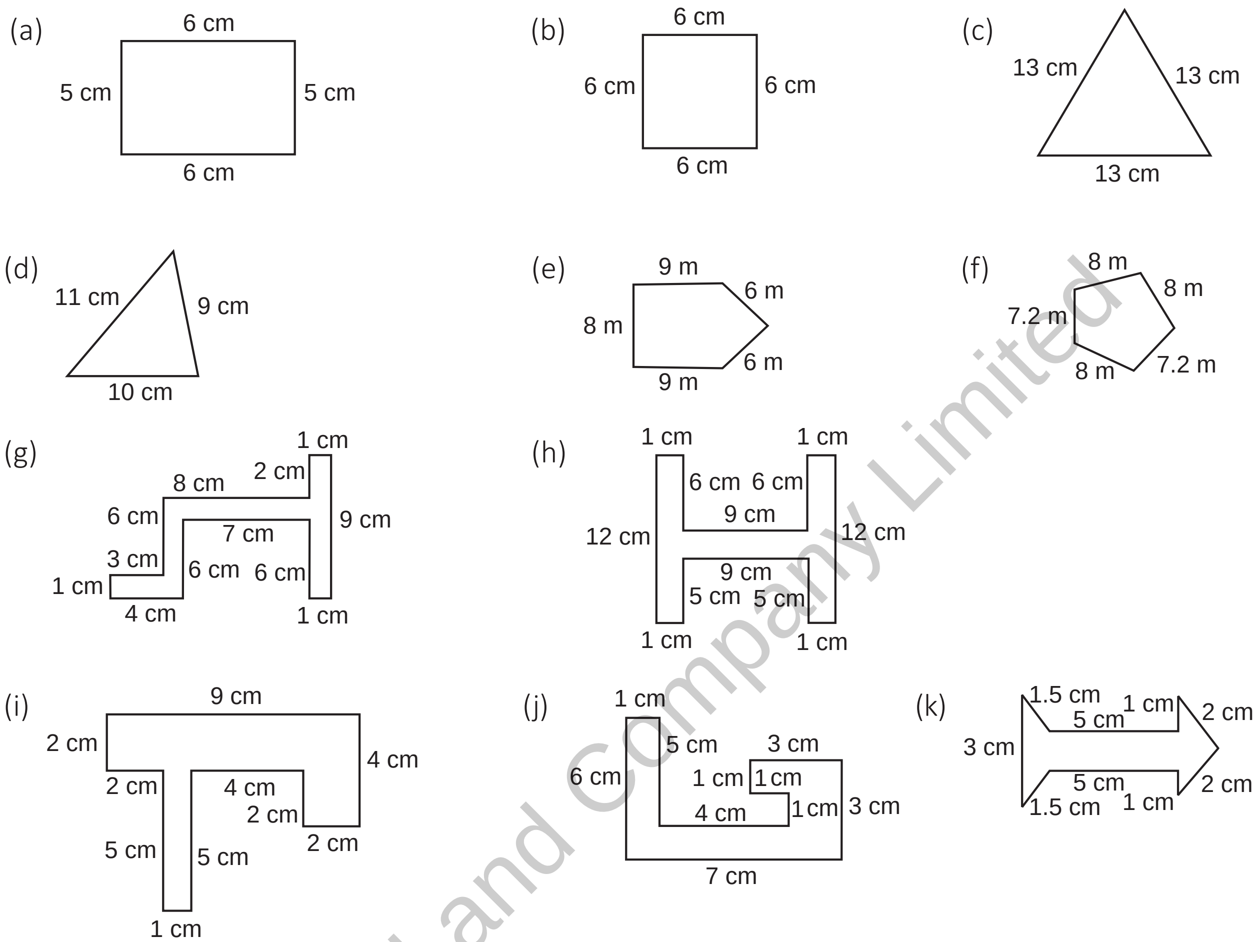
Perimeter of the square

$= 4 \times \text{side} = 4 \times \frac{3}{2}\text{ m} = 6\text{ m}$

Thus, the required perimeter = 6 m

Exercise 11.1

1. Determine the perimeter of the following figures with the given dimensions.



2. Find the perimeter of a rectangle whose dimensions are given below.

(a) l = 3 cm, b = 7 cm (b) l = 8 cm, b = 5 cm (c) l = 12.68 m, b = 6.23 m

(d) l = 13.5 cm, b = 14 cm (e) l = 36 cm, b = 36 cm

3. Determine the side of a regular polygon with the given perimeter.

(a) 24 cm; Hexagon (b) 66 cm; Equilateral triangle

(c) 18.9 cm, Nonagon (d) 1000 cm, Decagon

4. The perimeter of a rectangle is 60 cm. If the length of the rectangle is 20 cm, then find its width.

5. The lid of a rectangular box of sides 30 cm by 10 cm is stuck all around with the tape. Find the length of the tape used.

6. Determine the cost of fencing a square park with side 230 m at the rate of ₹15 per metre.

7. Determine the perimeter of the top of a table if the dimensions of the table top are 3 m 75 cm by 1 m 50 cm.

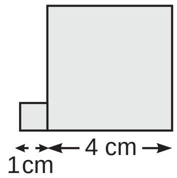
8. Kuldeep buys 16 square paving slabs, each of side of $\frac{1}{2}$ m. He lays them in the form of a square. Find the perimeter of his arrangement.

9. If Ritu takes 4 rounds of a square park of side 50 m, then find the total distance travelled by her.

10. The perimeter of a triangle is 56 cm. If two of its sides are 7 cm and 24 cm, then find the third side.

11. A rectangular field is 110 m long and 80 m wide. Find the length of wire needed to fence the field. Also, find the cost of the fence, if fencing costs ₹8.50 per metre.

12. A square of side 1 cm is joined to a square of side 4 cm. Find the perimeter of the new figure so formed.



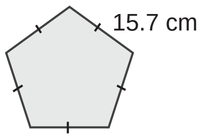
13. The cost of fencing a square field at the rate of ₹25 per metre is ₹1800. Determine the length of each side of the square field.

14. Gunjan runs around a square park of side 60 m. Radha runs around a rectangular park with length 50 m and breadth 40 m. Who covers less distance and how much?

15. The length and breadth of a rectangular field are in the ratio of 4 : 3. If its perimeter is 420 m, find its dimensions. [**Hint:** Assume the length and breadth of the rectangle as $4x$ and $3x$ respectively.]

16. The perimeter of a regular heptagon is 42 cm. Find the length of its side.

17. Rohan wants to buy a piece of cord to go once around his kite, as shown in the adjacent figure. Find the length of the cord he should buy.



18. If Akriti runs 600 m along the rectangular field whose dimensions are 35 m × 15 m, then find the number of times she goes along it.

19. If a wire in the shape of an equilateral triangle of side 12 cm is rebent to form a regular hexagon, then find its side.

AREA

In the previous section, recall region which included the curve and its interior.

The amount or measure of region enclosed by a closed figure is called its *area*.

Area of Figures Using Graph Paper

We can estimate the area of any simple closed figure by using a sheet of squared paper or graph paper where every square measures 1 unit × 1 unit or 1 sq unit. This square is known as a '**unit square**'.

For this, we trace the figure on to a transparent paper and place the same on a sheet of squared paper.

Follow these conventions to estimate area:

- The area of one full square is taken as 1 sq unit.
- Ignore portion of the area that is less than half a square.
- The area of more than half a square is counted as 1 sq unit.
- The area of exactly half the square is counted as $\frac{1}{2}$ sq unit.

Illustration: Let us estimate the area of the shaded portion in (Fig. 11.18) by counting squares.

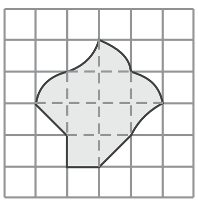
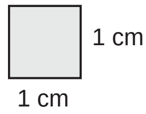


Fig. 11.18

	Cover	Number	Area Estimate (sq units)
(i)	Fully filled squares	5	5
(ii)	Half filled squares	2	$2 \times \frac{1}{2} = 1$
(iii)	More than half-filled squares	3	3
(iv)	Less than half-filled squares	2	0

∴ Area = (5 + 1 + 3 + 0) sq units = 9 sq units.

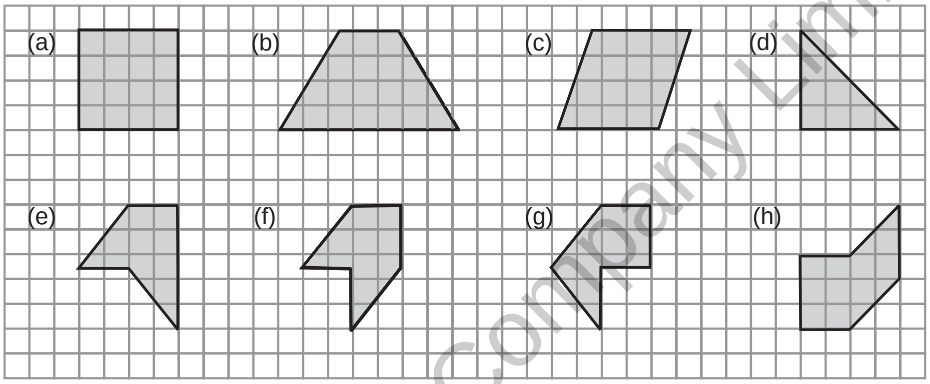
Some Standard Units of Area



The area of a square whose side is of length 1 cm is called a square centimetre and is written as 1 square centimetre, in short, 1 sq cm or 1 cm^2. Similarly, the area of a square whose side is of length 1 m is called a square metre and is written as 1 square metre, in short, 1 sq m or 1 m^2 and so on.

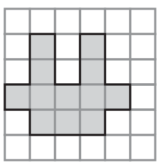
Exercise 11.2

1. **Find the area of the following figures. (Given 1 box = 1 sq cm)**

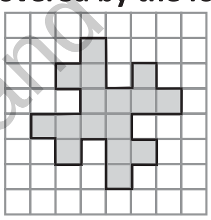


2. **Find the perimeter and total area covered by the following figures. (Given 1 box = 1 sq cm)**

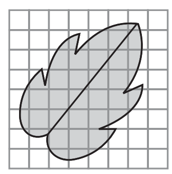
(a)



(b)



3. Find the area of a given figure on the square grid. Draw some irregular shapes on a square cm grid or a graph paper and find their areas (in sq cm).



Area of a Rectangle and a Square

Area of a rectangle

Consider the rectangular region which is of dimensions 8 cm by 4 cm. Divide the region into unit squares as shown in the Fig. 11.19.

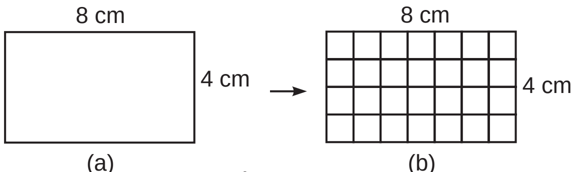


Fig. 11.19

Observe that there are 32 unit squares.

As each of these unit squares is of side 1 cm, so area of 1 unit square = 1 square centimetre. Therefore, area of the rectangle = 32 × 1 square centimetres

= 32 square centimetres

= 8 cm × 4 cm

= length × breadth

Try to find the area of the rectangles given in Fig. 11.20, by dividing into unit squares.

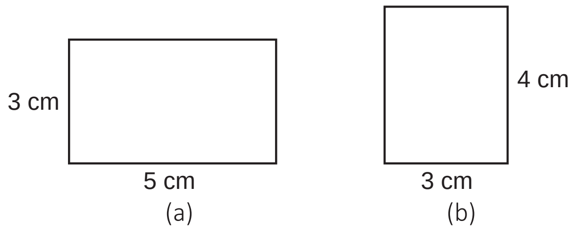


Fig. 11.20

In general, we can say

Area of a rectangle = length × breadth

Illustration 1: Let us find the area of a rectangle whose length is 8 cm and breadth is 5 cm.

Area of the rectangle = length × breadth

= 8 cm × 5 cm = 40 sq cm

Illustration 2: The area of a rectangle is 36 cm^2 and its length is 9 cm. To find the breadth of the rectangle, we do as follows:

Area of the rectangle = 36 cm^2

Length = 9 cm

Since the area of a rectangle = length × breadth

So, breadth = $\frac{\text{Area}}{\text{Length}} = \frac{36}{9}$ cm = 4 cm

Thus, breadth of the rectangle is 4 cm.

Area of a square

Consider the square region which is of dimensions 5 cm by 5 cm. Divide the region into unit squares as shown in the Fig. 11.21.

Observe that there are 25 unit squares.

Further, area of each of these unit squares is 1 square centimetre.

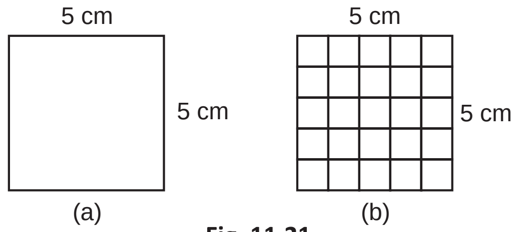


Fig. 11.21

So, area of the square

= 25 × 1 square centimetres

= 25 square centimetres

= 5 cm × 5 cm

= side × side

So, we can say:

Area of a square = side × side

Illustration 3: Let us find the area of a square whose side is 6 cm.

Area of the square = side × side

= 6 cm × 6 cm = 36 sq cm.

Illustration 4: The perimeter of a square field is 152 m. To find the area of the field, we follow these steps:

Perimeter of the square field, P = 152 m

So, side of the field = $\frac{P}{4}$ m

(Since perimeter (P) = 4 × side)

= $\frac{152}{4}$ m = 38 m

Thus, the area of the field = side × side

= (38 × 38) m^2 = 1444 m^2

Let Us Do

21st CS

Objective: (a) To draw figures having the same area but different perimeters

(b) To draw figures having the same perimeter but different areas

Materials required: Paper, pen, pencil, graph paper, etc.

Procedure: Part (a)

Step 1: Students will be encouraged to think of possible dimensions of the rectangle whose area is given, say 36 cm^2. All the possibilities will be discussed and drawn.

Step 2: They will be given graph paper and told to draw all the rectangles having area 36 cm^2.

Step 3: Next, they will compute the perimeter of each of these rectangles. Out of these which one is having least perimeter?

Example: Few samples are shown in Fig. 11.22 (a).

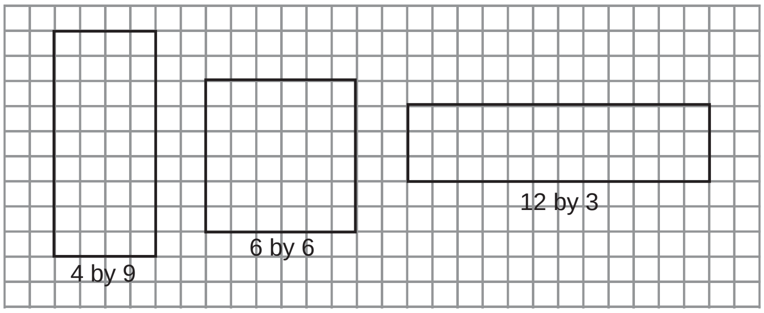


Fig. 11.22 (a)

Step 4: Further, they will be asked to draw any shape other than a rectangle and square having the same area.

Draw any 7 shapes and compute their perimeters (1 box = 1 sq cm). Colour the one with the least perimeter.

Example: Two of the samples are shown below in Fig. 11.22 (b).

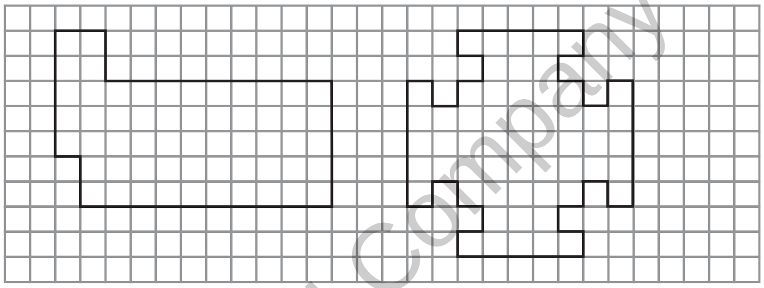


Fig. 11.22 (b)

Conclusion: ____

____

Procedure: Part (b)

Step 1: Draw rectangles and squares with given perimeter, say 24 cm. Also, compute the area of each one of them (1 box = 1 sq cm).

Step 2: Students will compute the area of the figures and colour the one with the greatest area.

Example: Few samples are shown in Fig. 11.22 (c).

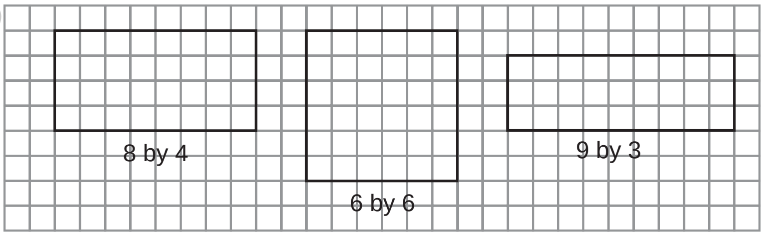


Fig. 11.22 (c)

Conclusion: ____

____

Applications of Area in Real life

Ex. 14. A room is 4 m 20 cm long and 3 m 65 cm wide. How many square metres of carpet is needed to cover the floor of the room?

Sol. Length of the room = 4 m 20 cm

= 4 m + 0.20 m = 4.2 m (∵ 1 m = 100 cm)

Breadth of the room = 3 m 65 cm

= 3 m + 0.65 m = 3.65 m

Carpet needed to cover the floor of the room = Area of the room

= length × breadth (Formula)

= 4.2 m × 3.65 m = 15.33 m^2

Thus, carpet needed to cover the floor of the room is 15.33 m^2.

Note

Here, length and breadth have been converted into the same unit, namely 'metres'.

Ex. 15. A floor is 5 m long and 4 m wide. A square carpet of side 3 m is laid on the floor. Find the area of the floor that is not carpeted.

Sol. Area of the floor = length × breadth (Formula)

= 5 m × 4 m = 20 m^2

Area of the square carpet = (side) × (side)

= 3 m × 3 m = 9 m^2

Area of the floor, that is, not carpeted

= Area of the floor – Area of the square carpet

= 20 m^2 – 9 m^2 = 11 m^2

Ex. 16. Four square flower beds each of side 4 m are dug in four corners on a piece of land 12 m long and 10 m wide. Find the area of the remaining part of the land.

Sol. Length of the land (l) = 12 m

Breadth of the land (b) = 10 m

Area of the whole land = $l \times b$

= 12 m × 10 m = 120 m^2

Side of a square flower bed = 4 m

Area of one square flower bed

= side × side = 4 m × 4 m = 16 m^2

Area of four square flower beds

= 4 × 16 m^2 = 64 m^2

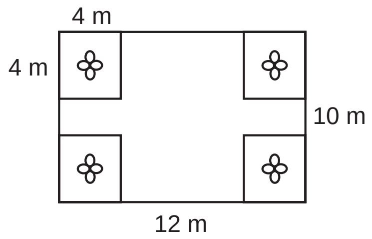


Fig. 11.23

Area of remaining part of the land

= Area of the whole land – Area of the 4 square flower beds

= 120 m^2 – 64 m^2 = 56 m^2

Thus, the area of the remaining part of the land is 56 m^2.

Ex. 17. How many tiles with dimensions 12 cm and 5 cm will be needed to cover a region whose length and breadth are 144 cm and 100 cm respectively?

Sol. Length of the region (l) = 144 cm

Breadth of the region (b) = 100 cm

Therefore, area of the region = $l \times b$

= 144 × 100 sq cm = 14,400 sq cm

Length of one tile = 12 cm

Breadth of one tile = 5 cm

Therefore, area of one tile = 12 × 5 sq cm

= 60 sq cm

Therefore, number of tiles required

$$= \frac{\text{Area of the region}}{\text{Area of one title}} = \frac{14{,}400}{60} = 240 \text{ tiles}$$

Ex. 18. What is the cost of tiling a rectangular piece of land, 500 m long and 300 m wide, at the rate of ₹10 per hundred sq metres?

Sol. Area of a rectangle = length × breadth

So, area of the rectangular piece of land

= 500 m × 300 m = 1,50,000 sq m

Cost of tiling 100 sq m = ₹10

So, cost of tiling 1 sq m = ₹ $\frac{10}{100}$

Thus, the cost of tiling 1,50,000 sq m

$$= ₹\,\frac{10}{100} \times 1{,}50{,}000 = ₹15{,}000$$

Area by Dividing Figures into Squares and Rectangles

Ex. 19. Find the area of the following figures.

(a)

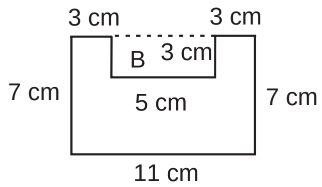


Fig. 11.24

(b)

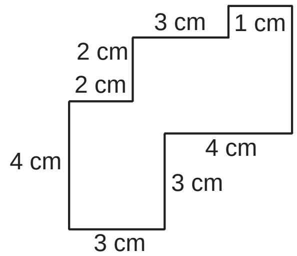


Fig. 11.25

Sol. (a) The given region is composed of one rectangular and two square regions as shown.

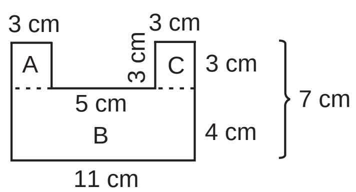


Fig. 11.26

The total area of the given figure= Area of a square A + Area of a rectangle B + Area of a square C

= (3 cm) × (3 cm) + (11 cm) × (4 cm) + (3 cm) × (3 cm)

= 9 cm^2 + 44 cm^2 + 9 cm^2

= 62 cm^2.

Alternate Solution:

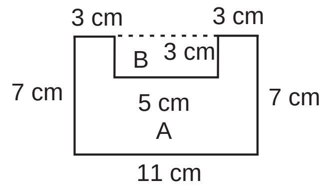


Fig. 11.27

The total area of the given figure

= (11 cm × 7 cm) – (5 cm × 3 cm)

= (77 – 15) cm^2 = 62 cm^2.

(b) We split the given figure into 2 rectangles and 2 squares [see Fig. 11.28].

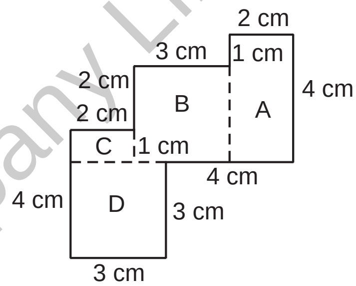


Fig. 11.28

Therefore, area of the given figure

= Area of rectangle A + Area of square B + Area of rectangle C + Area of square D

= (4 × 2) sq cm + (3 × 3) sq cm + (2 × 1) sq cm + (3 × 3) sq cm

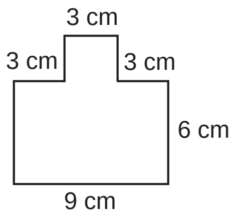
= 8 sq cm + 9 sq cm + 2 sq cm + 9 sq cm

= 28 sq cm.

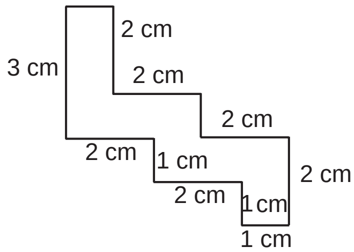
Exercise 11.3

1. **Find the missing value in each of the following.**
   (a) In a rectangle, length = 15 cm, breadth = 8 cm, area = ?
   (b) In a rectangle, perimeter = 5 m, breadth = 90 cm, length = ?, area = ?
   (c) In a square, side = 8 cm, perimeter = ?, area = ?
   (d) In a square, area = 81 cm^2, side = ?, perimeter = ?
2. If the area of a rectangle is 96 cm^2 and one of its side is 8 cm, then find the perimeter of the rectangle.
3. **Find the area of the figures given below.**

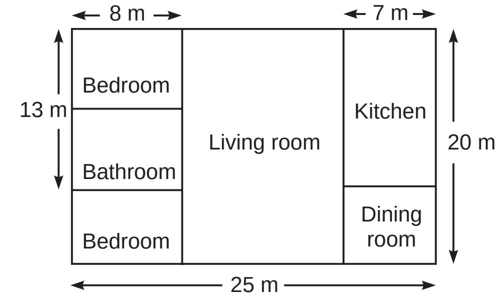
(a)



(b)



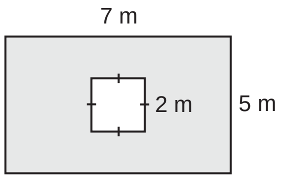
4. The diagram shows the floor plan of a house. Find the area of the kitchen and the living room.



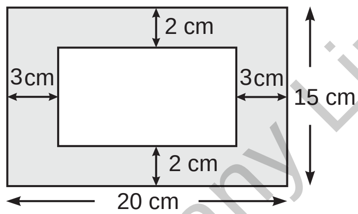
5. The total cost of flooring a room at ₹75 per m^2 is ₹6000. If the length of the room is 10 metres, find its breadth.

6. **Find the area of the shaded portions of the given figures.**

(a)



(b)



7. The area of a rectangle is 750 sq m and its breadth is 25 m. Find the perimeter of the rectangle.

8. The dimensions of a room are 24 m × 14 m. If a border with tiles of 1 m^2 are laid on all along its sides, then find the number of such tiles needed.

9. A marble tile measures 30 cm × 25 cm. Find the number of such tiles required to cover a wall of size 4 m × 3 m.

10. The floor of a bathroom is 5 m long and 3 m 50 cm wide. It is to be covered completely by square tiles of side 25 cm. Find the cost of the tiles at the rate of ₹60 per tile.

11. If the area of a square of side 20 cm is equal to the area of a rectangle of length 25 cm, find the breadth of the rectangle.

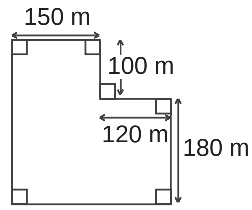
12. The length and breadth of a rectangular sheet of paper is 30 cm and 20 cm respectively. A square sheet of side equal to one-fourth of the breadth of the rectangular sheet is removed from each corner of the sheet. Find the area of the remaining portion of the sheet.

Competency Based Exercise

21st CS

1. **Tick (✓) the correct answer.**

(a) In the given figure, if the total cost of fencing the flower garden is ₹55,000, then the cost of fencing per metre is:

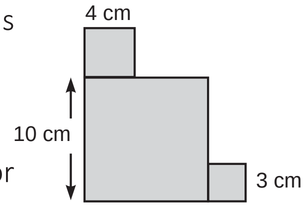


(i) ₹60
(ii) ₹55
(iii) ₹50
(iv) ₹255

(b) If three squares of sides 3 cm, 4 cm and 10 cm are joined together as shown in the given figure, then the area of the shaded region is:

(i) 100 cm^2 (ii) 125 cm^2

(iii) 169 cm^2 (iv) 196 cm^2



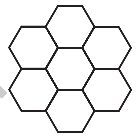
(c) The number of square tiles, each of width 90 cm, needed to cover a floor of area 8100 sq m is:

(i) 10,000 (ii) 9000 (iii) 900 (iv) 90

(d) If the ratio between the length and perimeter of a rectangular grassy plot is 3 : 8, then the ratio between its length and breadth is:

(i) 2 : 1 (ii) 3 : 1 (iii) 4 : 1 (iv) 8 : 3

2. The given figure is a part of a beehive consisting of regular hexagons. If the perimeter of this beehive is 72 cm, then what is the length of each of its sides?



3. A living room of Ashita's house is 13 m long and 9 m broad. If she wishes to carpet the floor with a carpet 4.5 m wide at the rate ₹950 per metre, then find the total cost of carpeting the floor.

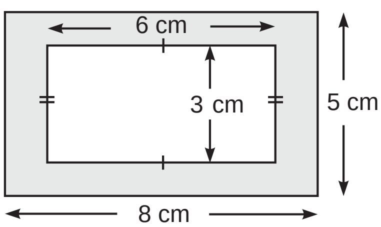
4. In a garden, 40 m × 15 m, two square shaped flower beds of side 5 m each are designed. Find:

(a) area covered by the flower beds. (b) remaining area of the garden.

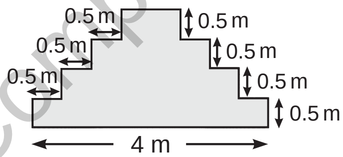
5. Dinesh wants to cover a floor 7 m wide and 4 m long by squared tiles. If each square tile is of side 0.25 m, then how many tiles are required to cover the floor of the room?

6. In the given figures, find the area of the shaded region.

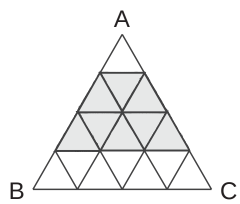
(a)



(b)



7. In the given figure, ΔABC is an equilateral triangle having each side of length 12 cm. If each side is divided into 4 equal parts and 16 equilateral triangles are formed, then find the perimeter of the shaded region.



8. The perimeter of a square and a rectangle is the same. If side of the square is 15 cm and one side of the rectangle is 18 cm, then find the area of the rectangle.

9. A wire is cut into several small pieces of equal length. Each of the small pieces is bent into a square of side 2 cm. If the total area of the small squares is 28 cm^2, then find the original length of the wire.

Let's Work in Mind

21st CS

1. Which is measured in cm^2 — capacity, weight, area or height?

2. What is the area of a rectangular plot of length 30 m and breadth 15 m?

3. The area of a rectangle is 120 cm^2 and its length is 12 cm. What is its breadth?

4. The length of a rectangular field is 2 m more than its breadth. If the perimeter of the field is 28 m, then what is the length of the field?

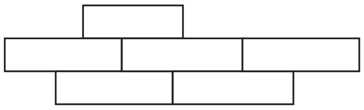
5. For what value of side, the perimeter and area of a square are numerically equal?

6. If side of a square is trebled, does its perimeter become nine times?

Challenge!

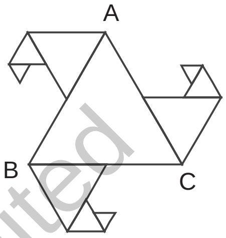
21st CS

1. The given figure has 6 rectangles of the same dimensions. If a single rectangle has a perimeter of 222 cm, what is the perimeter of the shape?



2. The area of each square on a chessboard is 4 cm^2. If during the game, 2 rooks and 2 bishops of one player and 4 pawns, and 1 bishop of the other player have been captured, then find the area of the squares left unoccupied.

3. In the given figure, all triangles are equilateral and AB = 8 cm. If other triangles are formed by taking the mid-points of the sides, then find the perimeter of the figure.



ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A) :** Perimeter of an equilateral triangle of length 7 cm is 21 cm.
   Reason (R) : Perimeter of a triangle is sum of the lengths of all its sides.
2. **Assertion (A) :** Perimeter of a regular polygon with 10 sides is $10 \times l$, if l is the length of the one side.
   Reason (R) : A regular polygon has the same length of all sides and perimeter is the sum of all lengths.
3. **Assertion (A) :** Area of a square of length 4 cm is 16 cm^2.
   Reason (R) : Perimeter of a square of side 4 cm is 16 cm.
4. **Assertion (A) :** Area of rectangle of sides 4 cm and 5 cm is 20 cm^2.
   Reason (R) : Area of rectangle with length l and breadth b is $2l + 2b$.
5. **Assertion (A) :** Area of rectangle with length 8 cm and breadth 2 cm is 16 cm.
   Reason (R) : Area of rectangle with length l cm and breadth b cm is lb cm^2.
6. **Assertion (A) :** Area can be measured in m^2.
   Reason (R) : Area can be measured in m^3.

12 Algebra

What Learners Will Achieve

- frame a rule for the given patterns using variables.
- identify expression and equation.
- express statements as algebraic expression.
- find the value of an expression.
- solve given equation in single variable.

DID YOU KNOW?

Algebra is a branch of Mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations.

The word 'algebra' is a Latin variant of Arabic word 'al-Jabr' which was taken from the book "Hisab al-jabrwal muqabala" written by a persian mathematician 'Muhammad ibn-musa al-khwarizmi'.

SOME BASIC TERMS

Variables and Constants

Let us consider the following situation.

Prachi is a student of class 6. On her birthday, she brings 4 packets, each containing 50 toffees, to distribute among classmates. When she distributes one packet in each of the four sections 6A, 6B, 6C, 6D, she is left with 8, 5, 4 and 7 toffees respectively. Think! How many toffees are distributed in these classes? We can express the above situation like this.

In section 6A, 8 + $\square$ = 50, which gives $\square$ = 42;

In section 6B, 5 + $\square$ = 50, which gives $\square$ = 45;

In section 6C, 4 + $\square$ = 50, which gives $\square$ = 46;

In section 6D, 7 + $\square$ = 50, which gives $\square$ = 43

Here, $\square$s are unknowns and obtain different values for different expressions. That means value of $\square$ varies from place to place. Therefore, we use different symbols, called **variables** to represent the unknowns.

Hence, a **variable** is a symbol which is used to represent an unknown number. It is denoted by a letter such as *x*, *y*, *z*, *a*, *b* and so on. It does not have a fixed value.

Remember

When letters are used to represent unknown numbers they are called literals, which are generally reffered to as variables.

A symbol having a fixed value is called a ***constant***. In the above case, 50, 4, 5, 7, 8 are constants. Also, 1, 2, 3, ..., – 10 etc., are examples of constants.

Think

Number of angles of a triangle and the number of sides of a quadrilateral are not variables because they are fixed numbers.

But number of angles in a polygon is a variable and can be represented as '*n*'. Why? Because number of angles in a polygon will depend upon the number of sides in the polygon.

Matchstick Patterns

Let us understand the meaning of a variable with the help of a matchstick pattern.

Take two matchsticks and arrange them to form the letter V. Again take two more matchsticks and arrange them in the shape of V and so on.

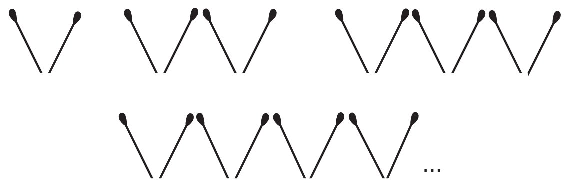


Fig. 12.1

By observing the pattern, we see that for one V, we need two matchsticks, for two Vs, we need four matchsticks, for three Vs, we need 6 matchsticks and so on.

Therefore, number of matchsticks required to form n Vs = 2 × Number of Vs = $2 \times n = 2n$.

Here, n is called a *variable*. It denotes the number of Vs and can take any of the values 1, 2, 3, 4, … .

More Matchstick Patterns

Take three matchsticks and arrange them to form the letter C. Repeat the process many times.

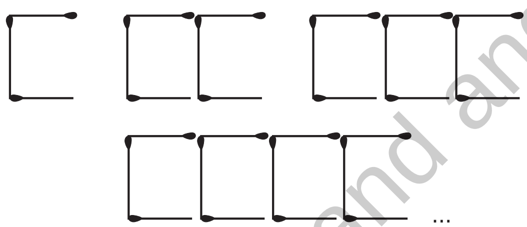


Fig. 12.2

Observe the pattern and make a table.

Number of Cs	1	2	3	4	...
Number of Matchsticks	3	6	9	12	...

Therefore, number of matchsticks required to form n Cs = 3 × Number of Cs = $3 \times n = 3n$.

To make 20 Cs, we need $3 \times 20 = 60$ matchsticks.

We see that the use of variables makes the work much easier.

Algebra of Literals

Since the literals (letters) are used to represent numbers, they must satisfy all the rules (including those for signs) regarding the operations of addition, subtraction, multiplication and division of numbers. For example, we have the following notations and rules.

(i) The sum of two literal numbers x and y is denoted by $x + y$.

(ii) If the literal number y is subtracted from the literal number x, we denote the difference by $x - y$.

(iii) The product of two literals x and y is denoted as $x \times y$. It is a convention to write $x \times y$ as xy, $2 \times y$ as $2y$, $x \times y \times z$ as xyz, $8 \times x \times z$ as $8xz$ and so on. Thus, $3x$ shall mean $3 \times x$.

(iv) The product of a literal number x with itself is written as $x \times x = x^2$, $x \times x \times x = x^3$, etc.

(v) If the number x is divided by $y (y \neq 0)$, it is written as $\frac{x}{y}$.

Let us study some more examples.

Ex. 1 **Find the rule, (using a variable) which gives the number of matchsticks required to make a matchstick pattern for the letter Z as**

Fig. 12.3

Sol. To make one Z, the number of matchsticks needed = 3 = 3 × 1

To make two Zs, the number of matchsticks needed = 6 = 3 × 2

To make three Zs, the number of matchsticks needed = 9 = 3 × 3 and so on.

So, we have

Number of Zs	Number of Matchsticks
1	3 (3 × 1)
2	6 (3 × 2)
3	9 (3 × 3)
4	12 (3 × 4)
5	15 (3 × 5)
6	18 (3 × 6)
...	...

Table 12.1

Therefore, number of matchsticks needed to form n Zs = $3 \times n = 3n$ (where n is a variable taking values 1, 2, 3, ...).

Ex. 2 **Observe the given matchstick pattern and find the rule that gives the number of matchsticks needed to form "x" number of squares.**

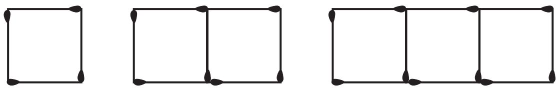


Fig. 12.4

> **Note**
>
> x is a variable here.

Sol. On observing the given matchstick pattern, we get

Number of Sqaures	Number of Matchsticks
1	4
2	7
3	10
4	13
...	...
x	$3x + 1$

Table 12.2

Therefore, number of matchsticks required to form x squares = $3x + 1$.

Ex. 3 **Cadets are marching in a parade. There are 5 cadets in a row. What is the rule, which gives the number of cadets, given the number of rows? (Use *n* for the number of rows.)**

Sol. In one row, the number of cadets = 5

Now, in two rows, the number of cadets = 10 = 5 × 2

Similarly, in three rows, the number of cadets = 15 = 5 × 3

Going by this pattern, if there are n number of rows, then the number of cadets = $5 \times n = 5n$.

Ex. 4 **A bird flies 200 m in one minute. Can you express the distance covered by the bird in terms of its flying time in minutes? (use *t* for flying time in minutes)**

Sol. Distance covered by a bird in 1 minute = (200 × 1) m = 200 m

Distance covered by the bird in 2 minutes = (200 × 2) m = 400 m

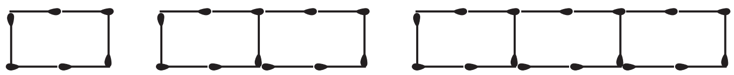
Distance covered by the bird in 3 minutes = (200 × 3) m = 600 m

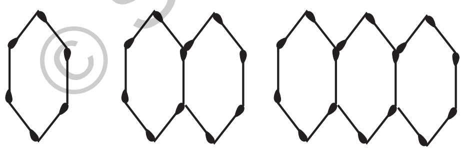
Therefore, distance covered by the bird in t minutes = $(200 \times t)$ m = $200t$ metres.

Exercise 12.1

1. **Observe the given matchstick pattern and find the rule that gives the number of matchsticks in terms of "*x*" number of polygons.**

(a) 

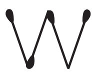
(b) 

(c) 

2. **Find the rule using a variable which gives the number of matchsticks required to make a matchstick pattern for the letters given below.**

(a) 

(b) 

(c) 

(d) 

3. If a bird flies a distance of t km in 1 minute, then find the distance covered by the bird in 7 minutes (in km).
4. Cadets are marching in rows. If there are 5 cadets in a row, then find the total number of cadets in p rows.
5. Ramu has x number of *laddus*. He gave some *laddus* to guests and family members, still 7 *laddus* remain. How many *laddus* did he give away?
6. If there are 40 oranges in a box, then what is the total number of oranges in x boxes?

USE OF VARIABLES IN COMMON RULES

In this section, we shall learn the use of variables in expressing relationship between the two quantities such as side of a square and its perimeter, etc.

Rules from Geometry

Perimeter of a rectangle

Consider a rectangle ABCD. We know that the opposite sides of a rectangle are ***equal in length***.

Let us denote length of the rectangle by l (*i.e.*, AB = CD = l) and breadth by b (*i.e.*, AD = BC = b).

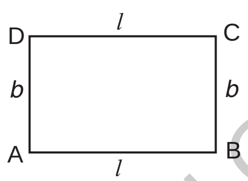


Fig. 12.5

Let us now find a rule which expresses the relationship between the ***perimeter*** and the ***sides of a rectangle***.

Perimeter of a rectangle = Sum of the lengths of the sides of the rectangle

$= l + b + l + b = 2l + 2b = 2(l + b)$

We represent perimeter of the rectangle with a variable, say P.

Therefore, we get the following rule which expresses the relationship between the perimeter and the sides of a rectangle.

$$\mathbf{P = 2(l + b)}$$

Here, l and b can take any values and P changes accordingly.

Perimeter of a square

Consider a square ABCD.

We know that all sides of a square are ***equal in length***.

Let us denote sides of a square by l (*i.e.*, AB = BC = CD = DA = l).

Now, find a rule which expresses the relationship between the ***perimeter*** and the ***side of a square***.

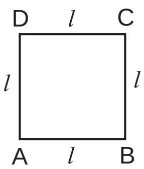


Fig. 12.6

Perimeter of a square = Sum of the lengths of the sides of the square

$= l + l + l + l = 4l$

We represent perimeter of the square with a variable, say P.

Therefore, we get the following rule which expresses the relationship between the perimeter and the side of a square.

$$\mathbf{P = 4l}$$

Here, l can take any value and P changes accordingly.

Skill Check

- If the side of a square is p units, then its area (in square units) is given by ________.
- Perimeter of a regular octagon of side 's' is ________.

Rules from Arithmetic

Commutativity of addition of two numbers

We know that while adding two numbers, order of numbers does not matter.

For example, 2 + 3 = 5 and 3 + 2 = 5

i.e., 2 + 3 = 3 + 2 = 5

This property is known as the **commutativity of addition of numbers**.

Use of variables helps us in expressing the above property in general form as follows:

If a and b are two numbers, then $\boldsymbol{a + b = b + a}$.

Commutativity of multiplication of two numbers

We know that for multiplication of two numbers, the order of the two numbers multiplied does not matter.

For example, 2 × 3 = 6 and 3 × 2 = 6

i.e., 2 × 3 = 3 × 2 = 6

This property is known as the **commutativity of multiplication of numbers**.

Again using the variables *a* and *b*, we can express the rule as follows:

$$a \times b = b \times a$$

Associativity of addition of numbers

Consider the sum of three numbers 10, 19 and 8.

We may first add 10 and 19 and then add 8 to the sum, to get the total sum of numbers or we may first add 19 and 8 and then add it to 10 to get the total sum.

i.e., (10 + 19) + 8 = 10 + (19 + 8) = 37

This can be done for any three numbers. This property is known as **associativity of addition of numbers**. Using the variables *a*, *b* and *c*, we can express this property as follows:

$$(a + b) + c = a + (b + c)$$

Associativity of multiplication of numbers

Consider the product of three numbers 4, 5 and 7. We can multiply them as follows:

(4 × 5) × 7 = 20 × 7 = 140

or 4 × (5 × 7) = 4 × 35 = 140

We observe that (4 × 5) × 7 = 4 × (5 × 7)

This property is known as the **associativity of multiplication of numbers.**

For any three variables *a*, *b* and *c*, it can be expressed as follows:

$$a \times (b \times c) = (a \times b) \times c$$

Distributivity of numbers

Suppose, we are asked to calculate 8 × 24. We can do it like this:

8 × 24 = 8 × (20 + 4)

= (8 × 20) + (8 × 4) = 160 + 32 = 192

Recall that 8 × (20 + 4) = (8 × 20) + (8 × 4)

This property is called the **distributivity of multiplication over addition of numbers** and can be expressed using the variables *a*, *b* and *c* as follows:

$$a \times (b + c) = (a \times b) + (a \times c)$$

Distributivity of multiplication over subtraction is also true.

For any three variables *a*, *b* and *c*, it can be given as:

$$a \times (b - c) = (a \times b) - (a \times c)$$

Let us study some more examples.

Ex. 5 The side of an equilateral triangle is denoted by *l*. Express the perimeter of the equilateral triangle using *l*.

Sol. Perimeter of an equilateral triangle = Sum of the lengths of the sides

= *l* + *l* + *l* = 3*l*

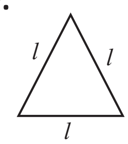


Fig. 12.7

Ex. 6 A cube is a three-dimensional figure, which has 6 faces and each face is a square (see Fig. 12.8). If length of an edge of the cube is *l*, find the formula for the total length of the edges of a cube.

Sol. Length of one edge = *l*.

A cube has 12 edges.

∴ Total length of 12 edges

= 12 × *l* = 12*l*

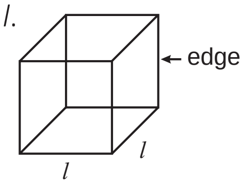
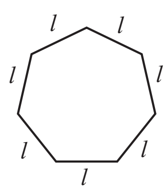


Fig. 12.8

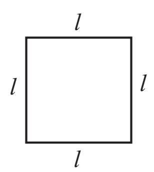
Exercise 12.2

1. **Express the perimeter of the following regular polygons using *l* as a side of the polygons.**

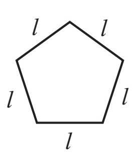
(a)



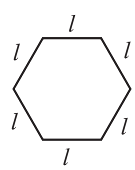
(b)



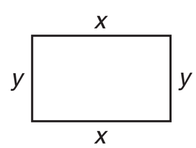
(c)



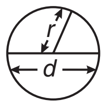
(d)



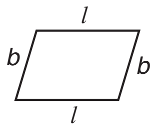
2. Express the perimeter of the rectangle using *x* and *y*.



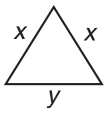
3. Express the diameter of the circle (d) in terms of its radius (r).



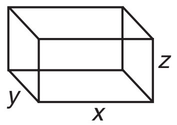
4. Express the perimeter of the parallelogram using l and b.



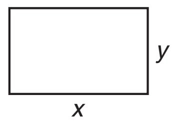
5. Express the perimeter of the isosceles triangle using x and y.



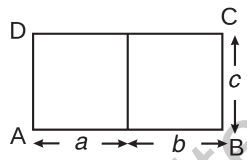
6. Write an expression for the sum of edges of the given cuboid.



7. Write an expression for area of a rectangle using x and y.



8. In the figure given, find the perimeter of the rectangular field ABCD in terms of a, b and c.



9. Check if the following are True (T) or False (F).

(a) $6 + 5 = 5 + 6$

(b) $x + y = y + x$

(c) $2 \times 3 \times 4 = 4 \times 3 \times 2$

(d) $a \times b \times c = c \times b \times a$

(e) $a(b - c) = (a \times b) - (a \times c)$

(f) $10 - 8 = 8 - 10$

UNDERSTANDING EXPRESSIONS

Number Expressions

Consider the expressions like $3 + 4 \times 5$, $3 \times 4 - 5$, $(4 + 5) \div 3$, $3 \times 4 \times 5$, etc., formed by using numbers 3, 4, 5 and the operations +, – , × and ÷.

Such expressions are called **number expressions**.

A number expression can be evaluated or simplified. For example,

$3 \times 4 \times 5 = 12 \times 5 = 60$, $(4 + 5) \div 3 = 9 \div 3 = 3$, etc.

Algebraic Expressions

Consider the expressions such as $2n$, $3b - 2a$, $3(x + 4)$, $\frac{d}{c} + 6$, $2l + 2b$. These expressions are formed by the combination of numbers, literals (variables) and arithmetical operations. These are called **algebraic expressions**. Some more examples of algebraic expressions are $y - x$, $x + y - z$, $3pq - 4qr$, $x + xy$.

Terms of an algebraic expression

Consider the algebraic expression $y - 5x$. Here, y and $-5x$ are called **terms**.

Similarly, in the expression $3pq - 4qr$, $3pq$ and $-4qr$ are terms, and in the expression $x + y - z$, x, y and $-z$ are terms.

Factors of terms

We know that when two or more numbers or literals are multiplied, the result is a product. Therefore, each number or literal is a factor of the product.

Let us consider the term $2x^2y$. As $2x^2y = 2 \times x \times x \times y$, so 2, x, x, y are the factors of $2x^2y$.

Similarly, factors of $-ab = -1$, a and b.

$$\text{Factors of } \frac{1}{2}xyz^2 = \frac{1}{2},\ x,\ y,\ z \text{ and } z.$$

Coefficient

Let us consider the term $5y$. In $5y$, 5 is constant and y is variable. Here, 5 is the coefficient of y.

A numerical factor is always considered as numerical coefficient of the term.

Illustration 1: In the term $6x^2yz$, the coefficient of x^2yz is 6, the coefficient of $6x^2y$ is z, the coefficient of $6yz$ is x^2, the coefficient of x^2z is $6y$ and so on.

Thus, we can say that the coefficient of any factor(s) of a term is the product of the remaining factor(s).

Like and Unlike Terms

Like terms

Two or more terms having the same literals and their exponents are called like terms.

For example, xy, $3xy$, $\frac{1}{7}xy$ are like terms.

Unlike terms

Two or more terms having different literals or their exponents are called unlike terms.

For example, x, y, z, ax^2, b^2x^2 are unlike terms.

Types of Algebraic Expressions

On the basis of number of terms (unlike), algebraic expressions are one of the following types:

Monomial

An expression having only one term is called a monomial. For example, $2x$, $\frac{1}{2}y$, $6x^2y$, 5, etc., are monomials.

Binomial

An expression having two terms is called a binomial. For example, $a + b$, $2x + 3y$, $x^2 - bc$, etc., are binomials.

Trinomial

An expression having three terms is called a trinomial. For example, $a + b + c$, $x^2 - y^2 + z^2$, $2x + 3y + z$, etc., are trinomials.

Polynomial

An expression having two or more terms is called a polynomial.
For example, $x^2 + y + z + 1$, $2x + 3y$, $6x - 1 + b$, etc., are polynomials.

Remember

Binomial, Trinomial are also known as Polynomials.

Value of an Algebraic Expression

As we now know that literals represent numbers. Thus, for an expression $x + 2y$, if we know the values of x and y, we can find the (numerical) value of this expression.

Remember

An algebraic expression cannot be evaluated or simplified without knowing the value of literals involved.

Illustration 2: If $x = 5$, $y = -2$, then the value of the algebraic expression

$x + 2y = 5 + 2 \times (-2) = 5 - 4 = 1.$

(Replacing the literals by numerical values is called substitution.)

Let us study some more examples.

Ex. 7 Write an algebraic expression which have the following terms:

(a) $a, 2b, c$ (b) $-5x, 6y^2, 7zx, 4z$

Sol. (a) There are three unlike terms. So, the expression is $a + 2b + c$.

(b) There are four unlike terms. So, the expression is $-5x + 6y^2 + 7zx + 4z$.

Ex. 8 Classify the following as monomials, binomials and trinomials.

(a) $x - 2y$ (b) $\frac{y}{3}$

(c) $x \times 2y \times 3z$ (d) $a + b - c$

Sol. (a) We know that terms are separated by the signs '+' and '–' only.

Therefore, $x - 2y$ is a binomial.

(b) $\frac{y}{3}$ is a monomial.

(c) $x \times 2y \times 3z$, *i.e.*, $6xyz$ is a monomial.

(d) $a + b - c$ is a trinomial.

Note

A constant term is also a monomial. For example, 5, 6 – 2, etc.

Ex. 9 Evaluate $2x + 3y + 5z - 6a$ for $a = 5$, $x = 3$, $y = 4$, $z = 4$.

Sol. $2x + 3y + 5z - 6a$

$= 2 \times (3) + 3 \times (4) + 5 \times (4) - 6 \times (5)$

$= 6 + 12 + 20 - 30 = 38 - 30 = 8$

Exercise 12.3

1. **Write the terms of the following expressions.**
   (a) $2a + 3b$ (b) $7x - 4x^2$ (c) $8 + 2x - y^2$ (d) $4a + 3b - c + \frac{1}{2}d$
2. **Write an algebraic expression which have the following terms.**
   (a) $x, 2y$ (b) $7a, 8b, -1$ (c) $x^2, y^2, 4z^3, 1$
3. **Write the factors of the following terms.**
   (a) $2xyz$ (b) $7a^2b$ (c) $\frac{1}{5}lmn^2$ (d) $\frac{6a^2bc}{13}$ (e) $125y^3$ (f) $-12a^3z^4x^6$
4. **Write the coefficient of '*x*' in each of the following.**
   (a) $12x$ (b) $3xy$ (c) $\frac{1}{2}yz^2x$ (d) $-x$
5. **Group the like terms in each of the following.**
   (a) $3a^2b, 2ab^2, -3ab, \frac{1}{2}ba^2$ (b) $2xy, 3yz, 7xz, -5zx, 4zy$
6. **Classify the following algebraic expressions as monomials, binomials and trinomials.**
   (a) $2 - 1$ (b) $6p$ (c) $l + m - n$ (d) $2q + 3t$ (e) $p \times q \times 2r$
   (f) $4x^2 + 7x + 9$ (g) $4a + 5a$ (h) $110 + 7 + 9z$ (i) $0.5x + 7.7x + 3x$
7. Evaluate $2a + b + 3c$ for $a = 2$, $b = 3$ and $c = 4$.
8. **Evaluate the following for $x = 1$, $y = 3$, $z = 10$ and $p = 8$.**
   (a) $x + y$ (b) $z - 3y$ (c) $y + z + 2p$ (d) $3p + 2x + y - z$

WRITING EXPRESSIONS

We can form an expression following some given instructions and using the operation(s) like '+', '–', '×' or '÷'.

Let us study the table given below.

Instruction	Expression
x added to 10	$10 + x$
y more than *x*	$x + y$
12 multiplied by *x*	$12 \times x$ or $12x$
3 times *y*	$3 \times y$ or $3y$
a subtracted from 8	$8 - a$
5 less than *x*	$x - 5$
exceeds 7 by *z*	$7 + z$
y divided by 6	$\frac{y}{6}$
n multiplied by 4 and then *y* added to the product	$4n + y$

Table 12.3

Watch Your Step!

$2x + 3x$ is not a binomial, it is a monomial as $2x + 3x = 5x$.

Similarly, $6y - y = 5y$; $2x \times 3y = 6xy$;

$$12x^2 \div 3x = \frac{\overset{4}{\cancel{12}} \times x \times \cancel{x}}{_1\cancel{3} \times \cancel{x}} = 4x$$

are also monomials.

Skill Check

Write an expression for each of the following.

(a) 7 more than *x* (b) 6 less than *x*
(c) exceeds 5 by *x* (d) 6 times *x* (e) *y* divided by 7

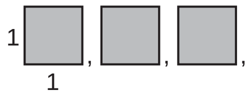
Remember

- Only like terms can be added or subtracted.
- Both unlike and like terms can be multiplied and divided.

Let Us Do

Objective: To demonstrate a given algebraic expression through combinations of algebraic tiles using the concept of area.

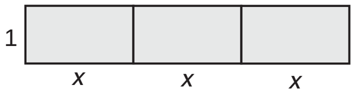
EL

Materials required: At least 20 tiles of 1 × 1 sq unit 

and

10 tiles of x × 1 sq unit 

Procedure:

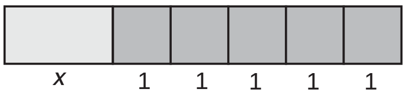
1. Show '3x' using algebraic tiles. 

Explanation: Here dimensions of a tile obtained by joining three tiles is as follows:

length = $x + x + x = 3x$ and breadth = 1

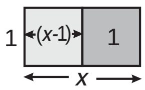
∴ Area of the tiles obtained (combined) = $(3x)(1) = 3x$ sq units.

2. Show $x + 5$.



Explanation: Here, length = $(x + 5)$ units; breadth = 1 unit

∴ Area of the tiles obtained combined = $(x + 5)$ sq units.

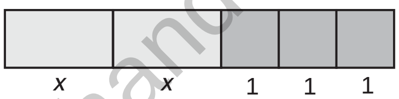
3. Show $x - 1$. 

Explanation: Tile of dimensions $1 \times x$ sq unit can be covered with tile of dimension 1 × 1 sq unit from one corner.

So, length of remaining portion = $(x - 1)$ units, breadth = 1 unit.

∴ Area = $(x - 1)\ 1 = (x - 1)$ sq units.

4. Show $2x + 3$.



> **Note**
>
> The teacher can also ask the students to create their own (linear) patterns combining different tiles and to write their algebraic expressions.

Explanation: Here, length = $x + x + 1 + 1 + 1 = (2x + 3)$ units, breadth = 1 unit.

∴ Area = $(2x + 3)(1) = (2x + 3)$ sq units.

Students shall repeat this activity with different algebraic expressions.

Ex. 10 Translate the phrases into an algebraic expression.

(a) Five times a number plus 4.

(b) Subtracting 5 from a number and then multiplying the result by 2.

(c) Adding 7 to 4 times the number of oranges.

Sol. (a) Let a number be x. Then five times a number plus 4 can be expressed as follows.

five	times	a number	plus	4
↓	↓	↓	↓	↓
5	×	x	+	4

or $5x + 4$.

(b) Let the number be x.

Subtracting 5 from the number

x – 5 or $x - 5$

then multiplying the result by 2.

(x – 5) × 2 or $2(x - 5)$

(c) Adding 7 to 4 times the number of oranges.

4 × x + 7

or $4x + 7$ oranges

Ex. 11 Write an algebraic expression for each of the following statements.

(a) 5 is multiplied by *y* and then 3 is subtracted.

(b) *y* is multiplied by (–5) and the result is added to 11.

Sol. (a) We have, 5 is multiplied by y and then 3 is subtracted.

5 × y – 3

Therefore, the algebraic expression will be $5y - 3$.

(b) We have, y is multiplied by (–5) and the result is added to 11.

–5 × y + 11

Therefore, the algebraic expression will be $-5y + 11$.

Using Expressions Practically

We have already learnt the art of writing expressions. We will now demonstrate how some practical situations can be translated into algebraic expressions.

Illustration:

	Situations (described in words)	The unknown or the variable	Statements using expressions
(a)	How old was Ritu 3 years ago?	Let x be Ritu's present age in years.	3 years ago, Ritu was $(x - 3)$ years old.
(b)	How old will Ritu be 3 years from now?	Let x be Ritu's present age in years.	3 years from now Ritu will be $(x + 3)$ years old.
(c)	The speed of a car is 20 km/h more than the speed of a truck.	Let the speed of the truck be s km/h.	The speed of the car is $(s + 20)$ km/h.
(d)	Sita has 6 marbles more than Radha.	Let m be the number of marbles Radha has.	Sita has $(m + 6)$ marbles.
(e)	Price of a mango is twice the price of an orange.	Let ₹p be the price of an orange.	Price of a mango is ₹$2p$.

Let us study some more examples.

Ex. 12. The length of a rectangular hall is 4 metres less than 3 times the breadth of the hall. What is the length, if the breadth is *b* metres?

Sol. Breadth of the rectangular hall = b

Three times the breadth of the hall

= $3 \times b = 3b$

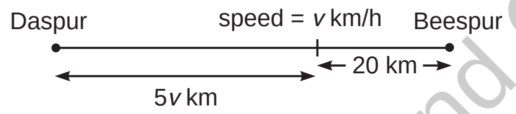
Since the length of the rectangular hall is 4 metres less than 3 times the breadth of the hall, therefore length of the hall = $(3b - 4)$ metres.

Ex. 13. A bus travels at a speed of *v* km per hour. It is going from Daspur to Beespur. After the bus has travelled for 5 hours, Beespur is still 20 km away. What is the distance from Daspur to Beespur? (Express in terms of *v*)

Sol. In one hour, the distance travelled by the bus = v km

In 5 hours, the distance travelled by the bus = $(5 \times v)$ km = $5v$ km

After the bus has travelled $5v$ km, Beespur is still 20 km away.



So, to find the distance from Daspur to Beespur, we have to add 20 km in $5v$ km.

Thus, the distance between Daspur and Beespur is $(5v + 20)$ km.

Ex. 14. Meena, Beena and Leena are climbing the steps to the hill top. Meena is at step S, Beena is 8 steps ahead and Leena is 7 steps behind Meena. Where are Beena and Leena?

Sol. Given, Meena is at step S. Beena is 8 steps ahead of Meena.

So, to find the position of Beena, we have to add 8 to S, hence we get S + 8.

Therefore, Beena is at step S + 8.

Also, Leena is 7 steps behind Meena.

So, to find the position of Leena, we have to subtract 7 from S, hence we get S – 7.

Thus, Leena is at step S – 7.

Ex. 15. Gayatri's present age is '*x*' years. Her grandfather's age is 5 times her present age and her grandmother is 2 years younger than her grandfather. What is her grandmother's age?

Sol. Given, Gayatri's present age = x years

Gayatri's grandfather's age = 5 times Gayatri's age = $(5 \times x)$ years = $5x$ years

Now, grandmother is 2 years younger than grandfather.

Therefore, grandmother's age = 2 years less than $5x$ years = $(5x - 2)$ years.

Ex. 16. A rectangular box has a height '*h*' cm. Its length is 5 times the height and breadth is 10 cm less than the length. Express the length and breadth of the box in terms of the height.

Sol. Given, height of the box = h cm

Length is 5 times the height.

Therefore, the length of the box

$= 5 \times h = 5h$ cm

Now, breadth of the box is 10 cm less than the length. Therefore, breadth of the box = $(5h - 10)$ cm.

Ex. 17. Change the following statements using expressions into statements in ordinary language.

(a) A notebook costs ₹P. A book costs ₹3P.

(b) Munnu's age is *x* years. His sister's age is (*x* + 3) years.

(c) Our class has *n* students. Our school has (20*n* – 13) students.

Sol. In ordinary language, we can express the given situations as follows:

(a) A book costs thrice the cost of a notebook.

(b) Munnu's sister is 3 years older than him.

(c) Our school has 13 students less than 20 times the students of our class.

Exercise 12.4

1. Translate the following phrases into an algebraic expression.

(a) 3 subtracted from a number.
(b) The product of 11 and a number.
(c) The quotient of a number and 4.
(d) The quotient of 4 and a number.

2. Translate the following into an algebraic expression.

(a) The difference between three times a number and twenty-two.
(b) Two times a number plus 5.
(c) The difference between three times a number and eleven.

3. Write an algebraic expression for each of the following statements.

(a) 3 times x to which 4 is added.
(b) 7 times x to which 3 is added.
(c) x is multiplied by 5 and the result is subtracted from 13.
(d) x is divided by 7 and the result is subtracted from 12.

4. Write an expression for the following statements.

(a) Adding 4 to a number and then dividing the result by 3.
(b) Subtracting 6 from a number and then multiplying the result by 4.
(c) Subtracting 3 from 2 times the number of oranges.
(d) Adding 5 to 3 times the number of oranges.

5. Match the following.

Set I

	Column A		*Column B*
(a)	20 more than x	(i)	$\frac{3x}{2}$
(b)	x decreased by 20	(ii)	$3x$
(c)	Half of $3x$	(iii)	$x + 20$
(d)	Thrice a number x	(iv)	$x - 20$

Set II

	Column A		*Column B*
(a)	10 less than x	(i)	$x + 10$
(b)	x increased by 10	(ii)	$x - 10$
(c)	Four times a number x	(iii)	$\frac{2x}{2} = x$
(d)	Half of $2x$	(iv)	$4x$

6. If the breadth of a rectangular hall is '*b*' metres, then how much is its length, when:

(a) length is 8 metres less than 4 times its breadth.
(b) length is 6 metres less than 5 times its breadth.

7. A bus is going from Gurdaspur to Bilaspur at the rate of v km per hour. How much is the distance between Gurdaspur and Bilaspur, if the bus has travelled:

(a) 4 hours and Bilaspur is 10 km away?
(b) 3 hours and Bilaspur is 15 km away?

8. Mini, Bini and Tini are climbing the steps to the hill top. Mini is at step s. Write in the terms of s where Bini and Tini are, if:

(a) Bini is 6 steps ahead and Tini is 3 steps behind.
(b) Bini is 5 steps ahead and Tini is 6 steps behind.

9. Payal's present age is 'x' years. What is her grandmother's age, if her grandfather's age is 4 times her present age and her grandmother is 3 years younger to her grandfather.

10. Express the length and breadth of a box in terms of the height, if its length is 7 times the height and breadth is 5 cm less than the length.

11. Change the following statements using expressions into statements in ordinary language.

(a) Shikhar scores r runs. Rohit scores $(r + 15)$ runs.

(b) Rony drops x coins in a savings bank. He has $6x$ coins in his wallet.

(c) Jenni is y years old. Her father is $3y$ years old and her mother is $(3y – 2)$ years old.

(d) There are z boys in our class. There are $(2z + 7)$ girls in our class.

12. Tick (✓) the correct answer.

(a) "The quotient of a and b added to the product of a and b" is written as:

(i) $a + ab$ (ii) $\frac{a}{ab}$ (iii) $\frac{a}{b}+ab$ (iv) $\frac{b}{a}-ab$

(b) Meena had x marbles and was given 12 more. Then she lost 5 marbles from her total. Which expression represents the amount she was left with?

(i) $(x + 12) – 5$ (ii) $(x – 12) + 5$ (iii) $(x \times 12) + 5$ (iv) $(x \div 12) – 5$

(c) Multiply x by 2 and add 15 to the result. Which mathematical expression describes this action?

(i) $2(x + 15)$ (ii) $x^2 + 15$ (iii) $\frac{2x}{2} + 15$ (iv) $2x + 15$

(d) If there are N boys and 25 girls in a class, then the strength of the class is:

(i) N – 25 (ii) N + 25 (iii) N × 25 (iv) N ÷ 25

(e) If Ishita's present age is y years, then her age 10 years ago was:

(i) $(y – 10)$ years (ii) $(y + 10)$ years (iii) $10y$ years (iv) $(10 \div y)$ years

UNDERSTANDING EQUATIONS

Earlier, we have learnt about algebraic expressions and have seen how they are formed.

An equation is a statement that expresses equality between two ***algebraic expressions***.

Recall the matchstick pattern (learnt earlier in this chapter) where we found that the number of matchsticks required to form n number of C's is $3n$. Now, the question is "If we have 24 matchsticks, how many C's can we form? Note that $3n$ is the total number of matchsticks for forming n C's. We are given that the total number of matchsticks = 24. So, $3n$ *is equal to* 24 or $3n = 24$.

This is an example of an ***equation***.

Thus, we can say:

An equation is a statement of equality which involves literal number(s).

Statement

$3n$ is equal to 24

$3n = 24$

n → Literal number (or variable)

= → Sign of equality

Literal number 'n' is also called '*unknown*'.

Some more examples of equations are as follows:

$x + 10 = 20$, $10y = 15$, $n - 16 = 4$, $\frac{y}{6} = 5$, $x + y = 2 - y$

Note

An equation has a sign of equality (=) between its two sides.

- The part to the left of the equality sign (=) is called **left hand side (LHS)** and the part to the right of equality sign (=) is called **right hand side (RHS)**.

For an equation, LHS = RHS.

- If the LHS is not equal to the RHS, we do not get an equation.

Illustration 1: Statements like $2n > 10$ or $x < 5$ are not equations.

Difference between an Expression and an Equation

Recall that an equation is a statement that expresses equality between two algebraic expressions.

$3x + 5 = 2$; $x^2 - x = 2x + 1$; $\frac{t}{5} = (7 - t)$, etc.,

are examples of equations.

The equality symbol (=) must be present if the statement is to qualify as an equation.

Now, let us consider the following phrases/ statements.

(a) The sum of four times a number and 7 is 91.

Since 'is' means '=', this will translate into an equation' as shown below:

four	times	a number	plus	seven	equals	91
↓	↓	↓	↓	↓	↓	↓
4	×	x	+	7	=	91

Thus, the given word statement translates to the equation as $4x + 7 = 91$.

(b) Two times the difference of a number and seven.

Since a word meaning "= " is not present in the sentence, this phrase translates an expression as $2(x - 7)$.

Thus, the difference between an expression and an equation is that an equation contains '=' sign and an expression does not contain '=' sign.

Solution of an Equation

The value of the variable in an equation which when substituted for the variable makes both sides equal, is called the **solution** of the equation. This value of the variable is said to **satisfy** the equation.

Illustration 2: Let us take an equation: $x + 3 = 11$.

The equation is *satisfied* when $x = 8$, because for $x = 8$,

LHS = $x + 3 = 8 + 3 = 11$ = RHS.

Thus, $x = 8$ is the solution to the equation $x + 3 = 11$.

Let us check if $x = 5$ is also a solution of this equation or not.

LHS = $x + 3 = 5 + 3 = 8$; RHS = 11

Since LHS ≠ RHS, $x = 5$ is not a solution of the equation.

Similarly, for the equation $x - 3 = 7$, the equation is *satisfied* when $x = 10$, because for $x = 10$,

LHS = $x - 3 = 10 - 3 = 7$ = RHS.

Thus, $x = 10$ is the solution to the equation $x - 3 = 7$.

SOLVING AN EQUATION

Trial and Error Method

In this method, we replace the variable in the given equation with some numbers and check if the equation is satisfied or not. If the equation is satisfied by a particular number, then that particular number is the solution of the equation.

Let us consider an equation $x + 5 = 9$. We make a table by replacing the variable x with numbers like 1, 2, 3, ... and finding corresponding values of $x + 5$.

x	$x + 5$	Is $x + 5 = 9$?	Equation Satisfied
1	6	$6 \neq 9$	No
2	7	$7 \neq 9$	No
3	8	$8 \neq 9$	No
4	9	**$9 = 9$**	**Yes**

We observe that for $x = 4$, equation $x + 5 = 9$ is satisfied.

Therefore, $x = 4$ is the solution of the equation $x + 5 = 9$.

Ex. 18 Pick out the solution of the equation $x - 4 = 2$ from the values (4, 6, 8, 10).

Sol. The given equation is $x - 4 = 2$.

For $x = 4$, LHS = $x - 4 = 4 - 4 = 0 \neq$ RHS

For $x = 6$, LHS = $x - 4 = 6 - 4 = 2$ = RHS

For $x = 8$, LHS = $x - 4 = 8 - 4 = 4 \neq$ RHS

For $x = 10$, LHS = $x - 4 = 10 - 4 = 6 \neq$ RHS

Therefore, $x = 6$ is the solution of the equation $x - 4 = 2$.

Ex. 19 **Complete the table and find the solution of the equation: $p + 7 = 12$.**

p	1	2	3	4	5	6
$p + 7$	□	□	□	□	□	□

Sol. The given equation is $p + 7 = 12$.

For $p = 1$, LHS $= p + 7 = 1 + 7 = 8 \neq$ RHS

For $p = 2$, LHS $= p + 7 = 2 + 7 = 9 \neq$ RHS

For $p = 3$, LHS $= p + 7 = 3 + 7 = 10 \neq$ RHS

For $p = 4$, LHS $= p + 7 = 4 + 7 = 11 \neq$ RHS

For $p = 5$, LHS $= p + 7 = 5 + 7 = 12 =$ RHS

For $p = 6$, LHS $= p + 7 = 6 + 7 = 13 \neq$ RHS

p	1	2	3	4	5	6
$p + 7$	8	9	10	11	12	13

On observing the table we find that for $p = 5$, LHS = RHS.

Therefore, $p = 5$, is the solution of the equation $p + 7 = 12$.

Ex. 20 **Solve the following equations.**

(a) $\frac{y}{2} = 1$ **(b)** $6 - z = 9 - 2z$

Sol. (a) Equation: $\frac{y}{2} = 1$

For $y = 0$, LHS = 0, RHS = 1, LHS $\neq$ RHS

For $y = 1$, LHS $= \frac{1}{2}$, RHS = 1 LHS $\neq$ RHS

For $y = 2$, LHS = 1, RHS = 1, LHS = RHS

For $y = 3$, LHS $= \frac{3}{2}$, RHS = 1, LHS $\neq$ RHS

Thus, $y = 2$ is the solution of the given equation.

(b) In equation $6 - z = 9 - 2z$, LHS $= 6 - z$ and RHS $= 9 - 2z$.

For $z = 0$, LHS $= 6 - 0 = 6$,

RHS $= 9 - 2 \times 0 = 9 - 0 = 9$, LHS $\neq$ RHS

For $z = 1$, LHS $= 6 - 1 = 5$,

RHS $= 9 - 2 \times 1 = 9 - 2 = 7$, LHS $\neq$ RHS

For $z = 2$, LHS $= 6 - 2 = 4$,

RHS $= 9 - 2 \times 2 = 9 - 4 = 5$, LHS $\neq$ RHS

For $z = 3$, LHS $= 6 - 3 = 3$,

RHS $= 9 - 2 \times 3 = 9 - 6 = 3$, LHS = RHS

Thus, $z = 3$ is the solution of the given equation.

Exercise 12.5

1. **Identify whether the following is an equation or an expression.**

   (a) $4x - 5y = 0$ (b) $7x = 9x - 12$ (c) $(3x + y) - 2y$ (d) $7x - 3$ (e) $6x - 9 = 28$

2. Is $x = 7$ a solution of the equation $x + 5 = 2$?
3. Is $x = 3$ a solution of the equation $x - 2 = 1$?
4. Is $x = 4$ a solution of the equation $x + 2 = 5$?
5. Pick out the solution of the equation from the given set of values.

   (a) $x + 3 = 7$; (2, 3, 4, 5) (b) $x - 5 = 3$; (4, 5, 7, 8)

6. **Complete the table and find the solution of the given equation.**

(a)

m	5	6	7	8	9	10
$m - 7$	□	□	□	□	□	□

; Equation $m - 7 = 3$

(b)

p	1	3	4	5	6	7
$p + 3$	□	□	□	□	□	□

; Equation $p + 3 = 7$

(c)

y	3	5	7	8	9	10
$\frac{y}{4}$	□	□	□	□	□	□

; Equation $\frac{y}{4} = 2$

7. Translate the following statements into equations and solve them.

(a) A number added to 3 is 5.
(b) Subtracting x from 9 gives 3.
(c) Thrice a number is 15.
(d) Quarter of a number is 6.
(e) The sum of a number and 1 is 12.
(f) Double of a number subtracted from 16 gives 10.

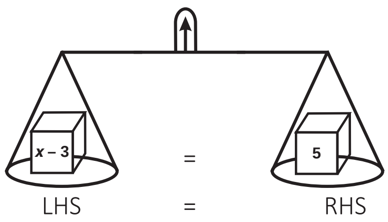
8. Solve the following equations.

(a) $8 = 5 + x$ (b) $x - 3 = 8$ (c) $x - 5 = 7$ (d) $3x - 2 = 10$

(e) $7x = 49$ (f) $9x = 20 - x$ (g) $\frac{y}{2} = 3$ (h) $\frac{y}{2} = 6 - y$

Systematic Method

Since trial and error method is time consuming, we need to learn a short and easier method to solve an equation. To understand the systematic method, let us compare an equation with a weighing balance.



We know that a balance has two pans and if equal weights are put on the two pans, the pans remain in balance. Similarly, if equal weights are removed from both the sides, the pans again remain in balance. Thus, we can also add or subtract the same things from both sides (any number of times) without distributing the balance. Similarly, we can apply the following rules in case of an equation.

Rule 1: We can add the same number to both the sides of an equation.

Illustration 1: Let us solve the equation $x - 3 = 5$.

We have $x - 3 = 5$.

$x - 3$ ↑ Subtraction

Adding 3 on both the sides, we get

$x - 3 + 3 = 5 + 3 \Rightarrow x = 8$

Check: For $x = 8$, LHS $= x - 3 = 8 - 3 = 5 =$ RHS

Thus, $x = 8$ is the solution of the equation $x - 3 = 5$.

Rule 2: We can subtract the same number from both the sides of an equation.

Illustration 2: Let us solve $y + 7 = 15$ and check.

We have $y + 7 = 15$.

$y + 7$ ↑ Addition

Subtracting 7 from both sides, we get

$y + 7 - 7 = 15 - 7 \quad \Rightarrow \quad y = 8$

Check: For $y = 8$, LHS $= y + 7 = 8 + 7 = 15 =$ RHS

Thus, $y = 8$ is the solution of the equation $y + 7 = 15$.

Rule 3: We can multiply the same number to both the sides of an equation.

Illustration 3: Let us solve $\frac{z}{3} = 2$.

$\frac{z}{3}$ or $z \div 3$ ↑ Division

We have $\frac{z}{3} = 2$.

Multiplying both the sides by 3, we get

$\frac{z}{\cancel{3}} \times \cancel{3} = 2 \times 3 \quad \Rightarrow \quad z = 6$

Check: For $z = 6$, LHS $= \frac{z}{3} = \frac{6}{3} = 2 =$ RHS

Thus, $z = 6$ is the solution of the equation $\frac{z}{3} = 2$.

Rule 4: We can divide both the sides of an equation by the same non-zero number.

Illustration 4: Let us solve $6x = 30$.

We have $6x = 30$

$6x$ or $6 \times x$ ↑ Multiplication

Dividing both the sides by 6, we get

$$\frac{6x}{6} = \frac{30}{6} \quad \Rightarrow \quad x = 5$$

Check: For $x = 5$, LHS $= 6 \times 5 = 30 =$ RHS

Thus, $x = 5$ is the solution of the equation $6x = 30$.

Skill Check

- Solve:

(a) $x - 1 = 3$ (b) $3 + p = 10$ (c) $\frac{1}{5}z = 2$

Ex. 21 Solve the following equations and check.

(a) $4x - 1 = 19$ (b) $\frac{3p}{4} = 15$

Sol. (a) We have $4x - 1 = 19$

$\Rightarrow 4x - 1 + 1 = 19 + 1$

(Adding 1 to both the sides)

$\Rightarrow 4x = 20$ or $\frac{4x}{4} = \frac{20}{4}$

(Dividing both the sides by 4)

$\therefore \quad x = 5$

Check: For $x = 5$, LHS $= 4x - 1 = 4 \times 5 - 1$
$= 20 - 1 = 19 =$ RHS

Thus, $x = 5$ is the solution of the equation $4x - 1 = 19$.

(b) We have $\frac{3p}{4} = 15 \Rightarrow \frac{3}{\cancel{4}} p \times \cancel{4} = 15 \times 4$

(Multiplying both the sides by 4)

$\Rightarrow \quad 3p = 60$

$\Rightarrow \frac{\cancel{3}p}{\cancel{3}} = \frac{60}{3}$ (Dividing both the sides by 3)

$\therefore \quad p = 20$

Check: For $p = 20$, LHS $= \frac{3}{4} p = \frac{3}{\cancel{4}} \times \cancel{20}^{5}$

$= 15 =$ RHS

Thus, $p = 15$ is the solution of the equation $\frac{3p}{4} = 15$.

From the above illustrations and examples, we observe that we do an inverse operation of the operation present between the number and variable.

Transposition Method

Let us consider the equation $x + 5 = 10$.

$\because x + 5 = 10 \Rightarrow x + 5 - 5 = 10 - 5$, *i.e.*, $x = 10 - 5$

+5 appears as – 5

Also, consider

$y - 7 = 15 \Rightarrow y - 7 + 7 = 15 - 7$, *i.e.*, $y = 15 + 7$

–7 appears as +7

Clearly, a number appears with opposite sign on the other side of '=' sign. We can use this property straight away to transfer a number or term in other side of the equation. This is called **transposition**.

Ex. 22 Solve: $2x - 5 = 7$

Sol. We have $2x - 5 = 7$

$\Rightarrow \quad 2x = 7 + 5$

(Transposing –5 from LHS to +5 on RHS)

$\Rightarrow \quad 2x = 12$

$\therefore \quad x = \frac{12}{2} = 6$

(Transposing × 2 from LHS to ÷ 2 on RHS)

Thus, $x = 6$ is the solution of $2x - 5 = 7$.

Application of Equations in Real Life

Ex. 23 Arun's mother gave him ₹10. Now, Arun has ₹25 in all. How much money was there with Arun initially?

Sol. Let Arun have ₹x.

After receiving ₹10, from his mother, he has ₹$(x + 10)$ in all. But, now he has ₹25.

So, $x + 10 = 25 \Rightarrow x = 25 - 10$

$\therefore \quad x = 15$

Thus, Arun had ₹15 initially.

Ex. 24 Solve the riddle: "Go round a square counting every corner thrice and no more. Add the count to me to get exactly thirty-four. What number am I"?

Sol. Let I be x. A square has four corners.

Counting every corner thrice, $4 \times 3 = 12$.

Add the count to me to get thirty-four, *i.e.*, $12 + x = 34$

$\Rightarrow \quad x = 34 - 12$ (Transposing 12 to RHS)

$\therefore \quad x = 22$

Check: LHS $= 12 + 22 = 34 =$ RHS

Therefore, I am the number 22.

Ex. 25 Solve the riddle. "For each day of the week, make an upcount from me. If you make no mistake, you will get twenty-three! Who am I"?

Sol. We know that the number of days in a week = 7.

Let I be x. For each day of the week making an upcount from x, we get $x + 7$.

If there is no mistake, we get 23.

Add the count to me to get twenty-three.

i.e., $x + 7 = 23 \Rightarrow x = 23 - 7 = 16$

(By Transposing 7 to RHS)

Check: LHS $= 16 + 7 = 23 =$ RHS

Therefore, I am the number 16.

Exercise 12.6

1. Solve the following equations by systematic method.

(a) $x - 3 = 2$ (b) $3 + y = 9$ (c) $2z = 18$ (d) $\frac{1}{3}x = 7$

(e) $3p - 1 = 5$ (f) $4 + 5t = 24$ (g) $\frac{4}{7}u = 28$ (h) $z - \frac{1}{6} = \frac{1}{3}$

2. Solve the following equations by transposition method and check your answer.

(a) $\frac{x}{3} = \frac{1}{15}$ (b) $4x - 7 = 5$ (c) $3x - \frac{1}{2} = 2\frac{1}{2}$

(d) $\frac{5}{7}z = 30$ (e) $16 - 3p = 1$ (f) $40 + \frac{1}{2}p = 45$

3. Answer the following questions.

(a) A number increased by 9 is 24. Find the number.

(b) For what value of p, $p \times 4 = 9 \times 4$?

(c) Find the solution of the equation $3x - 5 = 7$.

(d) If $5g + 12 = 27$, what is the value of g?

4. Tarun is 23 years old. After how many years will he be 30 years old?

5. Five times a number y decreased by 8 is equal to 22. What is the number y?

6. Twice a certain number minus 3 is equal to 4. Find the number.

7. "Go round a square counting every corner two times and no more. Add the count to me to get exactly fifty-five. Who am I?"

8. "For each day of the week, make an upcount from me. If you make no mistake, you will get thirty-four. Who am I?"

Competency Based Exercise

21st CS

1. Tick (✓) the correct answer.

(a) If the perimeter of a regular hexagon is p metres, then the length of each of its side is:

(i) $(p + 6)$ metres (ii) $6p$ metres (iii) $(6 \div p)$ metres (iv) $(p \div 6)$ metres

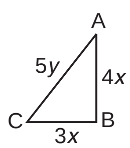
(b) The expression obtained when x is multiplied by 5 and then subtracted from 4 is:

(i) $5x - 4$ (ii) $5x + 4$ (iii) $4 - 5x$ (iv) $4x - 5$

(c) In the given figure, the perimeter of the triangle ABC is:

(i) $5y = 4x + 3y$ (ii) $7x + 5y$

(iii) $5y + 4x$ (iv) $5y + 3x$



(d) Which of the following statements is not correct?

(i) The difference between the ages of Varun's father and mother is not a variable.

(ii) t minutes are equal to $60t$ seconds.

(iii) $2x - 7 > 25$ is an equation. (iv) The additive inverse of x is $-x$.

(e) Translate the following into an algebraic expression:
"The product of a and b subtracted from their sum."

(i) $a + b + ab$ (ii) $a + b - ab$ (iii) $a + ab - b$ (iv) $ab - a - b$

(f) In a hall, there are x rows of chairs and each row contains y chairs. The total number of chairs in the hall is:

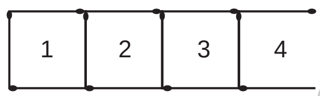
(i) $x + y$ (ii) xy (iii) x^2y (iv) y^2x

2. Solve the following equations.

(a) $x + 108 = 130$ (b) $x - 12 = 21$ (c) $\frac{n}{8} = 32$

(d) $3x + 4 = 19$ (e) $a \times 5 = 32.85$ (f) $45 - y = 19$

3. Observe the following matchstick pattern of Cs below:



How many matchsticks will be required to make twenty-five Cs?

4. Translate the following into equations and solve for the unknown.

(a) 36 divided by x gives 6. (b) p is a number. I add 15 to this number; the result is 26.

5. Sania's uncle is 3 times Sania's age. Sania's age is z years. Her aunt is 4 years older than her uncle. What are the ages of her uncle and aunt?

Challenge! 21st CS

1. Some oranges are to be transferred from a larger box into smaller boxes. When the large box is emptied, the oranges from it fill 2 smaller boxes and still 10 oranges remain outside. If the number of oranges in the small box are taken to be x, what is the number of oranges in the larger box?
2. A radio taxi charges ₹50 for the first kilometre and ₹12 for each subsequent kilometre. What are the total charge (in ₹) for 'x' kilometres?

Let's Work in Mind 21st CS

1. If each matchbox contains 50 matchsticks, then how many matchsticks are required to fill m such boxes?
2. Which variable is used in the equation $3p - 12 = 9$?
3. Harshita has a sum of ₹x. She spent ₹1500 on clothes, ₹500 on grocery and ₹400 on make up and received ₹800 as a gift. How much money (in ₹) is left with her?
4. What is the value of $2a + 3b$ for $a = 3$ and $b = 2$?
5. What is the coefficient of x^2y in $5x^2yz$?
6. Is $6x + 5y - 2x$ a trinomial?

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. Assertion (A) : $3xy$ and $2xy$ can be added.
Reason (R) : $3xy$ and $2xy$ are like terms.

2. Assertion (A) : $x^2 - y^2$ is a binomial.
Reason (R) : Binomials are expressed in two variables.

3. Assertion (A) : $x^2 + y^3 + 2$ is a trinomial.
Reason (R) : Binomials have two terms.

4. Assertion (A) : $xz + 2$ is a polynomial.
Reason (R) : $xz + 2 = 2z$ is an equation.

5. Assertion (A) : $\frac{x}{3} = \frac{1}{15}$ is an equation in variable x.
Reason (R) : An equation is a statement of equality which involves variables.

6. Assertion (A) : $2 = 3$ is an equation.
Reason (R) : An equation is a statement of equality which involves variables.

7. Assertion (A) : In a hall, there are x rows of chairs and each row contains y chairs. The total number of chairs in the hall is xy.
Reason (R) : Algebraic expression for the total number of chairs is $x + y$.

CASE STUDY

CC

Aruna's mother gifted a money bank to her, to inculcate the habit of saving money. She also asked the guests invited for birthday party to just contribute small money to her money bank instead of bringing costly gifts. Aruna's mother believes that every child should learn the value of money.

1. Few guests gave ₹10 for the money bank and few gave ₹20. Express money received by Aruna on her birthday as an algebraic expression.
2. If Aruna has saved ₹50 per month for next 10 months, what will be the total money at the end of 11 months?
3. If total money at the end of 11 months is ₹800, how much money was received by Aruna on her birthday?

13 Ratio and Proportion

What Learners Will Achieve

- understand the concept of ratio as comparing the quantities of the same unit.
- understand the difference between ratio and fraction.
- find the ratio between two given quantities with the same unit.
- express ratio in its simplest form.
- compare the ratios.
- understand the concept of proportion.
- check whether the two given ratios are in proportion or not.
- determine whether the four given numbers are in proportion.
- find unknown quantity when four numbers are in proportion.
- solve application based problems using unitary method.

COMPARING QUANTITIES

In our daily life, we are often required to compare two quantities of the same type in terms of their measurements/magnitudes. One way of comparison is by difference, the other is by division. Do the two give the same picture and represent the same idea?

Let us observe:

Rahul and Ruhi are twins and are studying in class VI in two different schools. In a test in his school, Rahul scores 10 marks out of 20 whereas Ruhi in a test of the same subject in her school scores 90 out of 100. Their mother tells Rahul that he will have to work harder to catch up with Ruhi. However, Rahul says that there is no difference between their performances as just like him she has also lost ten marks in her exam. Who is right, Rahul or his mother?

Let us examine:

Rahul's performance: Ruhi's performance:

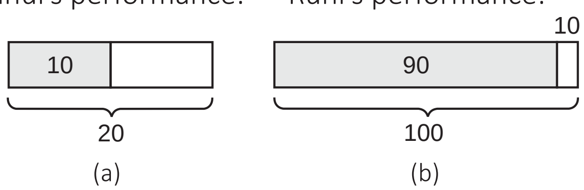


Fig. 13.1

We observe that, Rahul got half and he lost half while Ruhi got almost everything and lost very little compared to what she got.

If it was money, Rahul would have got only half of what was available, whereas Ruhi would have received a big chunk of what was available.

Conclusion: Rahul was wrong and his mother was right. We find that comparison by division gives a clearer picture than comparison by difference. Comparison by division is called **ratio**. It tells us how many times one quantity is of the other. We will study about ratios and its applications in this chapter.

RATIO

Let us compare the weight of Julie and her daughter.

Weight of Julie is 70 kg and weight of her daughter is 10 kg. So, we can say that weight of Julie is (70 – 10) kg = 60 kg *more than* the weight of her daughter. This is one way of comparing the weights of Julie and her daughter. This way of comparison of two quantities is known as **comparison by difference**.

Weight of Julie is $\frac{70}{10}$ = 7 *times* the weight of her daughter. This way of comparison of two quantities is known as **comparison by division**.

When two quantities of the same type (with respect to their magnitudes) are compared by division, they form a **ratio**.

Thus, the ratio of the weight of Julie to the weight of her daughter is 70 ÷ 10 or $\frac{70}{10}$ or 70 : 10, read as '70 is to 10' or '70 to 10'.

The symbol ':' is used to denote a ratio. In the ratio 70 : 10, 70 is the **first term** and 10 is the **second term**.

Thus,

> Ratio of two numbers ***a*** and ***b*** is written as $\frac{a}{b}$, ***a* : *b*** or ***a* ÷ *b***. These three expressions have the same meaning. Here, '***a***' is called the **first term** or **antecedent** and '***b***' is called the **second term** or **consequent.**

A ratio can be expressed in many ways.

For example, 70 : 10 can be written as 35 : 5, 14 : 2 or 7 : 1.

This is so because we can divide or multiply the numerator and denominator by the same non-zero number.

In a ratio *a* : *b*, order of terms is very important. $a : b \neq b : a$.

Note

Ratio 7 : 1 is different from 1 : 7.

In reference to Julie and her daughter,

7 : 1	1 : 7
means weight of Julie is 7 times the weight of her daughter.	means weight of the daughter is one-seventh of the weight of Julie.

Difference between a Fraction and a Ratio

A ratio is different from a fraction. A ratio is a comparison between two quantities while a fraction is used to denote a part of the whole.

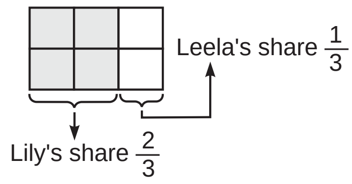


Fig. 13.2

For example, a chocolate is shared between Lily and Leela as shown in Fig. 13.2.

Clearly, Lily and Leela share the chocolate in the ratio 2 : 1. But, they got $\frac{2}{3}$ and $\frac{1}{3}$ part of the chocolate respectively.

Ratio of Like Measurements

Comparison of two quantities is *meaningful*, if they are of the same kind or in the same units (of length, weight, capacity, currency, etc.

For example, ratio of 20 rupees to 25 rupees is $\frac{₹20}{₹25}$ or $\frac{20}{25}$ or 20 : 25.

(The units are dropped, if they are the same.)

Similarly, the ratio of 36 metres to 27 metres is $\frac{36\text{ m}}{27\text{ m}}$ or $\frac{36}{27}$ or 36 : 27.

(The units are dropped, if they are the same.)

What is the ratio of 10 boys and 15 cows, 20 litres and 15 toys, etc.? Of course, we cannot compare them and hence there is no ratio between them.

Reducing Ratios to the Simplest Form

Since the ratio is expressed as a fraction, it can be reduced to its lowest terms.

For example, the ratio $\frac{24}{15}$ in lowest terms (or simplest form) is $\frac{24 \div 3}{15 \div 3} = \frac{8}{5} = 8:5$.

Ratio of 9 books to 5 books = $\frac{9 \text{ books}}{5 \text{ books}} = \frac{9}{5} = 9:5.$

Ratio of ₹20 to ₹25 = $\frac{₹20}{₹25} = \frac{20}{25} = \frac{4 \times 5}{5 \times 5} = \frac{4}{5} = 4:5.$

Similarly, ratio of 36 m to 27 m

$$= \frac{36 \text{ m}}{27 \text{ m}} = \frac{36}{27} = \frac{4}{3} = 4:3.$$

Thus, from the above we can say that:

A ratio $a : b$ is said to be in its *lowest* terms or *simplest* form if a and b have no common factors, other than 1.

In general, to write the ratio of two numbers or like measurements, we follow these steps:

Step 1: Write the ratio as a fraction.

Step 2: Drop the common units.

Step 3: Reduce the fraction.

Step 4: Write the fraction as a ratio.

Ex. 1. Write the ratio of 234 to 36 in lowest terms.

Sol. The ratio of 234 to 36 = $\frac{234}{36}$

(Write the ratio as a fraction)

$$= \frac{13 \times 18}{2 \times 18} = \frac{13}{2}$$

The ratio of 234 to 36 in the simplest form is $\frac{13}{2}$ or 13 : 2.

Ratio of Unlike Measurements (Quantities of the Same Kind, but Different Units)

For finding the ratio of two unlike measurements, convert them into like measurements, whenever possible.

Illustration 1: The ratio of 6 metres to 35 centimetres is $\frac{6 \text{ m}}{35 \text{ m}}$.

Note

Metres and centimetres are unlike measurements. So, we convert them into the same units of measurements.

So, $\frac{6 \text{ m}}{35 \text{ m}} = \frac{600 \text{ cm}}{35 \text{ cm}} = \frac{120}{7} = 120:7$ ($\because$ 1 m = 100 cm)

(The common units are dropped and the fraction is simplified.)

Ex. 2. Find the ratio of:

(a) 90 cm to 1.5 m.

(b) 250 mL to 1.5 litres.

Sol. (a) The two quantities have different units. Two quantities can be compared, only if they are in the same unit. So, we change them into the same unit.

1.5 m = 1.5 × 100 cm = 150 cm

($\because$ 1 m = 100 cm)

Therefore, the required ratio is $\frac{90 \text{ cm}}{1.5 \text{ m}}$

$$= \frac{90 \text{ cm}}{150 \text{ cm}} = \frac{10 \times 3 \times 3}{10 \times 5 \times 3} = \frac{3}{5} \text{ or } 3:5$$

(b) 1.5 L = 1.5 × 1000 mL = 1500 mL

($\because$ 1 L = 1000 mL)

Therefore, the required ratio is $\frac{250 \text{ mL}}{1.5 \text{ L}}$

$$= \frac{250 \text{ mL}}{1500 \text{ mL}} = \frac{250}{1500} = \frac{1}{6} \text{ or } 1:6$$

Building Equivalent Ratios

Consider the ratio 8 : 12 and 10 : 15. Reduce the ratios to the lowest terms (or simplest form).

$8:12 = \frac{8}{12} = \frac{2}{3} = 2:3$ and $10:15 = \frac{10}{15} = \frac{2}{3} = 2:3$

We observe that both these ratios reduce to 2 : 3. So, the ratios 8 : 12 and 10 : 15 are **equivalent ratios**.

Thus, it can be stated as follows:

To get equivalent ratios, multiply or divide the numerator and denominator by the same non-zero number.

Consider the ratio 4 : 6.

$$4:6 = \frac{4}{6} = \frac{4 \times 2}{6 \times 2} = \frac{8}{12} = 8:12$$

Therefore, 8 : 12 is an equivalent ratio of 4 : 6 or 4 : 6 is an equivalent ratio of 8 : 12.

Similarly, $4 : 6 = \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3} = 2 : 3$.

So, 2 : 3 is another equivalent ratio of 4 : 6.

Skill Check

State if the ratios are the same.

(a) 2 : 3 and 10 : 15 (b) 4 : 5 and 5 : 4

COMPARISON OF RATIOS

Since a ratio can be written as a fraction, two or more ratios can be compared as we compare fractions.

Illustration 2: Let us compare 5 : 8 and 7 : 12.

We have $5 : 8 = \frac{5}{8}$ and $7 : 12 = \frac{7}{12}$

LCM of 8 and 12 is 24.

So, $5 : 8 = \frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$ and

$7 : 12 = \frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$.

(Building equivalent ratios with the same denominator)

Since 15 > 14, *i.e.*, $\frac{15}{24} > \frac{14}{24}$.

Therefore, 5 : 8 > 7 : 12.

Illustration 3: To find the smaller ratio among 4 : 9 and 5 : 6.

We have $4 : 9 = \frac{4}{9}$ and $5 : 6 = \frac{5}{6}$

So, $\frac{4}{9} = \frac{4 \times 2}{9 \times 2} = \frac{8}{18}$ and $\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$.

($\because$ LCM of 9 and 6 is 18.)

Clearly, 8 < 15, *i.e.*, $\frac{8}{18} < \frac{15}{18}$. Therefore, 4 : 9 < 5 : 6.

Thus, 4 : 9 is a smaller ratio than the ratio 5 : 6.

From the above we say that, to compare the ratios, follow these steps:

Step 1: Write the ratios as fractions.

Step 2: Write the equivalent fractions (ratios) with the common denominator as LCM.

Step 3: Compare the numerators and arrange the fractions.

Step 4: Replace the fractions with the corresponding given ratios.

Let us study some more examples.

Ex. 3. From Fig. 13.3, find the ratio of the number of rectangles (marked R) to the number of squares (marked S).

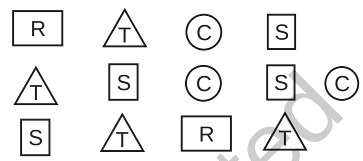


Fig. 13.3

Sol. Number of rectangles = 2,

Number of squares = 4

Therefore, ratio of the number of rectangles to the number of squares = $\frac{2}{4} = \frac{1}{2}$.

Thus, the required ratio is 1 : 2.

Ex. 4. The ratio of breadth and length of a hall is 2 : 5. Complete the Table 13.1 that shows some possible breadths and lengths of the hall.

Breadth	□	**8**	□
Length	**25**	□	**35**

Table 13.1

Sol. Given, ratio of breadth and length = 2 : 5

Length = 25 = 5 × 5

Therefore, breadth should be 2 × 5 = 10.

Now, Breadth = 8 = 2 × 4

Therefore, length should be 5 × 4 = 20.

Again, Length = 35 = 5 × 7

Therefore, breadth should be 2 × 7 = 14.

Thus, the possible breadths and lengths of the hall can be as shown in the Table 13.2.

Breadth	**10**	8	**14**
Length	25	**20**	35

Table 13.2

Ex. 5. A library has 8750 books, of which 1750 are Mathematics books and the rest are Literature books. Find the ratio of:

(a) the number of Mathematics books to the number of Literature books.

(b) **the number of Literature books to the total number of books.**

Sol. Total number of books in the library = 8750

Total number of Mathematics books = 1750

So, the number of Literature books

$= 8750 - 1750 = 7000$

(a) The ratio of the number of Mathematics books to the number of Literature books $= \frac{1750}{7000} = \frac{1}{4} = 1 : 4$ (Simplify)

Thus, the ratio of Mathematics books to the Literature books is 1 : 4.

(b) The ratio of the number of Literature books to the total number of books in the library $= \frac{7000}{8750} = \frac{4}{5} = 4 : 5$

So, the ratio of the Literature books to the total books is 4 : 5.

Ex. 6. Divide 20 pencils between Saumya and Sidhant in the ratio of 3 : 2.

Sol. Sum of the terms of the ratio = 3 + 2 = 5

If there are 5 pencils, Saumya will get 3 pencils and Sidhant will get 2 pencils.

Or Saumya gets 3 parts out of total 5 parts and Sidhant gets 2 parts of total 5 parts.

Therefore, Saumya's share $= \frac{3}{5}$ of 20

$= \frac{3}{5} \times 20$ pencils = 12 pencils

Sidhant's share $= \frac{2}{5}$ of 20 pencils

$= \frac{2}{5} \times 20$ pencils = 8 pencils.

Ex. 7. Two sums of money are in the ratio 4 : 5. If the larger amount is ₹460, what is the smaller amount?

Sol. Given, sums of money are in the ratio 4 : 5.

Let the sums of money be ₹$4x$ and ₹$5x$.

Also, the larger amount is ₹460.

So, $5x = 460$

or $x = \frac{460}{5} = 92$

Thus, the smaller amount = ₹$4x$

= ₹4 × 92 = ₹368

Ex. 8. The cost of 12 fountain pens is ₹180 and the cost of 8 ball pens is ₹56. Find the ratio of the cost of a fountain pen to the cost of a ball pen.

Sol. Cost of 12 fountain pens = ₹180

Therefore, cost of 1 fountain pen = ₹$\frac{180}{12}$

= ₹15

Cost of 8 ball pens = ₹56

Therefore, cost of 1 ball pen = ₹$\frac{56}{8}$ = ₹7

Ratio of the cost of a fountain pen to the cost of a ball pen $= \frac{15}{7}$ or 15 : 7.

Thus, the required ratio is 15 : 7.

Exercise 13.1

1. From the following figure:

(a) Find the ratio of the number of circles (marked C) to the number of triangles (marked T).

(b) Find the ratio of the number of circles (marked C) to the number of rectangles (marked R).

(c) Find the ratio of the number of squares (marked S) to the number of triangles (marked T).

2. Find the ratio of:

(a) 15 cm to 40 cm (b) 45 m to 3 m (c) ₹60 to ₹105 (d) 2 h to 45 min

(e) 300 mL to 2.5 L (f) 350 g to 3.5 kg (g) 65 m to 1.3 km (h) ₹5 to 25 paise

3. **Write the following ratios in the lowest terms.**

(a) 285 to 45 (b) 230 to 40 (c) 276 to 48 (d) 75 to 225

4. Find the ratio of the number of teachers to the number of students in a school, if there are 85 teachers and 1258 students.

5. Complete the table that shows some possible breadths and lengths of a hall, if their ratios are given along with.

(a) 3 : 5,

Breadth	18		27
Length		15	

(b) 5 : 7,

Breadth	25	45	
Length			21

6. **Compare the following ratios using >, < or = .**

(a) 3 : 4 ☐ 4 : 7 (b) 16 : 21 ☐ 14 : 15 (c) 3 : 8 ☐ 27 : 72

7. What is the ratio of the number of Mathematics books to the number of Literature books, if a library has 4200 books, of which 1400 are Mathematics books and the rest are Literature books?

8. Find the ratio of the cost of a fountain pen to the cost of a ball pen, if the cost of 10 fountain pens is ₹140 and cost of 8 ball pens is ₹72.

9. Divide 30 pencils between Manu and Kanta in the ratio 2 : 3.

10. Out of 45 students in a class, 8 like football, 15 like cricket and the remaining like tennis.

Find the ratio of the:

(a) number of students who like football to the number of students who like tennis.

(b) number of students who like cricket to the total number of students.

11. Present age of father is 40 years and his son is 10 years. Find the ratio of the age of the father to the age of son after 10 years.

12. Tina has ₹150 and Brij has ₹600. What is the ratio of the total amount of money to Brij's amount?

13. The sum of two numbers is 96. If the two numbers are in the ratio 3 : 5, find the bigger number.

14. A sum of money is shared between Geeta and Samita in the ratio of 3 : 2. If Samita receives ₹128, find the share of Geeta.

15. Sufiyan has 1 dozen eggs and Joginder has $1\frac{1}{2}$ dozen eggs. Find the ratio of Sufiyan's eggs to Joginder's.

Proportion

Let us consider a real life situation.

Ritu buys 10 litres of petrol for her car for ₹970 while Ria buys 3 litres of petrol for her scooty at the same petrol station for ₹291.

Ratio of two quantities of petrol = 10 : 3.

Ratio of the money spent on buying petrol

$$= 970 : 291 = \frac{970}{291} = \frac{10 \times 97}{3 \times 97} = \frac{10}{3} = 10 : 3$$

Thus, we observe that ratio of quantities of petrol = ratio of the amount spent

i.e., 10 : 3 = 970 : 291.

We can also conclude that both Ritu and Ria bought the petrol at the same rate.

Such an equality of ratios is known as a **proportion** and the numbers 10, 3, 970, 291 are said to be in proportion.

Thus, equality of two ratios $a : b = c : d$ is called a proportion and we say that the numbers a, b, c, d are in proportion.

We use the symbol ': :' to denote a proportion.

So, $a : b = c : d$ is the same as $a : b :: c : d$.

Illustration 1: Let us see whether the ratios 3 : 7 and 18 : 42 form a proportion or not.

$$3 : 7 = \frac{3}{7};\ 18 : 42 = \frac{18}{42} = \frac{2\times3\times3}{2\times3\times7} = \frac{3}{7}$$

As $\frac{3}{7} = \frac{18}{42}$, so 3 : 7 and 18 : 42 form a proportion,

i.e., 3 : 7 : : 18 : 42.

The proportion 3 : 7 : : 18 : 42 is read as '3 is to 7 is as 18 is to 42'.

Illustration 2: Let us check if 2 : 3 and 6 : 8 form a proportion.

$$\frac{6}{8} = \frac{2\times3}{2\times2\times2} = \frac{3}{4} \text{ which is } \textbf{\textit{not equal}} \text{ to } \frac{2}{3}.$$

So, 2 : 3 and 6 : 8 do not form a proportion.

Terms of a Proportion

In a statement of proportion, the four quantities involved taken in order are known as *the respective terms* of proportion.

For example, in 4 : 6 : : 8 : 12

4 ↓ First term; 6 ↓ second term; 8 ↓ third term; 12 ↓ fourth term

First and fourth terms are known as ***extreme terms*** **(or extremes)**.

Second and third terms are known as ***middle terms*** **(or means)**.

In the example of quantity of petrol and amount spent on buying petrol, we have seen that

10 : 3 : : 670 : 201

10 ↓ First term; 3 ↓ second term; 670 ↓ third term; 201 ↓ fourth term

We observe that

3 × 670 = 2010 (product of second and third terms)

10 × 201 = 2010 (product of first and fourth terms)

i.e., product of second term and third term = product of first term and fourth term

Thus, we can say that

product of middle terms (means)
= product of extreme terms (extremes)

In general, it can be stated as follows:

if $\underbrace{a}_{\text{extreme term}} : \underbrace{b :: c}_{\text{middle terms}} : \underbrace{d}_{\text{extreme term}}$

i.e., if a, b, c and d are in proportion, then

$$\underbrace{b \times c}_{\substack{\uparrow \\ \text{Product of} \\ \text{middle terms}}} = \underbrace{a \times d}_{\substack{\uparrow \\ \text{Product of} \\ \text{extreme terms}}}$$

Remember

If product of middle terms = product of extreme terms, the proportion is formed.

Or, if $a \times d = b \times c$, then $\frac{a}{b} = \frac{c}{d} \Rightarrow a : b :: c : d$.

To determine, if the ratios form a proportion or not, we follow these steps:

Step 1: Write the proportion using fractions for the ratios.

Step 2: Find the cross products.

Step 3: If the cross products are equal, then the given ratios form a proportion otherwise they do not form a proportion.

Ex. 9. Verify if the ratios 30 : 40 and 45 : 60 form a proportion.

Sol. $30 : 40 = \frac{30}{40} = \frac{3}{4};\ 45 : 60 = \frac{45}{60} = \frac{3}{4}$

So, 30 : 40 = 45 : 60.

Therefore, the ratios 30 : 40 and 45 : 60 form a proportion.

Alternate method: By cross-multiplication, we have 30 × 60 = 1800;

40 × 45 = 1800

$\frac{30}{40} \times \frac{45}{60}$

Since 30 × 60 = 40 × 45, the given ratios 30 : 40 and 45 : 60 form a proportion.

Ex. 10. Determine whether the ratios given below form a proportion or not.

(a) $\frac{3}{4}$ to 5; 6 to 40

(b) 2.4 to 5; 5 to 2.4

(c) 25 cm : 1 m; ₹40 : ₹160

Sol. (a) $\frac{\frac{3}{4}}{5} \times \frac{6}{40}$ $\rightarrow 5 \times 6 = 30$

$\rightarrow \frac{3}{4} \times 40 = 30$

(Find the cross products.)

So, $\frac{3}{4} : 5 = 6 : 40.$

(The cross products are equal.)

Thus, the given ratios are in proportion.

(b) $\frac{2.4}{5} \times \frac{5}{2.4}$ (Find the cross products.)

$25 \neq 5.76$ (The cross products are not equal.)

Thus, the given ratios are not in proportion.

(c) $25 \text{ cm} : 1 \text{ m} = \frac{25 \text{ cm}}{1 \text{ m}} = \frac{25 \text{ cm}}{100 \text{ cm}}$

$= \frac{25}{100} = \frac{1}{4}$

$₹40 : ₹160 = \frac{₹40}{₹160} = \frac{40}{160} = \frac{1}{4}$

Thus, 25 cm : 1 m = ₹40 : ₹160.

Therefore, the given ratios form a proportion.

Skill Check

Check if the following ratios are in proportion.
(a) 6 to 8 and 9 to 12 (b) 6, 9, 14 and 6

Watch Your Step! **LS**

1, 25, 2 and 50 are in proportion but ₹1 : 25 p ≠ 2 h : 50 min

$₹1 : 25\text{p} : \frac{100\text{ p}}{25\text{ p}} = \frac{4}{1} = 4:1$

and $2\text{ h} : 50\text{ min} = \frac{120\text{ min}}{50\text{ min}} = \frac{12}{5} = 12:5$

$4:1 \neq 12:5$

Ex. 11. The first, second and fourth terms of a proportion are 45, 63 and 35 respectively. Find the third term.

Sol. Let the third term of the proportion be x.

Given, 45, 63, x, 35 are in proportion.

So, Product of extreme terms = Product of middle terms

i.e., $45 \times 35 = 63 \times x$

or $x = \frac{45 \times 35}{63} = 25$

Thus, the third term of the proportion is 25.

Ex. 12. Find x, if the numbers 2, x, x, 8 are in proportion.

Sol. Given, 2, x, x, 8 are in proportion.

∵ Product of middle terms = Product of extreme terms

So, $x \times x = 2 \times 8$

i.e., $x \times x = 4 \times 4$ (∵ 16 = 4 × 4)

So, $x = 4$

We observe that the numbers 2, 4, 4 and 8 are in proportion. Also, it can be stated as three numbers 2, 4 and 8 are in continued proportion.

Thus, if a, b, c are in continued proportion, then:

$a : b : : b : c$

$\Rightarrow b \times b = a \times c$

or $b^2 = ac$

We conclude that, when a, b, c are in continued proportion, b is called the mean proportion and c is called the third proportion.

Ex. 13. Find the third proportion to 6 and 15.

Sol. Let the third proportion be x. Then,

$6 : 15 : : 15 : x$

or $6 \times x = 15 \times 15$

∴ $x = \frac{225}{6} = \frac{75}{2} = 37\frac{1}{2}$

or $x = 37.5$

Thus, the third proportion is 37.5.

Ex. 14. The cost of a dozen lemons is ₹48. What is the cost of 8 lemons?

Sol. Let the cost of 8 lemons be ₹x.

We know the ratio of number of lemons and the ratio of their cost are in proportion.

Therefore, $\frac{12}{8} = \frac{48}{x}$ (∵ 1 dozen = 12)

or $12 \times x = 48 \times 8$

∴ $x = \frac{48 \times 8}{12} = ₹32$

Thus, the cost of 8 lemons is ₹32.

Exercise 13.2

1. **Find out, if the given ratios are in proportion.**

   (a) 10 : 15 and 4 : 12 (b) 15 : 45 and 40 : 120 (c) 24 : 28 and 12 : 14 (d) 25 : 30 and 40 : 48

2. **Determine, whether the ratios given below are in proportion or not.**

   (a) $\frac{5}{15}$ and $\frac{10}{30}$ (b) $\frac{2}{7}$ is to 3 and 5 is to 42

   (c) 3.2 is to 3 and 4 is to 1.9 (d) 20 cm : 1 m and ₹25 : ₹100

   (e) 50 paise: ₹3 and 6 km : 36 km (f) 350 mL : 1 L and ₹21 : ₹54

3. **Find:**

   (a) the third term, if the first, second and fourth terms of a proportion are 12, 6 and 36 respectively.

   (b) the second term, if the first, third and fourth terms of a proportion are 60, 48 and 8 respectively.

   (c) the first term if the second, third and fourth terms of a proportion are 15, 90 and 30 respectively.

   (d) the fourth term if the first, second and third terms of a proportion are 80, 10 and 64 respectively.

4. **Find *x*, if the given numbers are in proportion.**

   (a) 5, 10, x, 20 (b) 30, 45, 50, x (c) 57, x, 51, 85

   (d) 21, x, 48, 528 (e) 9, x, x, 81

5. **Find the mean proportion for the following.**

   (a) 3, 27 (b) 4, 16 (c) 18, 2 (d) 8, 18

6. **Find the third proportion for the following.**

   (a) 8, 12 (b) 10, 20 (c) 24, 60

7. If 9, x and 81 are in continued proportion, find x.
8. If 28, 42 and y are in continued proportion, find y.
9. The weight of 18 litchis is the same as the weight of 6 oranges. How many oranges weigh as much as 45 litchis weigh?
10. If the cost of 30 pairs of gloves is ₹750, then find the cost of 16 pairs of gloves.
11. If a family of 8 persons is entitled for 6.4 kg of sugar per month, then find the entitlement of sugar for 10 persons.
12. A man earns ₹5600 in one week. Find the amount earned by him in 10 days.

UNITARY METHOD

Let us consider the following situation.

Bhavna went to the market to purchase notebooks. She purchased 3 notebooks and paid ₹21 for them.

A few natural questions arose in her mind. These were:

- What is the price of one notebook?
- How much it would have cost her, if she had purchased 5 notebooks?

Let us observe the solution of these questions.

Cost of 3 notebooks is ₹21.

Therefore, cost of 1 notebook = ₹21 ÷ 3 = ₹7.

Now, cost of 5 such notebooks

= ₹7 + ₹7 + ₹7 + ₹7 + ₹7 = ₹7 × 5 = **₹35**

We observe that in this method, first we found the value of one unit and then using that value of one unit, we found the value of required number of units. This method is known as **unitary method**.

Let us study some more examples.

Ex. 15. Rani pays ₹7500 as rent for 3 months. How much amount she has to pay for the whole year, if the rent per month remains the same?

Sol. Rent for 3 months = ₹7500

Therefore, rent for 1 month = ₹ $\frac{7500}{3}$

= ₹2500

So, rent for the whole year, *i.e.*, 12 months = ₹2500 × 12 = ₹30,000.

Thus, Rani has to pay ₹30,000 as rent for a whole year.

Alternate Method:

We can also solve the above problem using proportion.

Let the amount of rent for whole year be ₹x. Since number of months and amounts of rent form proportion,

∴ 3 months : 12 months : : ₹7500 : ₹x

⇒ $3 : 12 = 7500 : x$

⇒ $3 \times x = 12 \times 7500$

Thus, $x = \frac{12 \times 7500}{3}$ = ₹30,000.

Ex. 16. A truck requires 108 litres of diesel for covering a distance of 594 km. How much diesel will be required by the truck to cover a distance of 1650 km?

Sol. To cover a distance of 594 km, the diesel required = 108 litres

Therefore, to cover a distance of 1 km, the diesel required = $\frac{108}{594}$ litres

[Quantity of diesel on RHS]

Therefore, to cover a distance of 1650 km, the diesel required = $\frac{108}{594} \times 1650$ litres

= 300 litres

Ex. 17. The cost of 4 dozen bananas is ₹120. How many bananas can be purchased for ₹25?

Sol. We know that 1 dozen = 12 units

Therefore, 4 dozen = 4 × 12 = 48

Cost of 48 bananas = ₹120

Since we have to find the number of bananas, we change the statement as follows:

Number of bananas bought for ₹120 = 48

Number of bananas bought for ₹1 = $\frac{48}{120}$

Thus, number of bananas bought for ₹25

= $\frac{48}{120} \times 25 = 10$

Ex. 18. The temperature dropped 10 degree Celsius in the last 20 days. If the rate of drop in temperature remains the same, how many degrees will the temperature drop in the next 6 days?

Sol. Temperature dropped in last 20 days = 10°C

Temperature dropped in 1 day = $\frac{10}{20}$°C

∴ Temperature will drop in the next 6 days

= $\frac{10}{20} \times 6°C = 3°C$

Exercise 13.3

1. If the cost of 6 m of cloth is ₹150, find the cost of 4 m of cloth.
2. Cost of 7 kg wheat is ₹126. What will be the cost of 11 kg wheat?
3. Rani pays ₹8000 as rent for 4 months. How much rent she has to pay for a whole year, if the rent per month remains the same?
4. A truck requires 115 litres of diesel for covering a distance of 460 km. How much diesel will be required by the truck to cover a distance of 1460 km?
5. The cost of 7 kg rice is ₹168. What quantity of rice can be purchased for ₹312?
6. The cost of 2 dozen bananas is ₹80. How many bananas can be purchased for ₹20?
7. Kiran earns ₹3200 in 8 days. How much will she earn in 32 days?
8. The temperature dropped 5°C in the last 15 days. If the rate of drop in temperature remains the same, how many degrees will the temperature drop in the next 9 days?
9. The weight of 81 books is 27 kg. Find the weight of 54 such books.
10. If the cost of 20 kg sugar is ₹720, then find the cost of 7 kg sugar.
11. If the cost of 20 envelopes is ₹70, then find the number of envelopes which can be bought for ₹10.50.
12. If Vipul covers a distance of 28 km in 2 hours, then find the distance covered by him in 7 hours.
13. If the cost of 5 bars of washing soap is ₹82.50, then find the cost of one dozen such bars.
14. If a car travels 165 km in 3 hours, then how long will it take to travel 440 km with the same speed?
15. If the rent of a room for 3 months is ₹14,400, then find the room rent for a year.

Competency Based Exercise

21st CS

1. **Tick (✓) the correct answer.**

(a) There are '*b*' boys and '*g*' girls in a class. The ratio of the number of boys to the total number of students in the class is:

(i) $\frac{b}{b+g}$ (ii) $\frac{g}{b+g}$ (iii) $\frac{b}{g}$ (iv) $\frac{b+g}{b}$

(b) A textbook for class VI has 320 pages. The tenth chapter of the book runs from page 261 to 272. The ratio of the number of pages of this chapter to the total number of pages of the book is:

(i) 3 : 40 (ii) 3 : 80 (iii) 11 : 320 (iv) 272 : 320

(c) The first, third and fourth terms of a proportion are 27, 63 and 84 respectively. The second term of the proportion is:

(i) 36 (ii) 48 (iii) 63 (iv) 72

(d) 10 g caustic soda dissolved in 100 mL of water makes a solution of caustic soda. Amount of caustic soda required for 1 litre water to make a solution of same concentration is:

(i) 50 g (ii) 75 g (iii) 100 g (iv) 120 g

(e) If the ratio of two positive integers is 3 : 17 and their sum is 180, then the integers are:

(i) 17 and 163 (ii) 27 and 153 (iii) 29 and 151 (iv) 41 and 139

(f) A sum of money is divided into two parts in the ratio 3 : 5. If the sum of money is ₹504, then the smaller amount is:

(i) ₹168 (ii) ₹189 (iii) ₹252 (iv) ₹315

(g) A factory produces CFLs. If 1 out of every 1072 CFLs is defective and the factory produces 2,74,432 CFLs per day, then the number of defective CFLs produced each day is:

(i) 256 (ii) 272 (iii) 276 (iv) 356

(h) If an electric pole casts a shadow of length 30 m at the time when a 12 m high tree casts a shadow of length 16 m, then the height of the pole is:

(i) 36 m (ii) 22.5 m (iii) 42 m (iv) 48 m

2. Which is the greatest ratio among the ratios 2 : 3, 5 : 8, 75 : 121 and 40 : 25?

3. An office opens at 9 a.m. and closes at 6 p.m. with a lunch break of 45 minutes. What is the ratio of time period of the lunch break to the total time period in the office?

4. In a public school, the ratio of the number of non-classrooms to the classrooms is 3 : 17. If the number of non-classrooms is 30, then what is the total number of rooms in the school?

5. Jupiter and Saturn take 9 hours 56 minutes and 10 hours 40 minutes respectively for one complete spin about their respective axis. Find the ratio of the time taken by Jupiter to that by the Saturn in making one spin about their axes.

6. If the cost of 5 bars of washing soap is ₹177.50, then find the cost of one dozen such bars of washing soap.

7. Find the order of the numbers to get their ratios in proportion:

(a) 8, 25, 40, 5 (b) 12, 2, 3, 8

8. In a farmhouse, 1,11,360 tomato plants were planted in rows, each row having 64 plants. If 12 plants in each row were destroyed by a worm, then find the total number of tomato plants destroyed in the farmhouse.

9. Jeffrey and Sara shared 133 sweets in the ratio 4 : 3. How many less sweets did Sara get than Jeffrey?

10. Sex ratio is defined as the number of females per 1000 males in the population. If there are 13,720 females per 14,000 males in a town, then compute the sex ratio of the town.

11. Manav, Shourya and Gopesh share 450 stamps in the ratio 4 : 3 : 2. What are their shares?

12. Murli and Peter painted a wall in the ratio 9 : 11. Peter got ₹2310 for his work. How much did Murli get?

Challenge! 21st CS

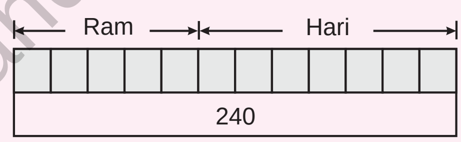
1. There are 72 vehicles in a parking lot. The number of cars, motorcycles and scooters are in the ratio 3 : 4 : 2. How many more motorcycles are there than the number of scooters in the parking lot?
2. Rajan and his sister Roohi share toffees in the ratio of 3 : 2. If Rajan gives 5 toffees to his sister, they would have the same number of toffees. How many toffees do they have altogether?
3. A tea merchant blends two varieties of tea costing him ₹396 per kg and ₹468 per kg in the ratio of their costs. If the weight of the mixture is 96 kg, then find the weight of each variety of tea.

Let's Work in Mind 21st CS

1. If a line segment of length 63 cm is divided in the ratio 4 : 5, then what is the length of the larger part?
2. If a quarterly fee for class VI in a public school is ₹7200, what is the monthly fee?
3. The earth rotates 360° about its axis in 24 hours. How many degrees will it rotate in 4 hours?
4. A car travels 240 km in 3 hours and a train travels 240 km in 2 hours. What is the ratio of the speed of the car to that of the train?
5. If 7 bowls cost ₹91, then what is the cost of 10 such bowls?

SMART TIME 21st CS

1. **Ram and Hari share 240 apples in the ratio 5 : 7.**



Ram's share = $\frac{5}{5+7} \times 240 = ?$

Hari's share = $\frac{7}{5+7} \times$ _______ = ?

2. Mala and Swati share 300 beads in the ratio 8 : 7. Find the share using the bar model.

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A)** : $7 : 1 \neq 1 : 7$

   Reason (R) : $7 : 1 = \frac{7}{1}$ and $1 : 7 = \frac{1}{7}$, and $7 > \frac{1}{7}$

2. **Assertion (A)** : Height of two ladders are in ratio 2 : 7. This means if height of one ladder is 200 cm, then height of another ladder is 700 cm.

   Reason (R) : $\frac{a}{b}$ can be expressed as $a : b$.

3. **Assertion (A)** : 40 : 60 is the same as 20 : 30.

   Reason (R) : $\frac{1}{3}$ and $\frac{2}{3}$ are equivalent ratios.

4. **Assertion (A)** : $4 : 6 :: 8 : 12$, then $4 \times 6 = 8 \times 12$.

   Reason (R) : Product of middle terms = Product of extreme terms.

5. **Assertion (A)** : 1 : 25 = 2 : 50

   Reason (R) : ₹1 : 25 p = 2 h : 50 min

6. **Assertion (A)** : Third proportion of 6 and 15 is 37.5.

   Reason (R) : $\frac{6}{15} = 6 : 15$

14 Symmetry

What Learners Will Achieve

- recognise symmetrical shapes, designs and objects in daily life.
- identify number of lines of symmetry for the given shapes, designs or objects.
- understand reflection symmetry.
- draw mirror image of a given figure.

Warm-up

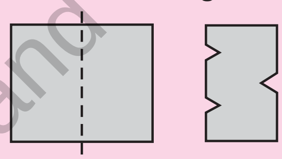
What we already know

AIL

When a piece of rectangular or square paper is folded into two halves, the two halves are symmetrical, *i.e.*, one half of the paper is the same as other half of the paper.

Now, try to solve the following.

1. Anil folds a rectangular sheet of paper, along the dotted line [see Fig. 14.1 (a)]. Then he makes the cuts in the paper as shown in Fig. 14.1 (b).



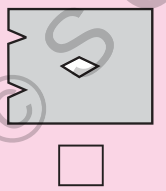
(a) (b)

Fig. 14.1

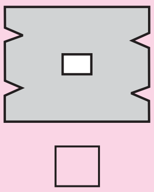
(a) What do you expect on unfolding the paper?

(b) Identify the correct paper Anil obtained on unfolding.

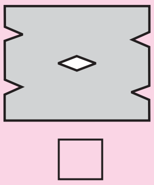
(i)



(ii)

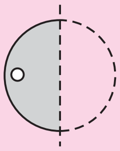


(iii)

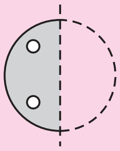


2. Manav takes some circular cut-outs and make holes using a punching machine. How will it appear when he unfolds? Mark holes on the other side of the paper.

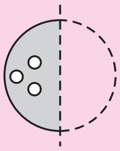
(a)



(b)



(c)



UNDERSTANDING SYMMETRY

Let us observe the following pictures.

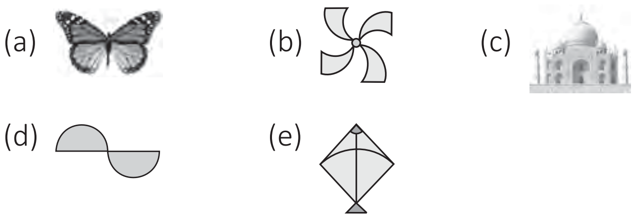


Fig. 14.2

These pictures look beautiful because of their ***symmetrical shapes***.

Some of the above pictures appear to be symmetrical about a line like (a), (c) and (e), while in other cases, no such line exists. In this chapter, we shall learn the concept of **line symmetry** also known as **linear symmetry**. The concept involves the study of pictures or figures symmetrical about a line.

Line Symmetry and Line of Symmetry

Consider the picture given in Fig. 14.2 (e). Can we find a line which divides the picture in two halves such that the two halves match exactly on both sides? From Fig. 14.3 (b), we observe that such a line does exist. In fact, if we fold the shape along this line the two halves coincide. If this happens, we say that the picture has a line of symmetry and both halves are the mirror image of each other.

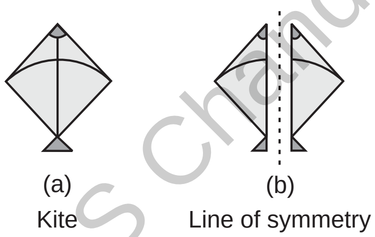


Fig. 14.3

Let us now try to do the same thing with the picture in Fig. 14.2 (b). We observe that, no matter how the picture is folded, it is not possible to split the picture into two halves which match exactly. In such cases, we say that the picture [14.2 (b)] has no line of symmetry.

Thus, if we can fold a picture (or figure) in two halves such that the two halves match each other exactly, then the picture (or figure) is said to have a line symmetry.

The line from where we folded the picture is called ***line of symmetry*** or ***axis* of symmetry** of the picture.

Illustration 1: Nature has plenty of things having line symmetry in their shapes. Look at the following symmetrical figures.

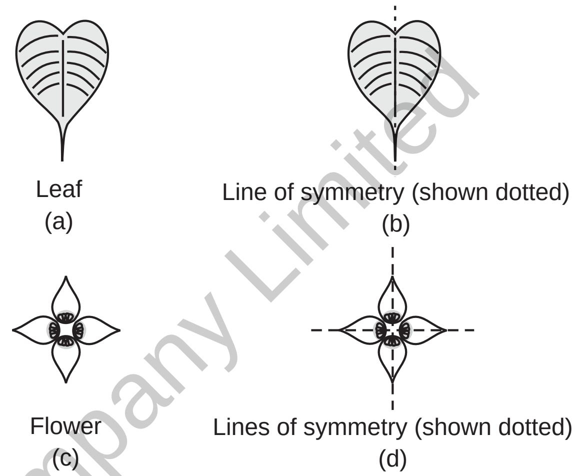


Fig. 14.4

Creating Symmetrical Figures

LS

(i) Using Ink: Take a piece of a paper. Fold it in halves and then unfold it [Fig. 14.5 (a)].

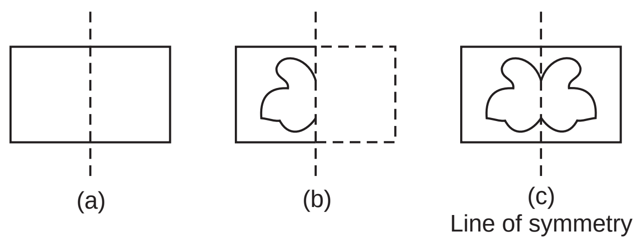


Fig. 14.5

Draw a picture on one half with ink and then press the halves together [Fig. 14.5 (b)]. Unfold and see that the resulting figure is symmetrical as shown in Fig. 14.5 (c). It has a line of symmetry. The line of symmetry for the figure is the line from where we folded the paper.

(ii) Using Thread: Instead of ink, we can also use a thread dipped in colour (paint) to draw a symmetrical figure (Fig. 14.6).

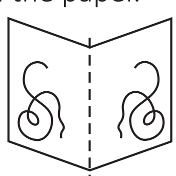


Fig. 14.6

Ex. 1. **Identify whether lines *l* and *m* are lines of symmetry of the given figure or not.**

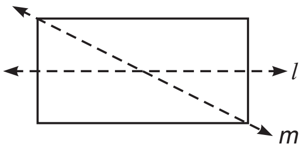


Fig. 14.7

Sol. The line *l* divides the figure in two halves such that both the halves match exactly.

The line *m* does not divide the figure in two halves such that both the halves match exactly.

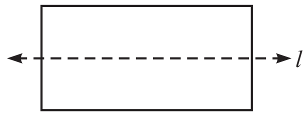


Fig. 14.8

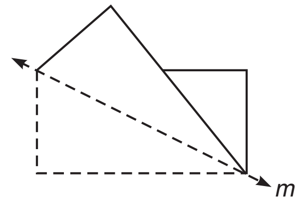


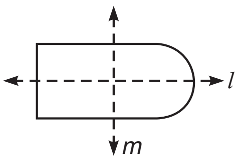
Fig. 14.9

Therefore, *l* is a line of symmetry of the figure but *m* is not a line of symmetry of the given figure.

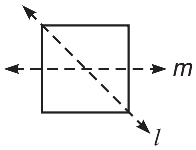
Exercise 14.1

1. **Identify whether the lines *l* and *m* are the lines of symmetry of the given figures or not.**

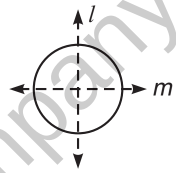
(a)



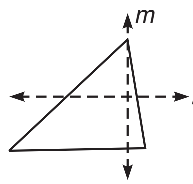
(b)



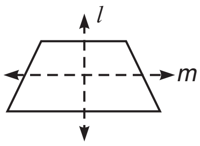
(c)



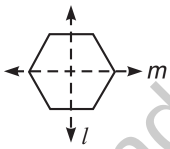
(d)



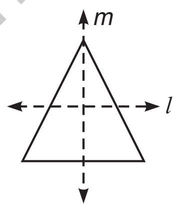
(e)



(f)

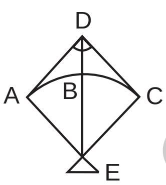


(g)

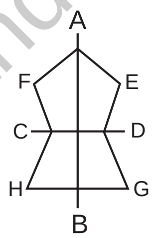


2. **Name the line(s) of symmetry about which the following figures are symmetrical.**

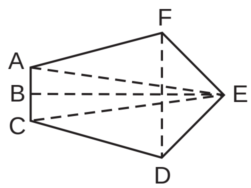
(a)



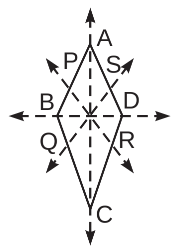
(b)



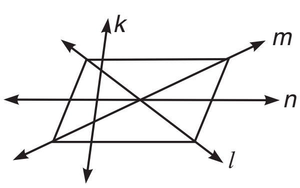
(c)



(d)



3. Can you name the line(s) of symmetry for the given figure?



4. How many instruments in your geometry box are symmetrical?
5. Can you create symmetrical shape using set squares?

NUMBER OF LINES OF SYMMETRY

Let us perform an activity to explore the number of lines of symmetry in geometrical shapes.

Let Us Do

EL

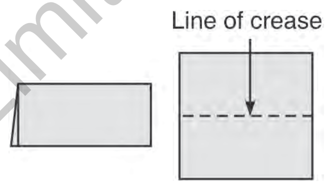
Objective: To examine the geometrical shapes for the number of lines of symmetry by paper folding.

(a) Square (b) Rectangle (c) Circle (d) Rhombus
(e) Trapezium (f) Kite (g) Equilateral triangle
(h) Isosceles triangle (i) Scalene triangle

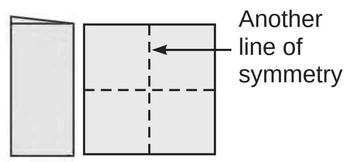
Materials required: Cut-outs of all geometrical shapes mentioned above.

Procedure:

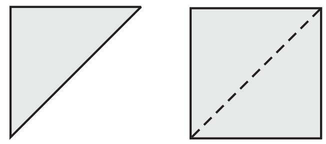
(a) For Square



Step 1: Fold the square along the length so that the opposite sides coincide. Unfold the square and notice the line of crease. Line of crease along the length is a line of symmetry of the square.

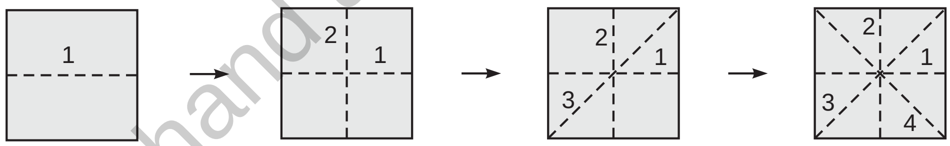


Step 2: Take the unfolded square and now fold it along the breadth so that opposite edges coincide. The new line of crease obtained is another line of symmetry.



Step 3: Now, fold the square along the opposite corners so that the remaining two opposite corners coincide. Repeat the process with the other two opposite corners.

Observation: A square has four lines of symmetry.



Hence, we can say that a square has _________ lines of symmetry.

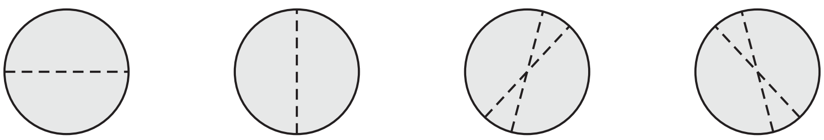
(b) For Rectangle

Repeat the same procedure with the rectangle and find the number of lines of symmetry.

(c) For Circle

Can you repeat the same with a circle?

A circle has no corners. Take any two points on the circumference of the circle on any diameter. Fold the circle along this diameter.



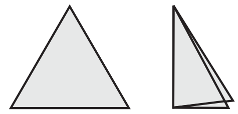
Is this diameter a line of symmetry?

Observation: Can you say every diameter of a circle is a line of symmetry of the circle?

(d) – (f) Repeat the same procedure with rhombus, trapezium and kite to determine the number of lines of symmetry.

(g) For Triangles

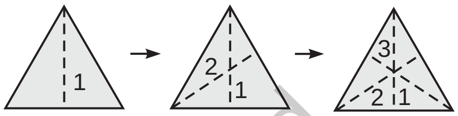
Take a cut-out of equilateral triangle. Fold it through any one vertex so that the other two vertices meet each other. Draw a dotted line along the crease of fold. Observe that a triangle is divided into two equal parts and they overlap each other when folded along the crease.



This line of fold is called the line of symmetry of the triangle.

Fold similarly through other two vertices too.

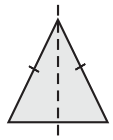
Observation: Equilateral triangle has three lines of symmetry.



(h) Repeat the same procedure with all vertices of isosceles triangle.

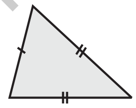
Observation: There is only one line of symmetry in isosceles triangle.

Fold along vertex common to equal sides divides the triangle into two equal halves.



(i) Repeat the same procedure with all the three vertices of scalene triangle.

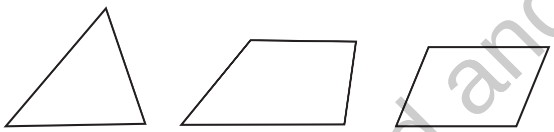
Observation: There is no line of symmetry in a scalene triangle.



Figures with No Lines of Symmetry

Every object (or figure) we see is not symmetric (or symmetrical). There are objects (figures) which have no lines of symmetry, *i.e.*, they are asymmetric (not symmetric).

Following figures have no lines of symmetry.



Scalene triangle (a) | Trapezium (b) | Parallelogram (c)

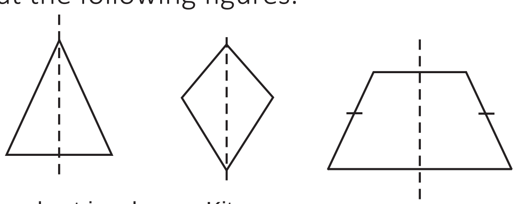
Fig. 14.10

Note

Some figures are symmetrical but still they do not have any line of symmetry [see Fig. 14.2 (b) and Fig. 14.2 (d)].

One Line of Symmetry

Look at the following figures.



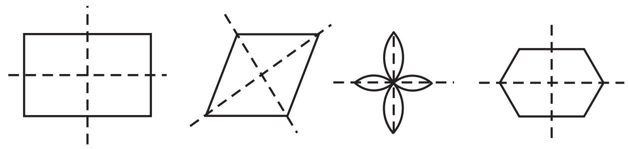
Isosceles triangle (a) | Kite (b) | Isosceles trapezium (c)

Fig. 14.11

The figures shown in Fig. 14.11 have only one line of symmetry. Dotted line shows the line of symmetry.

Two Lines of Symmetry

Look at the following figures.



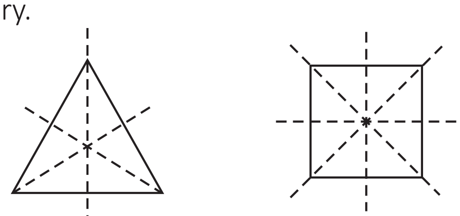
Rectangle (a) | Rhombus (b) | Design (c) | Hexagon (d)

Fig. 14.12

The figures shown above have two lines of symmetry. Dotted lines show the lines of symmetry.

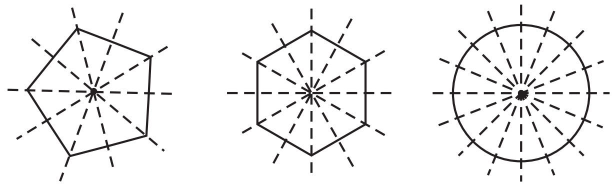
Multiple Lines of Symmetry

The figures which have more than two lines of symmetry are known to have multiple lines of symmetry.



An equilateral triangle has 3 lines of symmetry. (a)

A square has 4 lines of symmetry. (b)



A regular pentagon has 5 lines of symmetry.
(c)

A regular hexagon has 6 lines of symmetry.
(d)

A circle has infinite lines of symmetry.
(e)

Fig. 14.13

> **Note**
>
> Every line that passes through the centre of the circle is a line of symmetry.

Let us study some more examples.

Ex. 2. **Identify the line of symmetry in the letter A of the English alphabet.**

Sol. Vertical line of symmetry

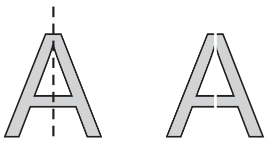


Fig. 14.14 (a)

Horizontal line of symmetry

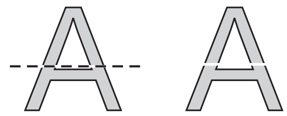


Fig. 14.14 (b)

Therefore, letter A has only vertical line of symmetry.

Ex. 3. **Identify the line(s) of symmetry in the letter H of the English alphabet.**

Sol. Vertical line of symmetry

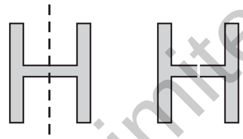


Fig. 14.15 (a)

Horizontal line of symmetry

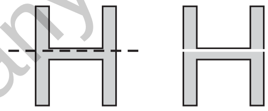
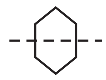
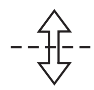


Fig. 14.15 (b)

Therefore, letter H has two lines of symmetry— vertical as well as horizontal.

Exercise 14.2

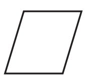
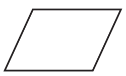
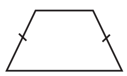
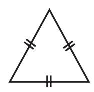
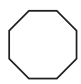
1. **Identify if the line divides the following figures into halves.**

(a)  (b)  (c)  (d)  (e) 

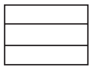
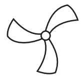
2. **Identify the line(s) of symmetry in the given letter of the English alphabets.**

(a) B (b) K (c) M (d) F (e) T
(f) Z (g) E (h) R (i) V (j) Q
(k) X (l) I (m) O (n) N (o) W

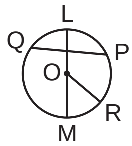
3. **How many line(s) of symmetry do the following figures have?**

(a)  (b)  (c)  (d)  (e)  (f)  (g) 

4. **Are the following pictures symmetrical? If yes, draw line(s) of symmetry.**

(a)  (b)  (c)  (d)  (e)  (f) 

5. In the given figure, identify the line of symmetry.



6. Answer the following questions.

(a) How many lines of symmetry does a parallelogram have?

(b) Name two polygons which have exactly two lines of symmetry.

(c) Which of the letters of the English alphabet has two lines of symmetry?

(d) How many lines of symmetry does a circle have?

(e) How many lines of symmetry does a kite have?

REFLECTION AND LINE SYMMETRY

Mirror reflection and **line symmetry** are closely related to each other.

The picture (see Fig. 14.16) shows the reflection of letter M in a mirror. Imagine that the mirror is placed along the dotted line [Fig. 14.16 (b)]. Observe the letter and its image again.

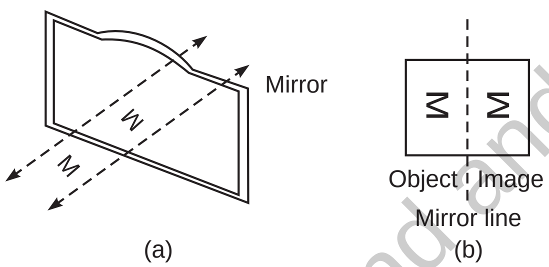


Fig. 14.16

On folding the picture along the dotted line, we see that the object and its image are **symmetrical** (or **symmetric**) with reference to the dotted line, where the mirror has been placed. This line is known as the **mirror line**. If an object is placed in front of a mirror, then it would appear as if the object is behind the mirror as well. The object appearing behind the mirror is called (virtual) **image** of the object in front of the mirror. The distance between the object and the mirror is called the **object distance** and the distance between the image and the mirror is called the **image distance**. We will learn in the higher classes that on plane mirrors **the object distance is equal to the image distance**. For example, if you are standing at a distance of 100 cm from the mirror, your image will appear to be made at a distance of 100 cm behind the mirror. If you come closer to the mirror, your image will also appear to come closer to the mirror. How can we use this to determine the line symmetry or to form objects (figure) that have a line symmetry?

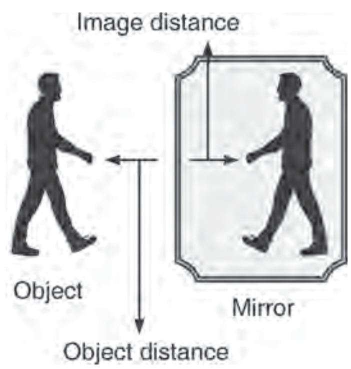


Fig. 14.17

REFLECTION OF SIMPLE FIGURES

Reflection of a Point

Let us see how to find reflection of a point.

Suppose, a point P is in front of a mirror line *m*. To find the image of point P after reflection, we follow these steps:

1. Draw a line segment, say PO, perpendicular to line *m*.
2. Produce PO to point P′ such that PO = OP′.

Thus, P′ is the image of P about line *m*.

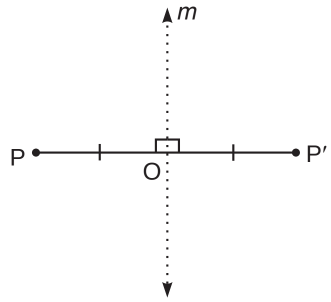


Fig. 14.18

Think

Line *m* is also the perpendicular bisector of line segment *PP'*, *i.e.,* the perpendicular bisector of a line segment is its line of symmetry.

Reflection of a line segment

Draw a line segment AB and a line *l,* as shown in the Fig. 14.19. Let us draw the reflection of AB about line *l* and explore its properties.

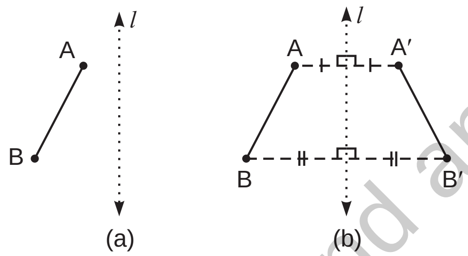


Fig. 14.19

Mark symmetric point A′ such that points A and A' are equidistant from *l*. Similarly, mark point B' such that B and B' are equidistant from *l* (see Fig. 14.19). On measuring the line segments AB and A'B', we observe that **AB = A'B'**.

Further, on folding the paper along *l*, we find that line segments AB and A'B' completely cover each other. In other words, line segments AB and A'B' are **symmetric** to each other with respect to the line *l*.

Let us study some more examples.

Ex. 4. **Complete the figure given on the right so that line "*l*" becomes the line of symmetry for the completed figure.**

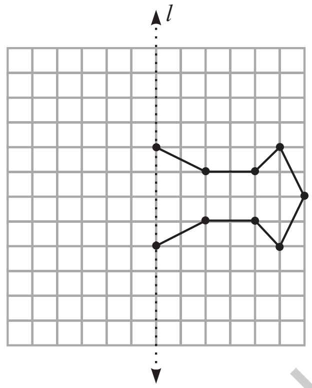


Fig. 14.20

Sol. The figure can be completed by drawing points symmetric to different corners (points) with respect to the line "*l*" as shown in Fig. 14.21.

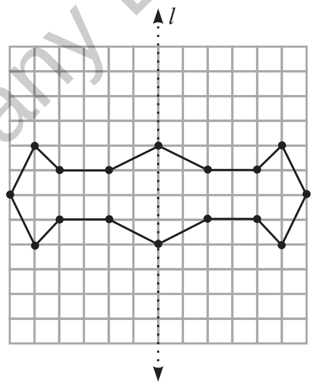
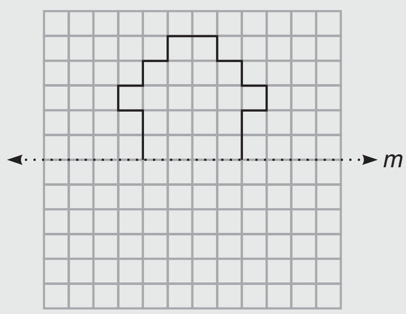


Fig. 14.21

Skill Check

Complete the following figure so that the line "*m*" becomes the line of symmetry of the whole figure.



Ex. 5. **Complete the given figure (see Fig. 14.22), so that the lines *l* and *m* become the two lines of symmetry for the completed figure.**

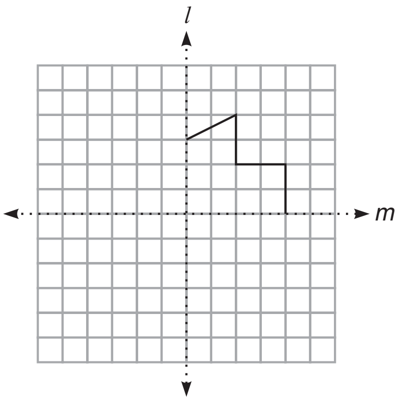


Fig. 14.22

Sol. First draw points symmetric to each corner with respect to line *l* and obtain the figure as shown in Fig. 14.23.

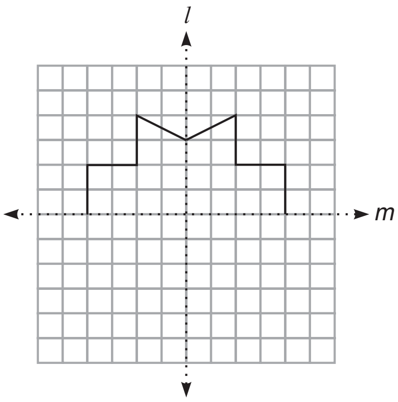


Fig. 14.23

Now, draw points symmetric to each corner of this figure with respect to line *m* shown in Fig. 14.23. The resulting Fig. 14.24 is the required completed figure having two lines of symmetry *l* and *m*.

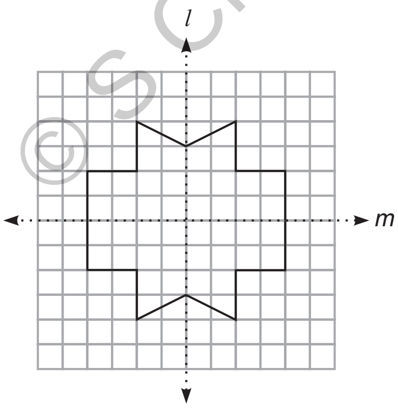


Fig. 14.24

Ex. 6. In the figure given below, draw points A', B' and C' symmetric respectively to vertices A, B and C of a ΔABC with respect to line *l*. Join A'B', B'C' and C'A'. What figure have you obtained? Measure AB, A'B', BC, B'C', CA and C'A'. Also, measure ∠A, ∠A', ∠B, ∠B', ∠C and ∠C'. What do you observe? Check your observations by folding the paper along *l*.

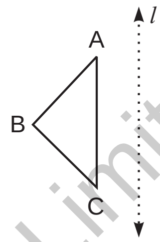


Fig. 14.25

Sol. Symmetric points A', B' and C' are marked such that A and A', B and B', C and C' are equidistant from the line *l* as shown in the Fig. 14.26.

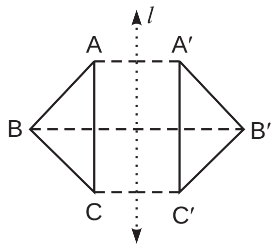


Fig. 14.26

On joining A'B', B'C' and C'A', we obtain ΔA'B'C'. On measurement, we find that AB = A'B', BC = B'C', CA = C'A'. Also, ∠A = ∠A', ∠B = ∠B' and ∠C = ∠C'. On folding the paper along *l*, we find that triangles ABC and A'B'C' completely cover each other.

In other words, triangles ABC and A'B'C' are symmetric to each other with respect to the line *l*.

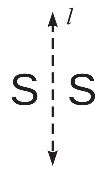
> **Note**
>
> The lengths and angles of an object (figure) and the corresponding lengths and angles of the image (symmetric figure) with respect to a line (mirror) remain the same.

Exercise 14.3

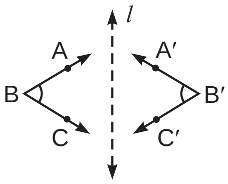
1. In which of the following, the figure on the left is not the image of the figure on the right with respect to the mirror line *l* ?

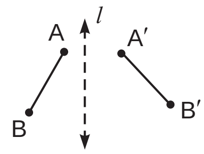
(a)

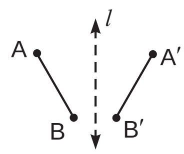

(b)

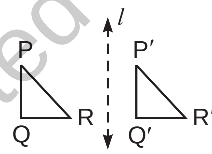

(c)


(d)

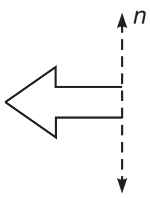

(e)


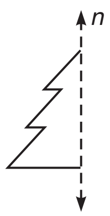
(f)


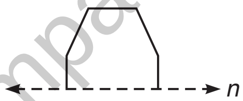
(g)


(h)


2. Complete the following figures so that line '*n*' becomes the line of symmetry of the completed figure.

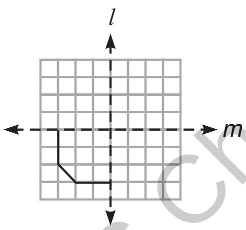
(a)


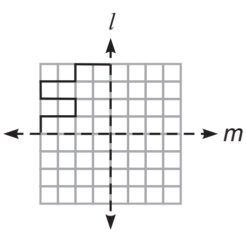
(b)


(c)


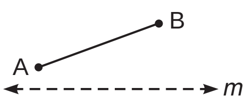
(d)

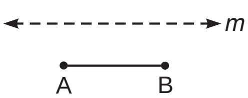

3. Complete the figures so that the lines *l* and *m* become the two lines of symmetry of the completed figure.

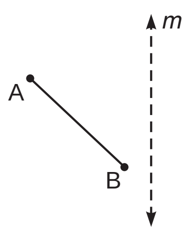
(a)


(b)


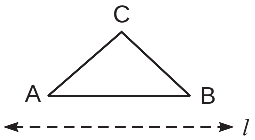
4. Draw points A' and B' symmetric to A and B respectively with respect to the line *m* in the following. Measure AB and A'B'. What do you observe?

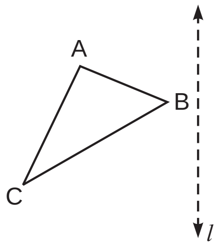
(a)


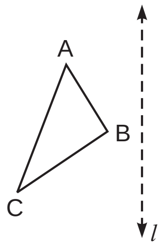
(b)


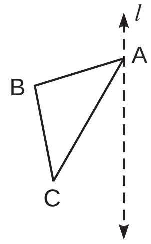
(c)


5. **Draw points A', B' and C' symmetric respectively to vertices A, B and C of a △ABC with respect to the line *l* in the following figures. Measure AB, A'B', BC, B'C' and CA, C'A'. What do you observe? Also, measure ∠A, ∠B, ∠C, ∠A', ∠B' and ∠C'. What do you observe?**

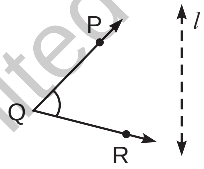
(a)


(b)


(c)


(d)


6. Draw points P', Q' and R' symmetric to points P, Q and R respectively lying on an angle PQR with respect to the line *l* in the given figure. Join Q'P' and Q'R', and measure angles PQR and P'Q'R'. What do you observe?



Competency Based Exercise

21st CS

1. **Tick (✓) the correct answer.**

(a) The number of line(s) of symmetry in a protractor is:
(i) 0 (ii) 1 (iii) 2 (iv) more than 2

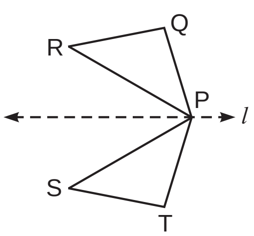
(b) Out of the digits 0, 1, 2 and 3, the digit having only one line of symmetry is:
(i) 1 (ii) 2 (iii) 3 (iv) 0

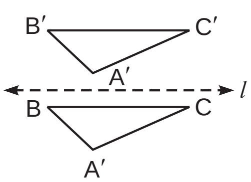
(c) The number of lines of symmetry in a regular hexagon is:
(i) 12 (ii) 6 (iii) 5 (iv) 4

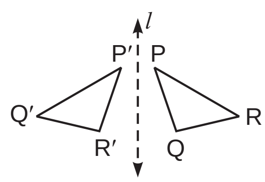
(d) The number of line(s) of symmetry in a 30° – 60° – 90° set square is:
(i) 0 (ii) 1 (iii) 2 (iv) 3

(e) Which of the following letters of the English alphabet has both horizontal and vertical lines of symmetry?
(i) R (ii) X (iii) T (iv) L

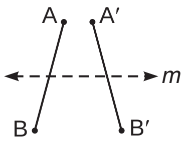
2. **In which of the following figures, the figure on one side of the line *l* is symmetric to the figure on the other side with respect to the line *l*?**

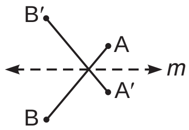
(a)


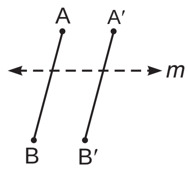
(b)


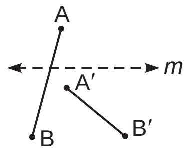
(c)


3. Which of the following figures depicts mirror images in the mirror line *m*?

(a) 

(b) 

(c) 

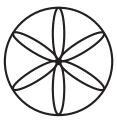
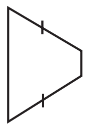
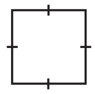
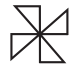
(d) 

4. In the word 'SYMMETRY', find the number of letters having no lines of symmetry.

5. In capital letters of the English alphabet, how many letters have more than one line of symmetry?

6. Name the two-digit number(s) having line(s) of symmetry.

7. In the following figures, find the number of line(s) of symmetry.

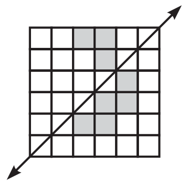
(a)  (b)  (c)  (d)  (e)  (f) 

8. Which letters of the word MATHEMATICS are not the mirror images of itself in a mirror line? 

Challenge!

21st CS

1. If the measure of an angle between two adjacent front-wheel spokes is 30°, then how many spokes are there in the wheel? How many axes of reflection symmetry does the wheel have?
2. How many minimum blocks should to be shaded to make the design symmetrical about the given line?



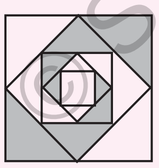
SMART TIME

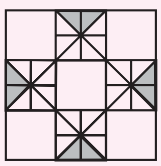
AIL

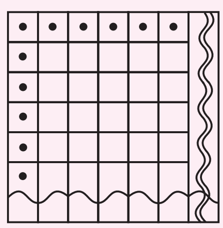
1. A die has six faces as shown below:

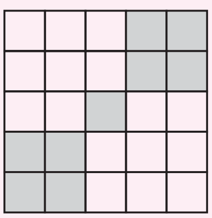
How many axes of reflection symmetry are there for each face?

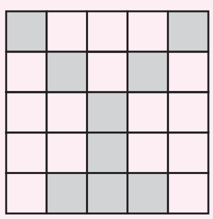
2. Below are given some quilt patterns. How many axis of symmetry are there?

(a) 

(b) 

(c) 

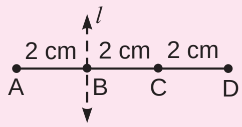
(d) 

(e) 

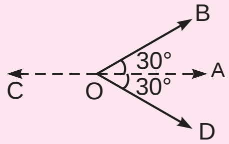
Let's Work in Mind

21st CS

1. Which line segment is symmetrical about line *l* in the given figure?



2. Is line AC the axis of symmetry of ∠BOD?



3. How many line(s) of symmetry does an isosceles obtuse triangle have?
4. Name any instrument which is not symmetric in your geometry box.

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A)** : A rectangle has three lines of symmetry.
   Reason (R) : Line of symmetry divides the rectangle into two equal halves.
2. **Assertion (A)** : Diameter is line of symmetry for a circle.
   Reason (R) : Line of symmetry divides the rectangle into two equal halves.
3. **Assertion (A)** : A square has two lines of symmetry.
   Reason (R) : Line of symmetry divided the rectangle into two equal halves.
4. **Assertion (A)** : Scalene triangle has no lines of symmetry.
   Reason (R) : All sides of a scalene triangle are equal.
5. **Assertion (A)** : Circle has infinite lines of symmetry.
   Reason (R) : In a circle, infinite number of diameters can be drawn.
6. **Assertion (A)** : Rhombus has two lines of symmetry.
   Reason (R) : Diagonals of a rhombus are not lines of symmetry.
7. **Assertion (A)** : A polygon of six sides has six lines of symmetry.
   Reason (R) : Line of symmetry divides the polygon into two equal halves.
8. **Assertion (A)** : A regular polygon of six sides has six lines of symmetry.
   Reason (R) : Line of symmetry divides the polygon into two equal halves.

15 Practical Geometry

What Learners Will Achieve

- recognise and name each instrument in geometry box and its use.
- construct a circle when radius is given.
- construct the copy of a given line segment and a line segment of the given length.
- construct a perpendicular and a perpendicular bisector of a line segment.
- construct an angle of a given measure using protractor.
- construct a copy of an angle of unknown measure using compasses and ruler.
- construct angle bisector using compasses.
- construct angles of measures 60°, 30°, 120°, 90° and 45°.

Warm-up

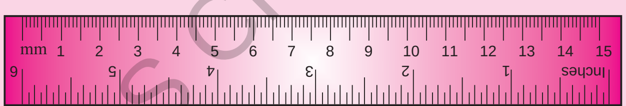
Architectural drawing of some buildings or machines to construct any building or to create even small part of a machine, drawing are required to project complete layout. Drawing should be accurate so that exact measure of each part of machine or building can be produced with great accuracy.

Architects, engineers and technicians have to make exact drawings of shapes that may include line segment, angle and their bisectors, circles, arcs and other similar shapes. We need certain special tools (or instruments) for the same.

Now, try to solve the following.

Write the names of the geometrical instruments shown below.

1.



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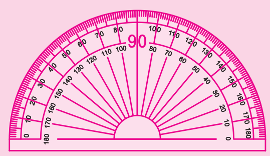
2.



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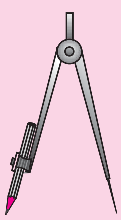
3.



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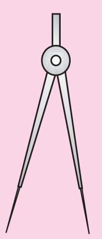
4.



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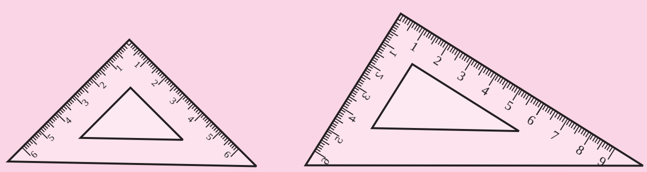
5.



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GEOMETRICAL INSTRUMENTS

Geometrical instruments are the tools to draw different types of geometric shapes.

The Ruler

A **ruler** is a rectangular strip of variable length. The straight edges along the length are used to draw line segments. The ruler in our geometry (instruments) box is graduated in centimetres along one edge and in inches along the other edge (Fig. 15.1).

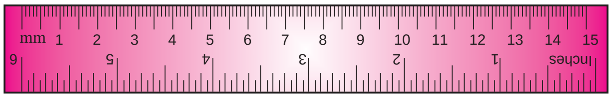


Fig. 15.1

It is used to measure lengths of the given line segments and also to construct them.

The Compasses

A **pair of compasses** or simply **compasses** is an instrument that has a pointer on one end and provision for placing a pencil on the other (Fig. 15.2).

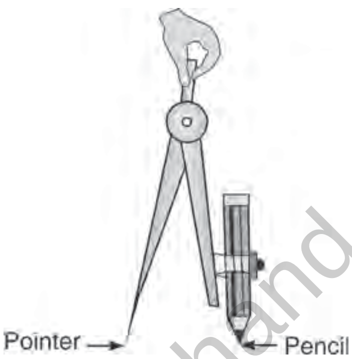


Fig. 15.2

It is used to mark off given lengths and to draw circles and arcs.

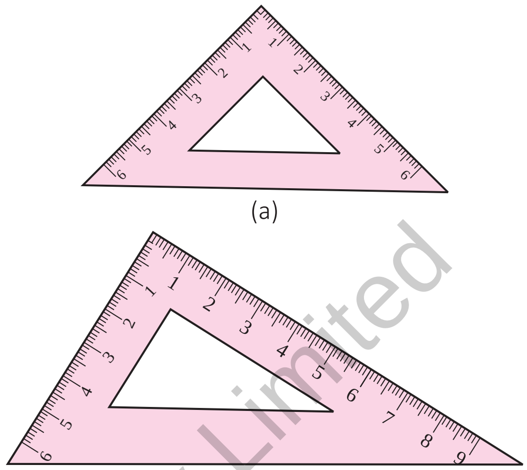
The Divider

A **divider** looks similar to a pair of compasses. It has a pair of pointers which can be adjusted as the requirement (Fig. 15.3). It is used to compare lengths.

Pair of pointers
Fig. 15.3

Set Squares

The following instruments found in a geometry box are known as **set squares**.



(a)

(b)

Fig. 15.4

- Measures of angles of set square in Fig. 15.4 (a) are 45°, 90° and 45°.
- Measures of angles of set square in Fig. 15.4 (b) are 30°, 90° and 60°.

Clearly, the graduated edges of set squares make a right angle and so are perpendicular to each other.

We may construct perpendicular lines using set squares.

The Protractor

A **protractor**, shown in Fig. 15.5, is an instrument used for measuring a given angle or for constructing an angle of the given magnitude (measure).

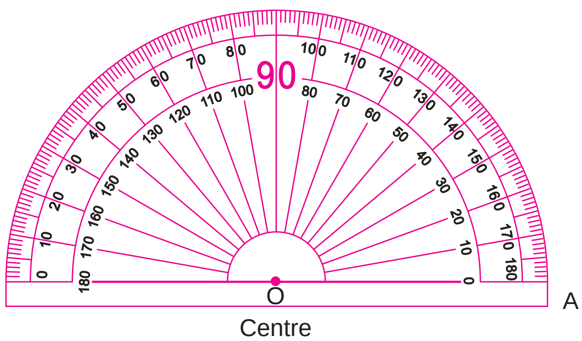


Fig. 15.5

It is semicircular in shape and is usually made of transparent plastic so that markings are visible when it is placed over straight lines.

It has degree marks on the curved edge from 0° to 180° and from 180° to 0° which enables us to read the measure of an angle or construct an angle of a given measure.

Note

There are two scales along the edge of the protractor the inner scale (0° – 180°) and the outer scale (180° – 0°).

DID YOU KNOW?

A bevel protractor is a graduated circular protractor with one pivoted arm, used for measuring or marking off angles. It has wide application in architectural and mechanical drawing. It is also used by footmakers.

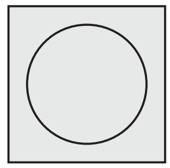
Let Us Do EL

Objective: To create a protractor by paper folding

Materials required: A4 size coloured sheet, a pair of scissors and scale

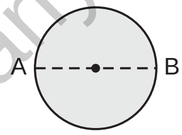
Procedure:

Step 1: Draw a circle of any diameter on a coloured sheet of paper [see Fig. 15.6 (a)].



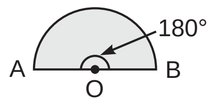
(a)

Step 2: Draw its diameter and cut the circle [see Fig. 15.6 (b)].



(b)

Step 3: Fold this circle along diameter AB. Get a semicircle with diameter AB [see Fig. 15.6 (c)].



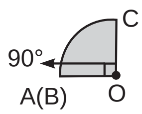
(c)

Observe:

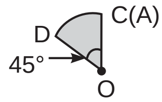
Angle at O is a straight angle.

$\therefore \angle AOB = 180°$

Step 4: Fold the semicircle so that A and B coincide.



(d)



(e)

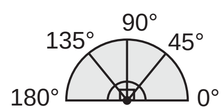
Observe:

$\angle AOC = 90°$ [see Fig. 15.6 (d)]

Step 5: Fold the quarter circle obtained so that A and C coincide.

Observe:

$\angle AOD = 45°$ [see Fig. 15.6 (e)]



(f)

Fig. 15.6

Step 6: Unfold all folds and draw lines along each crease.

Conclusion:

With the help of this paper protractor, you can now draw/measure angles of 45°, 90°, 135° and 180°.

CONSTRUCTING A CIRCLE

Let us construct a circle when its radius is known.

Given: Radius = 2 cm.

To construct a circle of radius 2 cm, follow these steps:

Step 1: Open the compasses, legs to measure the required radius of 2 cm.

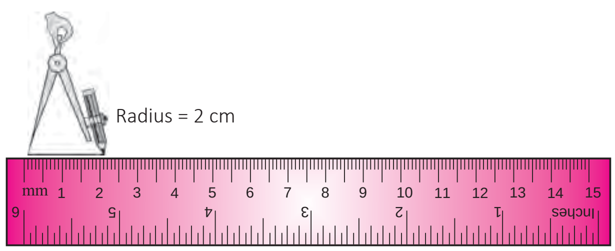


Fig. 15.7 (a)

Step 2: Mark a point with a sharp pencil where we want the centre of the circle to be and name it as O.

Fig. 15.7 (b)

Step 3: Place the pointer of the compasses at the point O.

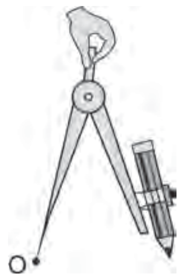


Fig. 15.7 (c)

Step 4: Turn the compasses slowly to draw the circle. Be careful to complete the movement around in one instance.

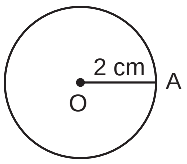


Fig. 15.7 (d)

Thus, we have drawn a circle with centre O and radius OA = 2 cm.

CONSTRUCTING A LINE SEGMENT

Constructing the Copy of a Given Line Segment

On a line *m*, let us construct a line segment equal in measure to a given line segment.

Given: Line segment AB.

Fig. 15.8 (a)

To construct line segment CD such that CD = AB, follow these steps:

Step 1: Draw a line *m*.

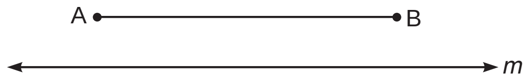


Fig. 15.8 (b)

Step 2: With the pointer of the compasses at A, open it for the required radius AB.

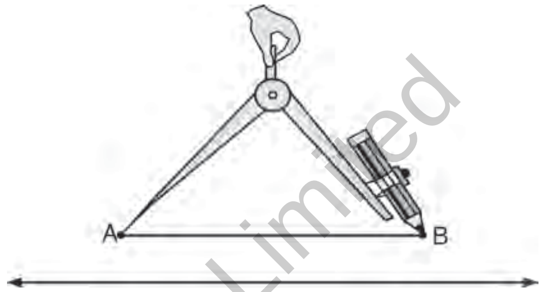


Fig. 15.8 (c)

Step 3: Locate any point on the line *m* and label it as C.

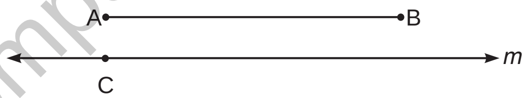


Fig. 15.8 (d)

Step 4: With the pointer of the compasses at C and using radius AB, draw an arc intersecting line *m* at D [see Fig. 15.8 (e)].

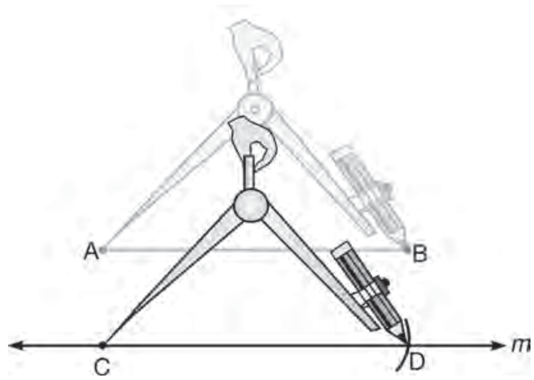


Fig. 15.8 (e)

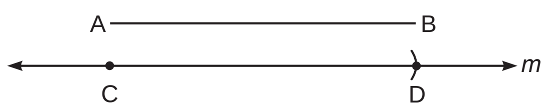


Fig. 15.8 (f)

Thus, CD = AB [see Fig. 15.8 (f)].

Constructing a Line Segment of the Given Length

Let us construct a line segment of a given length, say 4.5 cm.

Given: Length of the line segment is 4.5 cm.

To construct a line segment of length 4.5 cm, follow these steps:

Step 1: Draw a line *l*. Mark a point A on it as shown below.

Fig. 15.9 (a)

Step 2: Place the compasses' pointer on the zero mark of the ruler. Open it to place the pencil point up to the 4.5 cm mark.

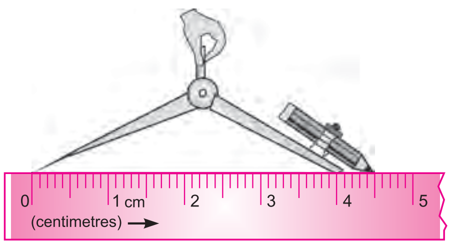


Fig. 15.9 (b)

Step 3: Without changing the opening of the compasses, place the pointer on A and swing an arc to cut line at B.

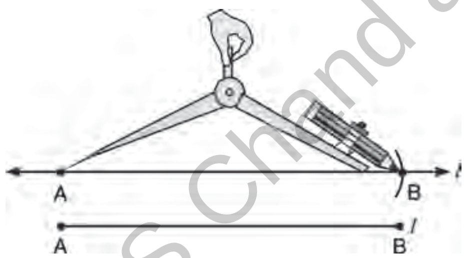
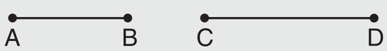


Fig. 15.9 (c)

Thus, AB is a line segment of the required length.

Skill Check

- Draw a circle of radius 3 cm.
- Draw a line segment of length 6.5 cm.
- Draw a line segment PQ such that PQ = AB + CD.



- Given AB = 4 cm and CD = 1.6 cm. Draw a line segment XY such that XY = AB – CD.

CONSTRUCTING PERPENDICULAR AND PERPENDICULAR BISECTOR

Perpendicular Lines

We know that two lines in a plane either intersect or do not intersect (*i.e.*, parallel) each other. When two lines intersect each other, four angles are formed.

If one of these angles is a right angle, the remaining three angles are also right angles and the lines are said to intersect at a right angle.

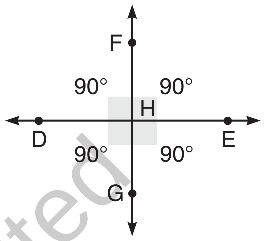


Fig. 15.10

These lines intersecting at a right angle are called **perpendicular lines**.

In Fig. 15.10, DE is perpendicular to FG and FG is perpendicular to DE.

Symbolically, it is written as DE ⊥ FG, read as line DE is perpendicular to line FG; likewise FG ⊥ DE.

Constructing Perpendicular to a Line through a Point on it

Let us construct a line perpendicular to a given line through a point on the given line.

Using ruler and compasses

Given: Line *l* and a point P on this line.

Fig. 15.11 (a)

To construct a perpendicular PQ to the line *l*, follow these steps:

Step 1: With given point P as centre and a convenient radius, draw an arc intersecting the line *l* at two points A and B.

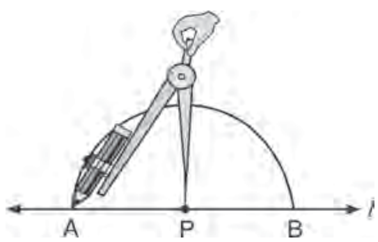


Fig. 15.11 (b)

Step 2: With A and B as centres and a radius greater than AP, draw two arcs, which cut each other at Q.

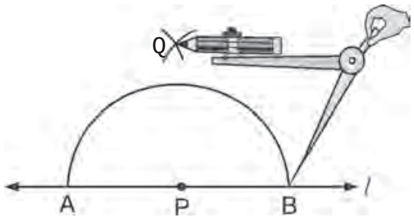


Fig. 15.11 (c)

Step 3: Join PQ.

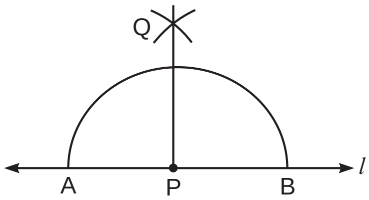


Fig. 15.11 (d)

Thus, PQ ⊥ *l*.

Using ruler and set square

Given: A line XY and point A on it.

Fig. 15.12 (a)

To construct a perpendicular AB to the line XY, follow these steps:

Step 1: Put the right angle vertex of a set square at point A on XY in such a way that another vertex of the set square fits on part of XY towards Y (or towards X).

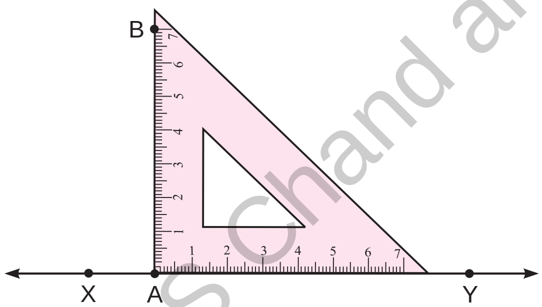


Fig. 15.12 (b)

Step 2: On the vertical side of the set square, select a point B and then remove the set square.

Step 3: Using a ruler, draw AB passing through points A and B.

So, AB ⊥ XY.

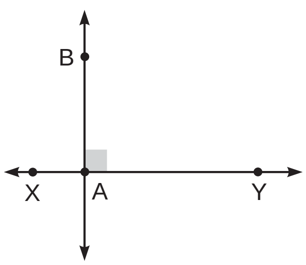


Fig. 15.12 (c)

Thus, AB is the required line perpendicular to the line XY at A.

Constructing Perpendicular to a Line through a Point not on it

Let us construct a line perpendicular to a given line through a point not on the given line.

Using ruler and compasses

Given: Line *l* and point P not on line *l*.

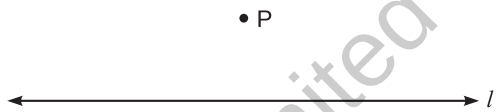


Fig. 15.13 (a)

To construct line PQ perpendicular to the line *l*, follow these steps:

Step 1: With P as centre, draw an arc intersecting the line *l* at two points, D and E.

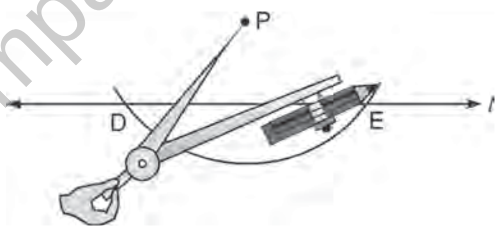


Fig. 15.13 (b)

Step 2: With E as centre and more than $\frac{1}{2}$ DE as radius, draw an arc below line *l*.

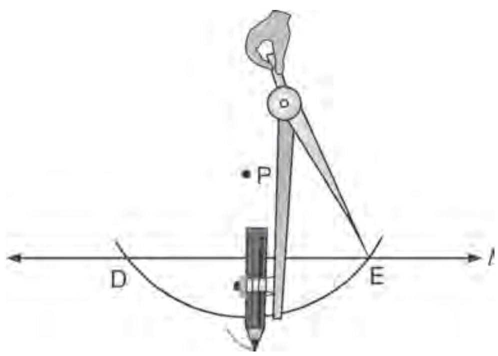


Fig. 15.13 (c)

Step 3: With D as centre and the same radius as in step 2, draw another arc intersecting the previous arc at a point Q.

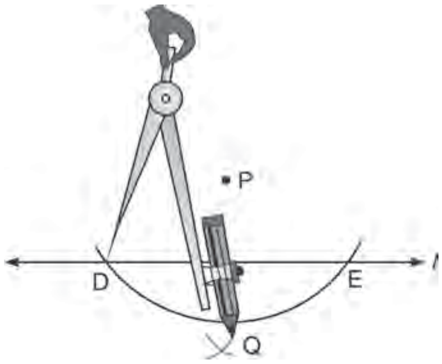


Fig. 15.13 (d)

Step 4: Join P and Q to draw line PQ.

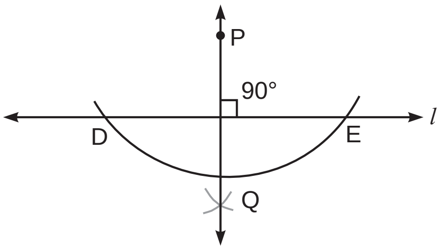


Fig. 15.13 (e)

Thus, line PQ is the required line perpendicular to line *l* from P.

Using ruler and set square

Given: PQ and point C outside PQ.

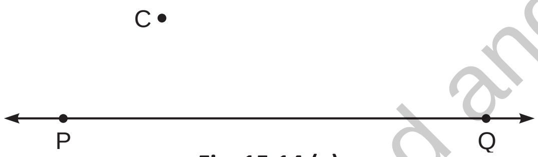


Fig. 15.14 (a)

To construct perpendicular CD to line PQ, follow these steps:

Step 1: Place a set square on PQ in such a way that its one side forming a right angle falls on PQ and other side passes through the point C, outside PQ.

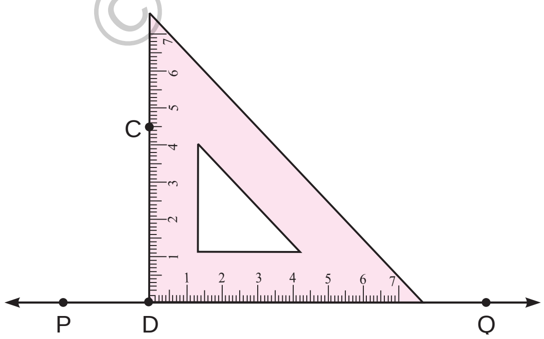


Fig. 15.14 (b)

Step 2: At the right angle vertex of the set square, mark the point D and then remove the set square.

Step 3: Using a ruler, draw line CD passing through points C and D. Then, CD ⊥ PQ.

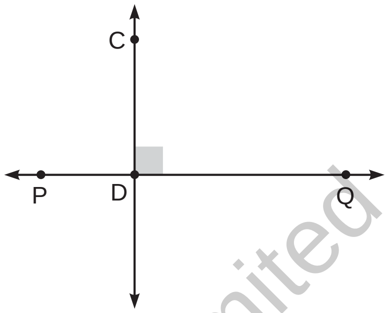


Fig. 15.14 (c)

Thus, CD is the required perpendicular line.

Constructing Perpendicular Bisector of a Line Segment

Let us construct the perpendicular bisector of a given line segment.

Given: Line segment AB.

Fig. 15.15 (a)

To construct the perpendicular bisector CD of the line segment AB, follow these steps:

Step 1: With A as centre and more than $\frac{1}{2}$ AB as radius, draw arcs above and below line segment AB.

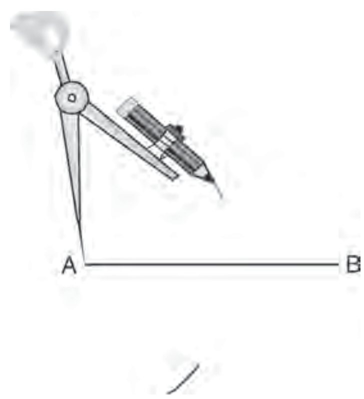


Fig. 15.15 (b)

Step 2: With B as centre and the same radius as in step 1, draw arcs that intersect the previous arcs at points C and D [see Fig. 15.15 (c)].

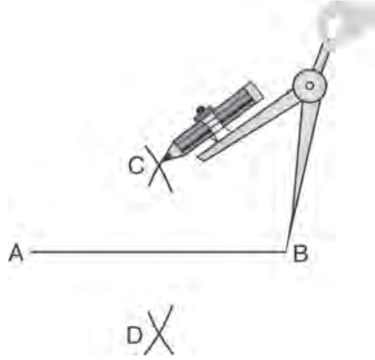


Fig. 15.15 (c)

Step 3: Draw line CD.

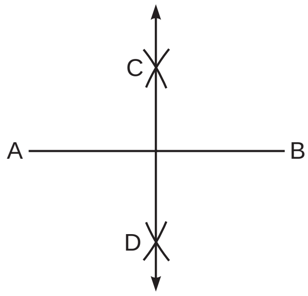


Fig. 15.15 (d)

Thus, line CD is the required perpendicular bisector of the line segment AB.

Exercise 15.1

1. Draw a circle of radius 2.8 cm.
2. Draw a circle of diameter 7 cm.
3. With the same centre, draw two circles of radii 4.2 cm and 5.7 cm.
4. **Draw line segments equal to the given line segments.**

   (a) 

   (b) 

5. **Draw line segments of the following measurements.**

   (a) 5.4 cm (b) 6.5 cm (c) 3.8 cm (d) 7.7 cm

6. Draw a line segment of length 10.5 cm. Now, draw a line which divides it into two equal parts. Check whether it is perpendicular or not.
7. Draw a line segment AB. Mark a point P outside AB. Then draw a perpendicular PQ from point P.
8. Draw a line segment of length 6.8 cm. Now, divide it into four equal parts.
9. Draw a circle of radius 3.6 cm. Take any three points A, B and C on it. Join AB and BC. Draw perpendicular bisectors of AB and BC. Where do the bisectors meet?
10. Draw a line segment PQ = 5 cm. Draw perpendiculars AP ⊥ PQ and BQ ⊥ PQ such that AP = BQ = 4 cm. Join AB and measure it. What is the figure ABQP so formed?

CONSTRUCTING ANGLE AND ANGLE BISECTOR

Constructing an Angle of a Given Measure

Let us construct an angle of a given measure.

Given: An angle of measure 40°.

To construct an angle of measure 40°, follow these steps:

Step 1: Draw a ray AB.

Fig. 15.16 (a)

Step 2: Place the centre of the protractor at A and the zero edge along AB.

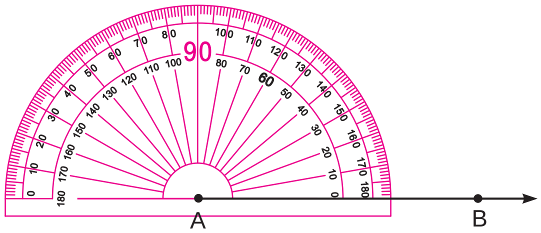


Fig. 15.16 (b)

Step 3: Start with zero (0) near B. Mark point C at 40° along the same scale as 0 near B.

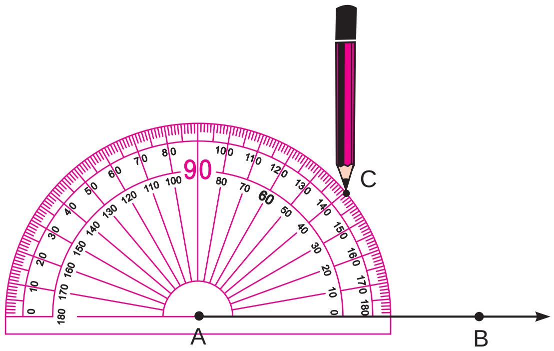


Fig. 15.16 (c)

Step 4: Remove the protractor and join AC.

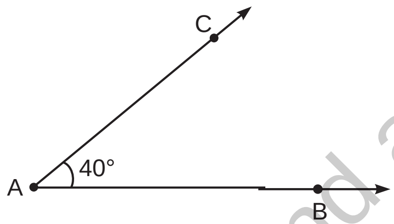


Fig. 15.16 (d)

Thus, ∠BAC is the required angle.

Construct a Copy of an Angle of Unknown Measure

Let us construct an angle equal in measure to a given angle.

Given: ∠ABC

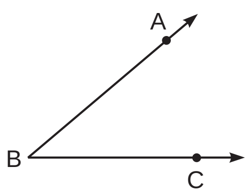


Fig. 15.17 (a)

To construct ∠FDE so that *m*∠FDE = *m*∠ABC, follow these steps:

Step 1: Consider the given angle ABC, with B as centre, draw an arc intersecting ray BA and ray BC at R and S respectively.

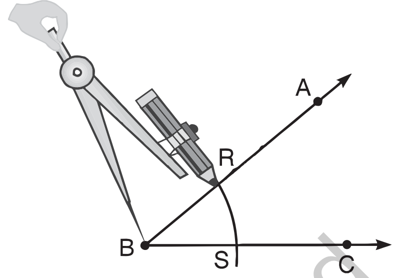


Fig. 15.17 (b)

Step 2: Now, draw a ray DE.

Fig. 15.17 (c)

Step 3: With D as centre and the same radius of step 1, draw an arc intersecting ray DE at Q.

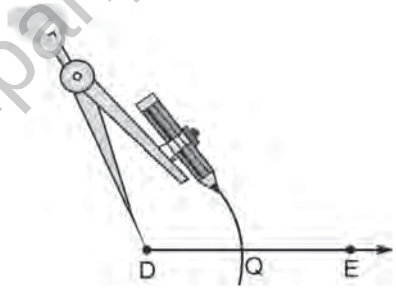


Fig. 15.17 (d)

Step 4: With S as centre, open the compasses up to the length SR.

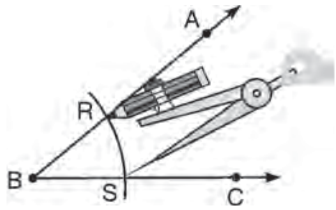


Fig. 15.17 (e)

Step 5: With Q as centre and length SR as radius, draw an arc intersecting arc drawn on step 3 at P.

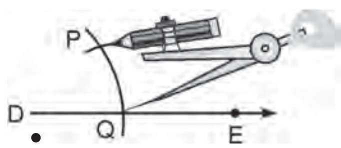


Fig. 15.17 (f)

Step 6: Draw ray DF through P.

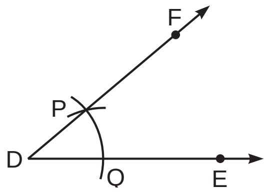


Fig. 15.17 (g)

Thus, $m\angle FDE = m\angle ABC$.

Constructing an Angle Bisector

Let us learn to construct an angle bisector.

Given: ∠ABC

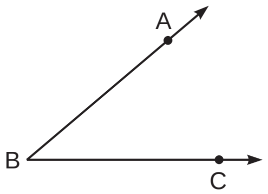


Fig. 15.18 (a)

To construct the angle bisector BD of ∠ABC, follow these steps:

Step 1: With B as centre, draw an arc that intersects ray BA at E and ray BC at F.

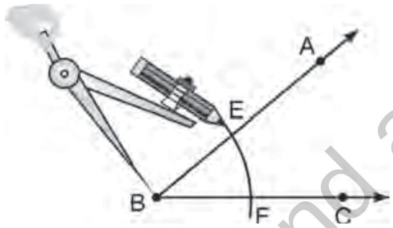


Fig. 15.18 (b)

Step 2: With F as centre, draw an arc whose radius is more than half the length of EF in the interior of ∠ABC.

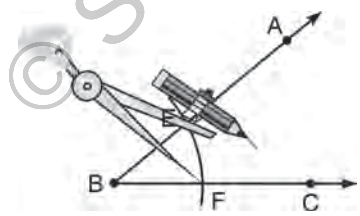


Fig. 15.18 (c)

Step 3: With E as centre and the same radius as in step 2, draw an arc that intersects the previous arc at a point. Label this point as D.

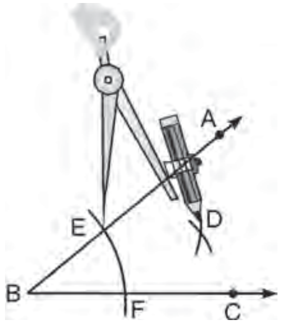


Fig. 15.18 (d)

Step 4: Draw ray BD.

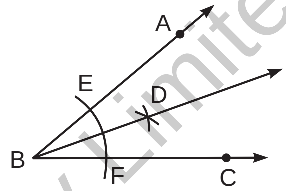


Fig. 15.18 (e)

Thus, ray BD bisects ∠ABC.

Note

An angle is symmetrical about its angle bisector. Ray BD is a line of symmetry for ∠ABC.

Skill Check

- Draw an angle of the measure 70° with the help of a protractor and bisect it using compasses.
- Draw an angle of the measure 125° with the help of a protractor and make a copy of it.

CONSTRUCTING ANGLES OF SPECIAL MEASURES

In this section, we will learn to construct angles of special measures without using a protractor.

Constructing a 60° Angle

To construct an angle of 60°, follow these steps:

Step 1: Draw a line *l* and mark a point O on it.

Fig. 15.19 (a)

Step 2: Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line ***l*** at a point, say A.

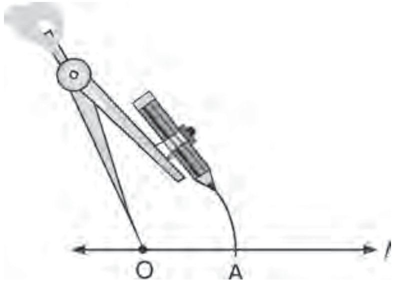


Fig. 15.19 (b)

Step 3: With the pointer at A (as centre) and the same radius, draw an arc that intersects the previous arc at B.

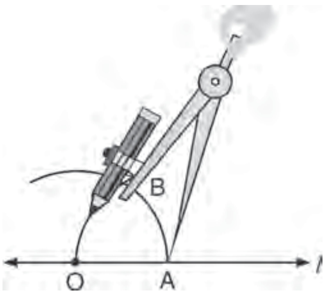


Fig. 15.19 (c)

Step 4: Join OB.

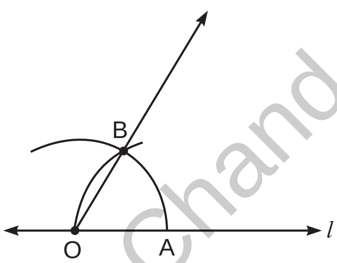


Fig. 15.19 (d)

Thus, we get ∠BOA whose measure is 60°.

Constructing a 30° Angle

We know that an angle of 30° is half of an angle of 60°. So, we will first construct an angle of 60° and then bisect it to obtain an angle of 30°.

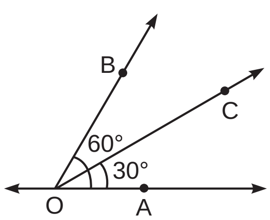


Fig. 15.20 (a)

To construct an angle of 30°, follow these steps:

Step 1: Construct an angle ∠AOB = 60°.

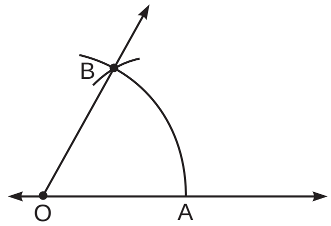


Fig. 15.20 (b)

Step 2: Draw the bisector OC of ∠AOB.

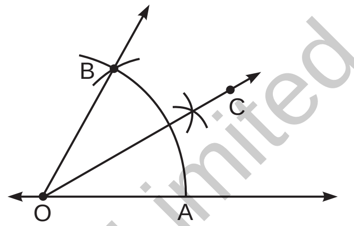


Fig. 15.20 (c)

Thus, ∠AOC = 30°.

Constructing a 120° Angle

To construct an angle of 120°, follow these steps:

Step 1: Draw any line PQ and take a point O on it.

Fig. 15.21 (a)

Step 2: Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line at A.

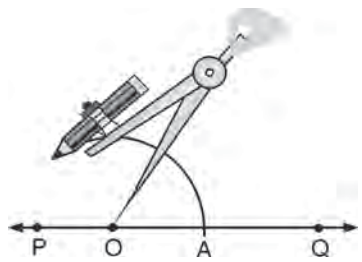


Fig. 15.21 (b)

Step 3: Without disturbing the radius on the compasses, draw an arc with A as centre which cuts the previous arc at B.

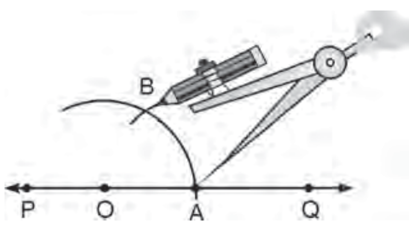


Fig. 15.21 (c)

Step 4: Again, without disturbing the radius on the compasses and with B as centre, draw an arc which cuts the first arc at C.

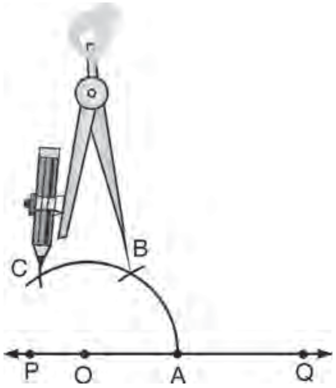


Fig. 15.21 (d)

Step 5: Join OC to form ray OC.

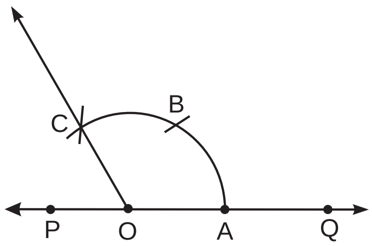


Fig. 15.21 (e)

Thus, ∠COA is the required angle of 120°.

Constructing a 90° Angle

To construct an angle of 90°, follow these steps:

Step 1: Draw a ray OP.

Fig. 15.22 (a)

Step 2: With O as centre and a convenient radius, draw an arc, intersecting OP at A.

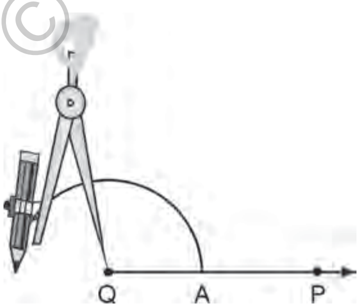


Fig. 15.22 (b)

Step 3: With A as centre and the same radius, draw an arc which cuts the previous arc at B.

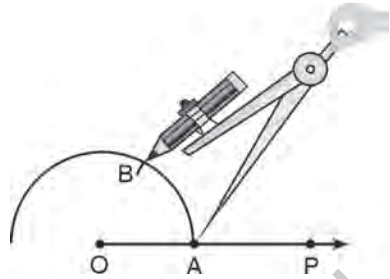


Fig. 15.22 (c)

Step 4: With B as centre and the same radius, draw an arc cutting the first arc at C.

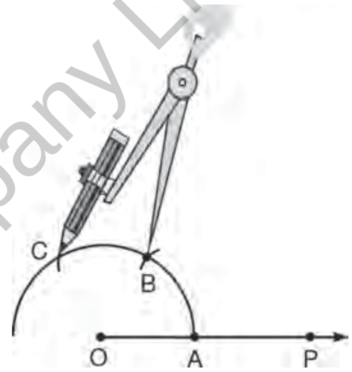


Fig. 15.22 (d)

Step 5: With B and C, as centre and convenient radius (more than $\frac{1}{2}$ BC), draw arcs intersecting at D.

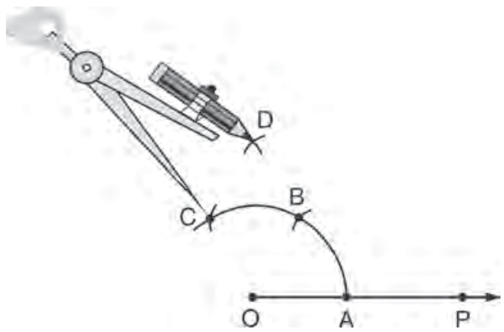


Fig. 15.22 (e)

Step 6: Join OD to form ray OD.

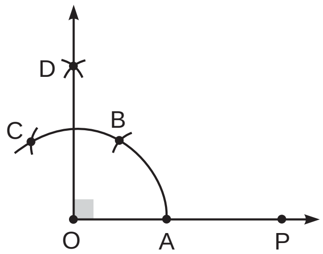


Fig. 15.22 (f)

Thus, ∠POD = 90°.

Constructing a 45° Angle

We know that an angle of 45° is half of an angle of 90°. So, we will first construct an angle of 90° and then bisect it to obtain an angle of 45°.

To construct an angle of 45°, follow these steps:

Step 1: Construct an angle ∠AOB = 90°.

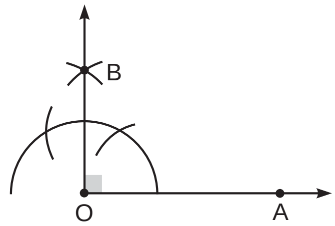


Fig. 15.23 (a)

Step 2: Draw an angle bisector OC of ∠AOB.

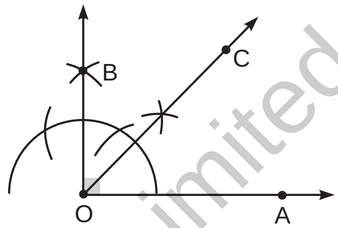


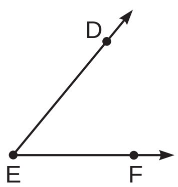
Fig. 15.23 (b)

Thus, ∠AOC = 45°.

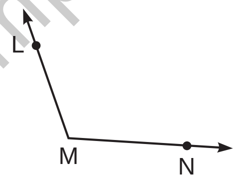
Exercise 15.2

1. **Construct a copy of the following angles and then bisect it.**

(a)



(b)



2. **Construct angles of the given measurements using a protractor.**

(a) 50° (b) 65° (c) 72° (d) 145°

3. Construct an angle of 30° with ruler and compasses. Then bisect the angle.
4. With the help of a ruler and compasses, draw angles 75°, 45° and 105°.
5. Construct an angle of 135° and draw its line of symmetry.
6. Draw a line segment AB = 6 cm. Make ∠ABC = 60° and ∠BAD = 90°. Let rays AD and BC meet at point E. Measure ∠AEB.

Let's Work in Mind

21st CS

1. There are two points P and Q on line segment AB, such that PA = PQ = QB. If AB = 6.6 cm, what is the measure of AQ?
2. Ray OQ is the angle bisector of ∠POR and ∠POQ = 15°. What is the measure of ∠ROP?
3. Diameter of a circle is 8 cm. A point P is 5 cm away from the centre of the circle. Where does the point P lie?

Competency Based Exercise

21st CS

1. **Fill in the blanks.**
   (a) If two lines are perpendicular, then the angle formed by them is ________.
   (b) The shape of a set square is ________.
   (c) While bisecting an angle of 55°, we get each angle of measure ________.
   (d) A ________ is used to measure an angle.
   (e) Using a ruler, we can draw a ________.
2. Draw a line segment of length 6.5 cm.
3. Draw a line segment of length 4.7 cm and bisect it.
4. Draw circles of radii 3.4 cm and 4.6 cm with the same centre O.
5. Draw a perpendicular bisector to a line segment of length 8.7 cm.
6. Draw an angle of measure 45° and bisect it.
7. Construct an angle of 150°, using a:
   (a) protractor (b) ruler and compasses
8. Construct a line segment PQ = 4 cm. Make right angles ∠PQX = 90° and ∠QPY = 90° at points P and Q. Cut off PS = 4 cm from PY and QR = 4 cm from QX. Join RS and name the figure PQRS so formed.

Challenge!

21st CS

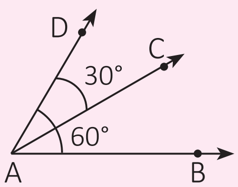
1. Draw a line segment XY = 6.8 cm. Taking X and Y as centres, draw circles of radius 4 cm. Let them intersect at A and B. What can you say about line segment AB?
2. Draw an angle ABC of any measure and its angle bisector BD. Take any point P on it and draw PM ⊥ BA and PN ⊥ BC. Check, if PM = PN.

ASSERTION – REASONING QUESTIONS

21st CS

Directions: Below are Assertion and Reason based questions. Two statements are given, one is labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion (A)** : 

AC is the bisector of ∠BAD.

   Reason (R) : Angle bisector divides the given angle into two equal angles.
2. **Assertion (A)** : Two lines perpendicular to each other intersect at 90°.
   Reason (R) : Two lines are parallel when they do not intersect at any point.

Self Assessment - 2

1. Write True (T) or False (F) for the following statements.

(a) The numbers 11,400 and 46,285 are divisible by both 3 and 5.

(b) 0 is the predecessor of 1 in the collection of natural numbers.

(c) A cuboid has 12 edges, 8 faces and 6 vertices.

(d) The Roman numeral CCXLIV is equivalent to 244.

2. Fill in the blanks.

(a) Rounding off 35.482 to the nearest tenths is _________.

(b) In a bar graph, bars of _________ width are drawn vertically.

(c) 10 times the difference of a number x and 3, where x is greater than 3, is written as _________.

(d) The diagonals of a rhombus bisect each other at ___________ angles.

3. Tick (✓) the correct answer.

(a) The number of line(s) of symmetry in a 30° – 60° – 90° set square is:

(i) 0 (ii) 1 (iii) 2 (iv) 3

(b) $\frac{37}{5}$ in the decimal form is given by:

(i) 0.74 (ii) 7.4 (iii) 70.4 (iv) 7.04

(c) The perimeter of a triangle is 56 cm. If two of its sides are 7 cm and 24 cm, the third side is:

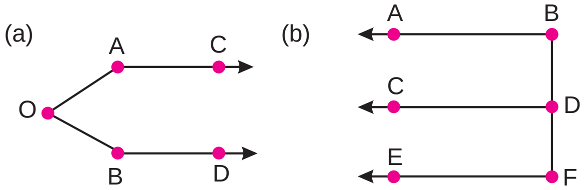
(i) 22 cm (ii) 23 cm (iii) 25 cm (iv) 26 cm

(d) The sum of the successor and predecessor of –10 is:

(i) 20 (ii) –20 (iii) –9 (iv) –11

4. Specify the type of a quadrilateral ABCD, given the following information:
AB || CD, BC || AD, $\angle DAB = 60°$.

5. Name the line segment(s) and ray(s) in the given figures.



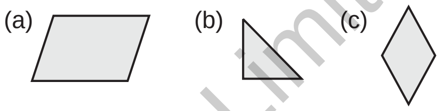
6. Harshita has a sum of ₹x. She spent ₹1500 on clothes, ₹500 on grocery and ₹400 on make-up and received ₹800 as a gift. How much money (in ₹) is left with her?

7. Subtract the sum of –250 and 1250 from the sum of –278 and –1222.

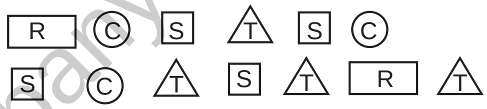
8. Solve the following equations.

(a) $3x + 4 = 19$ (b) $5x - 12 = 23$

9. Are the following pictures symmetrical? If yes, draw line(s) of symmetry.



10. From the following figure:



(a) Find the ratio of number of circles (marked C) to the number of triangles (marked T).

(b) Find the ratio of number of squares (marked S) to the number of triangles (marked T).

11. Draw an angle of measure 150° and bisect it.

12. The number of scouts in different classes of a school is given in the following table.

Class	VI	VII	VIII	XI	X
No. of scouts	30	15	35	20	10

Draw a pictograph for the above data.

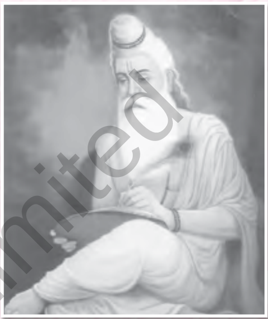
13. Solve the following problems.

(a) A marble tile measures 30 cm × 25 cm. Find the number of such tiles required to cover a wall of size 4.5 m × 3 m.

(b) Sujata and Debashish have bookshelves of the same size filled with books. Sujata's shelf is $\frac{3}{5}$ full of books and Debashish's shelf is $\frac{4}{7}$th full. Whose bookshelf has more books and by how much?

Great Mathematicians

Baudhayana



The Baudhayana Sulba Sutra is one of the oldest known mathematical texts in the world and is part of the larger corpus of ancient Indian texts called Sulba Sutras. These texts are a collection of ancient mathematical treatises that primarily deal with geometry, particularly the construction of altars and fireplaces used in Vedic rituals. The Baudhayana Sulba Sutra, along with other Sulba Sutras, contains valuable mathematical insights and geometric principles. These ancient texts represent an important milestone in the history of mathematics and demonstrate the advanced mathematical knowledge possessed by ancient Indian scholars during the Vedic period.

The Baudhayana Sulba Sutra is attributed to Baudhayana, an ancient Indian mathematician and scholar who lived around 800 BCE. It is believed to have been composed in the Vedic period, making it one of the earliest extant works on geometry.

The Sulba Sutras were not developed as standalone mathematical treatises but were part of the larger body of Vedic literature, which includes religious texts and rituals. The mathematical knowledge contained in these sutras was applied practically in constructing various geometric shapes for sacrificial altars.

The main focus of the Baudhayana Sulba Sutra is on geometric constructions related to the construction of fire altars for performing Vedic rituals. It provides rules and methods for constructing different types of altars, including circular, square, rectangular, and more complex geometric shapes. These constructions involved using ropes, stakes, and other simple tools, without the use of measuring instruments like rulers or compasses.

The Baudhayana-Pythagoras Theorem

The Baudhayana-Pythagoras theorem is a mathematical result that predates the well-known Pythagorean theorem by several centuries. This theorem is one of the earliest known statements of the Pythagorean theorem.

The Baudhayana-Pythagoras theorem states that in a right-angled triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. In mathematical terms: $c^2 = a^2 + b^2$

where 'c' is the length of the hypotenuse, and 'a' and 'b' are the lengths of the other two sides of the right-angled triangle.

Brahmagupta

Born in 598 CE, Brahmagupta has been described as Ganak Chakra Chudamani (jewel among the circle of mathematicians). Through his book, Brahma-sphuta-siddhanta, he was the first to list the rules and properties of zero as a number. Brahma-sphuta-siddhanta has 24 chapters. His discoveries are not given like mathematical equations but like poetry throughout the book, which makes his work one of a kind! The book is also one of the first mathematical books to provide concrete ideas on positive numbers (fortune) and negative numbers (debt). To see zero as a number was a breakthrough moment in history, which further made many advancements in mathematics and science possible.

Brahmagupta has devoted several chapters of Brahma-sphuta-siddhanta to mathematics. In chapters twelve and eighteen, he laid the foundations of two main areas of Indian mathematics — "Pati-ganita (Mathematics of procedures or algorithms)" and "Bija-ganita (Mathematics of seeds or equations)", which roughly correspond to arithmetic (Including measurements) and algebra, respectively.

Brahmagupta discovered the law of gravity 1000 years before Isaac Newton did. This statement caused quite a stir everywhere. He had quoted — "A body falls towards the Earth as it is the nature of the Earth to attract bodies just as it is in the nature of the water to flow."

Brahmagupta played a crucial role in Indian astronomy and made various astronomical calculations using mathematical formulas to predict the movements of the planets and Solar eclipses. He expressed the number Pi approximately as '3.1416'.

As a result, Brahmagupta is a valuable figure in Indian mathematics and astronomy and has contributed to many topics in modern mathematics and science.

Answers

Chapter 1: Knowing Our Numbers

Warm-up

1. (a) 900, 100 to 999
   (b) 90,000; 10,000 to 99,999
   (c) ones (d) 500, 5
   (e) 9999; 10,000 (f) 76,900; 76,902
2. 30,000 + 8000 + 200 + 90 + 8
3. No 4. 90,000
5. (a) LXVII (b) 65

Exercise 1.1

1. (a) True (b) False (c) True (d) False
2. (a) > (b) > (c) = (d) <
3. (a) 2873 (b) 4709 (c) 9939 (d) 7625
4. (a) Smallest—1357; Greatest—7531
   (b) Smallest—2379; Greatest—9732
5. (a) 47,808; 48,807; 84,087; 87,084
   (b) 32,960; 32,999; 92,360; 98,632
6. (a) 98,570; 7541; 1257; 628; 235
   (b) 85,237; 75,328; 73,258; 25,837
7. 87,630 8. 66,632
9. (a) Smallest—2507; Greatest—7502
   (b) Smallest—3567; Greatest—6537
   (c) Smallest—2604; Greatest—4620
   (d) Smallest—5691; Greatest—9651
   (e) Smallest—2235; Greatest—7765
10. 11,174

Exercise 1.2

1. (a) 10,000 (b) 1000 (c) 100 (d) 1
2. Smallest 7-digit number = 10,00,000; Greatest 5-digit number = 99,999; Difference = 9,00,001
3. (a) 2,25,78,093 → Two crore twenty-five lakh seventy-eight thousand ninety-three
   (b) 7,99,85,786 → Seven crore ninety-nine lakh eighty-five thousand seven hundred eighty-six
   (c) 30,05,08,599 → Thirty crore five lakh eight thousand five hundred ninety-nine
4. (a) 56,842,372 → Fifty-six million eight hundred forty-two thousand three hundred seventy-two
   (b) 666,666,666 → Six hundred sixty-six million six hundred sixty-six thousand six hundred sixty-six
   (c) 38,607,500 → Thirty-eight million six hundred seven thousand five hundred

5.

	HM	TM	M	HT	TTh	Th	H	T	O
(a)			8	0	2	3	8	0	0
(b)		6	2	0	8	3	5	6	1
(c)	2	0	0	0	0	0	3	5	4

6. (a) 6,15,00,022
   = 60000000 + 1000000 + 500000 + 20 + 2
   (b) 70,48,00,083
   = 700000000 + 4000000 + 800000 + 80 + 3
   (c) 94,52,000 = 9000000 + 400000 + 50000 + 2000
7. One lakh fifty-five thousand fifteen
8. (a) 42,268; 47,371; 49,669; 52,491; 52,918
   (b) In 2022, there were the greatest number of spectators and in 2002, there were the smallest number of spectators, *i.e.*, 42,268.
9. (a) < (b) < (c) < (d) =
10. (a) Smallest = 10,56,789; Largest = 98,76,510
    (b) Smallest = 1,02,34,567; Largest = 9,87,65,432
    (c) Smallest = 5,00,079; Largest = 9,99,750
    (d) Smallest = 20,00,003; Largest = 33,33,320
11. (a) 1,26,583 = 1,00,000 + 20,000 + 6000 + 500 + 80 + 3
    (b) 87,77,688 = 80,00,000 + 7,00,000 + 70,000 + 7000 + 600 + 80 + 8
    (c) 93,00,580 = 90,00,000 + 3,00,000 + 500 + 80
    (d) 86,88,888 = 80,00,000 + 6,00,000 + 80000 + 8000 + 800 + 80 + 8

Exercise 1.3

1. (a) 1,00,000 mm (b) 5000 m (c) 4000mm
   (d) 205 cm; 2050 mm
   (e) 20,000 g (f) 8010 g (g) 5100 mL
   (h) 4001 L; 4,001,000 mL
2. 55,876 = Fifty-five thousand eight hundred seventy-six
3. 5,00,112 4. 90,001 5. 52,30,165 students
6. 3,49,000 passengers 7. 6227 tonnes
8. 60 classes 9. 1,25,82,865 children
10. 57,60,00,000 plastic bags 11. 98,82,000 trees
12. 86 km 13. 99899

Exercise 1.4

1. (a) 7830 (b) 69,870 (c) 5,37,050
2. (a) 28,000 (b) 34,800 (c) 43,100

3. (a) 67,000 (b) 93,000 (c) 1,50,000

4.

	Nearest lakhs	Nearest ten lakhs	Nearest crores	Nearest ten crores
(a)	1,01,00,000	1,00,00,000	1,00,00,000	×
(b)	79,00,000	80,00,000	1,00,00,000	×
(c)	5,12,00,000	5,10,00,000	5,00,00,000	10,00,00,000
(d)	45,60,00,000	45,60,00,000	46,00,00,000	50,00,00,000

5. (a) 2800 (b) 6300 **6.** (a) 2500 (b) 5300

7. (a) 15,00,000 (b) 8,00,000

8. (a) 400 (b) 250 (c) 4

9. 14,600; 14,700

We observe that estimating the sum is close to the rounded value of the actual sum, but not equal.

10. 12,00,000 children **11.** 10,000 children

12. 16 reams; 96 reams **13.** 25 trees per person

14. 80,00,000 **15.** 53 runs

Exercise 1.5

1. (a) (11 – 4) ÷ 5 (b) (17 + 22) ÷ 2

(c) 48 ÷ (23 – 7) (d) (6 + 28) × 7

(e) (19 – 4) ÷ 3

2. (a) 4 (b) 20 (c) 3060 (d) 11,554

3. 21 hours **4.** Do it yourself.

Exercise 1.6

1. (a) LXXXIV (b) CVII (c) LXIX (d) CLV

(e) CCCLI (f) CXXXVIII (g) LVI (h) CCXXXI

(i) CCXX (j) CXCVIII (k) CLXXVIII (l) CCXXXIII

(m) CML (n) LXXV (o) CDXIII (p) CCLII

(q) CCCXXI (r) DXIV (s) MC

2. (a) 256 (b) 1106 (c) 156 (d) 1156

(e) 900 (f) 1100 (g) 1110 (h) 970

(i) 31 (j) 44 (k) 225

3. 494 **4.** (d)

Competency Based Exercise

1. (a) iii (b) iv (c) ii (d) i

(e) iii

2.

	Number	Indian System of Numeration	Number	International System of Numeration
(a)	6,58,23,425	Six crore fifty-eight lakh twenty-three thousand four hundred twenty-five	65,823,425	Sixty-five million eight hundred twenty-three thousand four hundred twenty-five
(b)	10,05,895	Ten lakh five thousand eight hundred ninety-five	1,005,895	One million five thousand eight hundred ninety-five
(c)	7,00,00,007	Seven crore seven	70,000,007	Seventy million seven
(d)	9,99,99,989	Nine crore ninety-nine lakh ninety-nine thousand nine hundred eighty-nine	99,999,989	Ninety-nine million nine hundred ninety-nine thousand nine hundred eighty-nine

3. (a) A.O. : 5,43,326; 5,54,326; 25,43,260; 5,55,43,326

D.O.: 5,55,43,326; 25,43,260; 5,54,326; 5,43,326

(b) A.O. : 0; 1005; 10,00,538; 10,12,708; 11,11,110

D.O.: 11,11,110; 10,12,708; 10,00,538; 1005; 0

(c) A.O. : 2 lakhs; 20 lakhs; 20 millions; 2 billions

D.O.: 2 billions; 20 millions; 20 lakhs; 2 lakhs

(d) A.O. : CXLI; MCXI; MCMXL; MMCD

D.O.: MMCD; MCMXL; MCXI; CXLI

4. 75,555

5. (a) 30,38,821 (b) 38,04,592

(c) 53,10,996 (d) 21,905

6. 23,456 **7.** (a) 8742 (b) 2478

8. 550 **9.** 7900 **10.** (a) and (d)

11. (91 – 7) ÷ 6; 14 **12.** 216 litres

13. 523

Challenge!

1. 5 frogs **2.** 10,011,010

Let's Work in Mind

1. Ten million three hundred twenty-five thousand four hundred thirty-six

2. 4 **3.** 10 millions **4.** 4000 mm

5. Greatest = 9876; Smallest = 1023

Assertion-Reasoning Questions

1. (a) **2.** (c) **3.** (a) **4.** (a)

5. (a) **6.** (a) **7.** (c) **8.** (d)

9. (a) **10.** (b) **11.** (d)

Chapter 2: Whole Numbers

Warm-up

1. 999; 1001 **2.** 10,100; 10,101; 10,102

3. (a) 5 x 5 = 25 (b) 5 x 4 = 20

(c) 4 × 7

4. (a) 13,21,34
(b) 10,10,101 x 10,10,101 = 10,20,30,40,30,201
(c) 1234 × 9 + 1 = 11,110

Exercise 2.1

1. (a) 5235 (b) 40,998 (c) 1,07,999 (d) 90,000
2. (a) 636 (b) 5000 (c) 25,010 (d) 9,87,451
3. (a) 452; 425 < 452 (b) 9530; 9305 < 9530
(c) 1,48,326; 1,46,832 < 1,48,326
4. 4,25,962; 4,25,961; 4,25,960; 4,25,959; 4,25,958
5. 32 6. 21
7. (a) 11 (b) 11 (c) 8
8. (a) 3 (b) 5 (c) 8
9. (a) 20 (b) 8 (c) 12
10. (a) 5 (b) 3 (c) 3
11. 10

Exercise 2.2

1. Do it yourself.
2. (a) 2344 (b) 1906 (c) 1689
3. (a) 23,600 (b) 4,32,000 (c) 7,50,000
4. (a) 37,83,400 (b) 21,77,500
(c) 18,27,600 (d) 9,31,700
5. (a) 83,325 (b) 7,29,456 (c) 17,266 (d) 4100
6. ₹9600
7. (a) Commutative (b) Additive identity
(c) Multiplicative identity (d) Associativity of addition
(e) Associativity of multiplication
(f) Distributive property of multiplication over addition
8. (a) 9,37,500 (b) 17,6800
(c) 73,60,000 (d) 0

Exercise 2.3

1. 9 2. 26 3. 2 4. 37
5. 5 6. 3 7. 25 8. 23
9. 5466 10. 108

Exercise 2.4

1. (a) 626 (b) 6435 (c) 2469 (d) 122877
2. (a) square (b) rectangular or triangular
(c) square (d) triangular
3. (a) (i) 9876 x 9 + 4 = 88888 (ii) 98765 x 9 + 3 = 888888
(b) (i) 54321 x 9 – 1 = 488888
(ii) 7654321 x 9 – 1 = 68888888
(c) (i) 66667 x 66667 = 4444488889
(ii) 666667 x 666667 = 444444888889
(d) (i) $\frac{1}{10\times10\times10\times10\times10\times10} = 0.000001$
(ii) $\frac{1}{10\times10\times10\times10\times10\times10\times10} = 0.0000001$

4. (a) 37 x 9 = 333; 37 x 12 = 444
(b) 44442222; 66667
(c) 1234321; $\frac{55555\times55555}{1+2+3+4+5+4+3+2+1}$

Competency Based Exercise

1. (a) ii (b) i (c) i (d) ii
(e) iii
2. Do it yourself.
3. 56 4. 8 5. 15
6. (a) Triangular, Rectangular (b) Square
(c) Triangular, Rectangular (d) Rectangular
7. Do it yourself.
8. (a) 8 (b) 23

Challenge!

1. Missing page numbers: 7, 8, 13, 14
2. 214

Let's Work in Mind

1. 2475 2. 1427 3. 1135 4. 2 – 2
5. False

Assertion-Reasoning Questions

1. (a) 2. (a) 3. (c) 4. (a)
5. (c) 6. (a) 7. (c) 8. (d)
9. (a) 10. (a) 11. (a)

Chapter 3: Playing with Numbers

Warm-up

1. 18 = 3 x 6 = 9 x 2 = 18 x 1
2. 24 = 4 x 6 = 3 x 8 = 12 x 2 = 1 x 24; 8; 24
3. 1, 2, 3, 6

Exercise 3.1

1. (a) 1, 2, 3, 5, 6, 10, 15, 30
(b) 1, 2, 3, 6, 7, 14, 21, 42
(c) 1, 2, 4, 17, 34, 68
(d) 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
2. (a) No (b) Yes (c) No
3. (a) 4: 4, 8, 12, 16, 20 (b) 7: 7, 14, 21, 28, 35
(c) 16: 16, 32, 48, 64, 80 (d) 23: 23, 46, 69, 92, 115
(e) 28: 28, 56, 84, 112, 140
4. (a) composite (b) prime (c) composite
(d) prime (e) prime
5. (a) 17 + 19 (b) 23 + 29 (c) 31 + 53
6. (a) 51 = 11 + 17 + 23 (b) 93 = 23 + 29 + 41
(c) 107 = 29 + 31 + 47
7. (a) 31 (b) 73 (c) 43
8. (a) 11, 13, 17, 19, 23, 29, 31, 37
(b) 29, 31, 37, 41, 43
9. (2, 3); (3, 7); (7, 13); (13, 17); (2, 23); (2, 13)
10. (59, 61); (71, 73)

11. (17, 71); (37, 73); (79, 97)
12. 1
13. (3, 17); (7, 13)
14. 4 (23, 37, 53, and 73)

Exercise 3.2

1. (a) iv (b) ii (c) iii (d) iii
2. (a) Divisible by 2, 4 (b) Divisible by 2
(c) Divisible by 2, and 4 (d) Divisible by 2, 4, and 8
3. (a) Divisible by 3 (b) None
(c) Divisible by 3 and 9 (d) Divisible by 3 and 6
4. (a) Divisible by 5 (b) None
(c) Divisible by 5 and 10
5. (a) No (b) Yes (c) Yes
6. 21 and 63 (Answers may vary)
7. 14 and 21 (Answers may vary)
8. (a) Yes (b) Yes (c) Yes (d) Yes
(e) Yes (f) Yes
9. No; Since, the actual production of ball bearings per minute was 171.
10. (a) 0 (b) 9
11. (a) 7 (b) 8
12. (a) 9 (b) 0
13. 10 and 15 (Answers may vary)

Exercise 3.3

1. (a) ii (b) ii (c) i (d) iv
(e) iv
2. (a) 90 = 2 x 3 x 3 x 5 (b) 749 = 7 x 107
(c) 625 = 5 x 5 x 5 x 5 (d) 1480 = 2 x 2 x 2 x 5 x 37
(e) 6182 = 2 x 11 x 281
3. 999999; 3 x 3 x 3 x 7 x 11 x 13 x 37
4. 1000; 2 x 2 x 2 x 5 x 5 x 5

Exercise 3.4

1. (a) 1, 5 (b) 1, 2, 7, 14 (c) 1, 5 (d) 1,2,5,10
2. (a) 6 (b) 2 (c) 4 (d) 8
3. (a) 15 (b) 21 (c) 18 (d) 12
4. (a) 10 (b) 9 (c) 35
5. 17 fruits 6. 130 litres 7. 10
8. 64 cm 9. 4 m
10. Square of side 30 cm

Exercise 3.5

1. (a) 100 (b) 240, 270, 300, 330 (c) 270
2. (a) 42 (b) 48 (c) 300
3. (a) 1080 (b) 720 (c) 480
4. (a) 1008 (b) 540 (c) 300
5. 50 6. 76 7. 12 midnight
8. 720 cm 9. 64 10. 3 am
11. At 09:03:30 (am)

12. (a) 960 (b) 990 (c) 936
13. 1440 14. 999720 15. 60 plants

Competency Based Exercise

1. (a) iii (b) iii (c) ii (d) iii
(e) ii (f) iv (g) iv
2. 25 3. 297 4. 49 years 5. 1 kg
6. 452 7. 3000 days 8. 15

Challenge!

1. 24 and 25 2. 600 m 3. 23 and 77
4. 27, 36, and 37

Let's Work in Mind

1. 0 2. 63 3. 105 4. 9
5. 99

Assertion-Reasoning Questions

1. (a) 2. (a) 3. (a) 4. (d)
5. (a) 6. (c) 7. (c) 8. (a)
9. (a) 10. (a) 11. (a)

Chapter 4: Basic Geometrical Ideas

Warm-up

1. (a) ray (b) line segment (c) angle
(d) line
2. (a) angle (b) rectangle (c) triangle
3. 2, 4 and 3 in (a), (b) and (c) respectively.

Exercise 4.1

1. (a) 4 points, 3 line segments
(b) 3 points, 2 line segments
(c) 4 points, 3 line segments
(d) 4 points, 3 line segments
2. (a) (i) parallel (ii) perpendicular
(b) (i) intersecting (ii) parallel
3. (a) Ray AC, ray BD, line segment AB
(b) Ray AC, ray BD, line segment OA, line segment OB
(c) Ray BA, ray DC, ray FE, line segment BD, line segment DF, line segment BF
(d) Ray BA, ray DE, line segment BC, line segment DC
4. PQ ⊥ QR, QR ⊥ RS
5. AB, BC, AC, CD, BD, AD, DE, CE, BE, AE (Ten)
6. (a) line *l* and line *a* (b) line *b* and line *n*
(c) *l* || *m*; *m* || *n*; *l* || *n* (d) I, K, E, B, and J
(e) A
7. (a) 4 (b) BA, BC, CD
(c) (*l*, *r*, *n*; A); (*p*, *q*, *r*; C); (*m*, *q*, *n*; B)
8. A line segment has two end points, a ray has one end point and a line has no end points. A line segment has fixed length but a line and a ray do not have fixed length.
9. 3; rays OQ, OP and OR 10. AB and DC; E

Exercise 4.2

1. Do it yourself.
2. Do it yourself.
3. GH
4. Do it yourself.
5. Point Q will lie between P and R
6. Do it yourself.

Exercise 4.3

1. (a) Points A, D, O, N and B
   (b) Points R and E (c) Points F and C
2. (a) Vertex: O; arms: OX and OY; Angle: angle XOY
   (b) Vertex: X; arms: XA and XB; Angle: angle BXA
   (c) Vertex: A; arms: AX and AY; Angle: angle XAY
3. (a) 2 angles; $\angle ABC$, $\angle BCD$
   (b) 3 angles; $\angle AOB$, $\angle BOC$, $\angle AOC$

Exercise 4.4

1. (a) Acute angles: 5; Right angles: 6
   (b) Acute angles: 9; Right angles: 16
2. (a) 2 (b) 1 (c) 3
3. (a) acute (b) right (c) obtuse (d) reflex
4. Do it yourself.
5. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
6. (a) At 11 (b) At 5
7. (a) East (b) East
8. (a) 3 right angles (b) 1 right angle
9. Obtuse angle
10. (a) Right angle (b) Straight angle
11. (a) Right angles: $\angle OPQ$;
    Obtuse angles: $\angle SRQ$, $\angle SOP$, $\angle PQR$
    (b) Right angles: $\angle PTR$, $\angle PTS$, $\angle TPQ$;
    Obtuse angles: $\angle QRT$, $\angle QPS$
    (c) Right angles: $\angle DCB$, $\angle DCA$, $\angle ACE$, $\angle ECB$;
    Obtuse angles: None
12. Right angle
13. Acute angle, approx. 45°

Competency Based Exercise

1. (a) iii (b) iv (c) iv (d) iv
   (e) iii (f) ii (g) ii
2. 10
3. 9
4. $a \parallel b$; $b \parallel d$; $a \parallel d$
5. 6: *a*, *p*; *a*, *q*; *a*, *r*; *b*, *p*; *b*, *q*; *b*, *r*
6. 210°
7. $\angle EAB$, $\angle EDC$
8. line *q*

Challenge!

1. 15
2. *t*, *s*, *p*

Let's Work in Mind

1. 0
2. Yes
3. Yes
4. Three-fourths

Assertion-Reasoning Questions

1. (c)
2. (a)
3. (c)
4. (a)
5. (d)
6. (c)
7. (a)

Chapter 5: Understanding Elementary Shapes

Warm-up

1. (a) (i) horizontal line segment
   (ii) vertical line segment
   (b) (i) parallel line segments
   (ii) intersecting line segments
   (iii) angle
   (c) (i) angles (ii) triangle
2. (a) square, rhombus (b) Hexagon

Exercise 5.1

1. (a) closed (b) open (c) closed (d) open
2. (a), (c) and (e)
3. (a) boundary of a wheel (b) boundary of a book
   (c) book, handkerchief, signboard, *etc*.
   (d) Arrowhead, star or starfish
4. (a) C: Interior (b) D: Exterior
   (c) E: on the polygon, G: Exterior
   (d) B: Exterior, A: Interior. C: on the polygon
5. (a) pentagon, regular (b) octagon, regular
   (c) heptagon, regular

Exercise 5.2

1. (a) 6, 3, 3 (b) median (c) triangular
2. (a) 5 (b) 8 (c) 6
3. Do it yourself.
4. (a) AM (b) BN (c) Points Q and R
   (d) Points A, Q, P and C
   (e) 5 triangles; ΔABN, ΔBMN, ΔAMC, ΔABM, ΔABC
5. Do it yourself.

Exercise 5.3

1. (a) different (b) 180° (c) 60°
   (d) equal (e) scalene triangle (f) longest
2. (a) vi (b) v (c) iv (d) ii
   (e) i (f) iii
3. (a) equilateral (b) scalene (c) scalene
   (d) isosceles
4. (a) obtuse-angled (b) acute-angled
   (c) right-angled (d) acute-angled
5. (a) right-angled, isosceles
   (b) equilateral, acute-angled
   (c) isosceles, acute-angled
   (d) obtuse-angled, scalene
6. (a) 1 (b) 2 (c) 1 (d) 8
7. (a) equilateral triangle (b) isosceles
   (c) right-angled triangle

Exercise 5.4

1. (a) rectangle (b) square
(c) isosceles trapezium (d) parallelogram
(e) rhombus (f) trapezium
(g) kite

2. (a) square (b) rectangle (c) rhombus

3. (a) False (b) False (c) False (d) True

4. (a) square (b) right (c) trapezium
(d) square or rhombus

5. (a) A, U, R, D, C, H (b) E, K, I, L, T, G
(c) (EK, TI); (KI, ET) (d) (∠K, ∠T); (∠E, ∠I)
(e) (EK, ET); (KI, TI) (f) (∠E, ∠K); (∠E, ∠T)

6. (a) i (b) ii (c) ii

7. Kite **8.** Isosceles trapezium

Exercise 5.5

1. (a) OP, OQ, OR (b) QP, QR, PR
(c) PR (d) P, Q, R, S
(e) O, N, L (f) T, M

2. (a) (i) X, Y (ii) P, Q (iii) A, B
(b) (i) X, Y (ii) T, S (iii) A, B, C
(c) (i) C (ii) D (iii) A, B

3. (a) major segment (b) minor sector
(c) minor segment (d) Semicircle

4. (a) chord/diameter (b) minor arc
(c) chord (d) radii

5. (a) diameter (b) diameter (c) circumference
(d) segment (e) sector

Competency Based Exercise

1. (a) iv (b) iii (c) ii (d) i
(e) iv (f) iii (g) iv

2. (a) obtuse-angled triangle (b) right-angled triangle
(c) acute-angled triangle

3. (a) scalene triangle (b) equilateral triangle
(c) isosceles triangle

4. Pentagon; 5 diagonals

5. (a) ΔPSU, ΔSQT, ΔUTR, ΔSUT, ΔPQR; 5
(b) ΔABC, ΔACD, ΔABD, ΔADE, ΔACE, ΔABE; 6
(c) ΔHOE, ΔHOG, ΔGOF, ΔEOF, ΔHEG, ΔEFG,
ΔHGF, ΔHEF; 8

6. (a) radius (b) diameter (c) chord (d) minor arc
(e) major arc

7. 4 right angles **8.** Do it yourself.

Challenge!

1. ∠BCA **2.** 60°

Let's Work in Mind

1. 3 **2.** Square **3.** 180°
4. Equilateral triangle **5.** Square
6. Diameter **7.** 0

Assertion-Reasoning Questions

1. (a) **2.** (d) **3.** (a) **4.** (a)
5. (a) **6.** (a) **7.** (c) **8.** (a)

Chapter 6: Three-Dimensional Shapes

Warm-up

1. Cuboid **2.** Rectangle

3. A square has all sides equal but a rectangle has only opposite sides equal. **4.** 12 **5.** 8

Exercise 6.1

1. (a) ii (b) i (c) iv (d) iii

2. (a) Cube (b) Cylinder and sphere
(c) sphere (d) faces = 4, edges = 6, vertices = 4

3. (a) edges = 12, vertices = 8, faces = 6
(b) edge = 1, vertex = 1, faces = 2
(c) edges = 2, vertices = 0, faces = 3
(d) edges = 6, vertices = 4, faces = 4
(e) edges = 10, vertices = 6, faces = 6

4. (a) faces = 5, edges = 8, vertices = 5
(b) faces = 5, edges = 9, vertices = 6
(c) faces = 6, edges = 12, vertices = 8

Exercise 6.2

1. (a) Cuboid (b) Hexagonal prism
(c) Cylinder (d) Triangular prism

2. (a)-(iii)-(C), (b)-(i)- (D), (c)-(iv)-(B), (d)-(ii)-(A)

Competency Based Exercise

1. (a) iii (b) i (c) iv (d) i

2. (a) faces = 5, edges = 9, vertices = 6
(b) faces = 6, edges = 12, vertices = 8
(c) faces = 6, edges = 12, vertices = 8
(d) faces = 2, edge = 1, vertex = 1
(e) faces = 3, edges = 2, vertices = 0
(f) faces = 4, edges = 6, vertices = 4

3. Open cube

4. (a) cuboid (b) sphere (c) cube (d) cylinder

5. (a) 2 (b) 4

6. No, as prism is named on the shape of its base. A square prism also can be a cuboid.

Challenge!

1. A sector of a circle **2.** Sphere

3. (a) Pyramid and cuboid (b) faces = 9, edges = 16

Let's Work in Mind

1. Cylinder and sphere
2. Yes, prism and pyramid
3. Ice cube and dice
4. Rectangle

Assertion-Reasoning Questions

1. (d) 2. (a) 3. (c) 4. (c)
5. (a) 6. (b)

Chapter 7: Integers

Exercise 7.1

1. (a) ii (b) ii (c) iii
2. (a) –5 (b) 7 (c) –15 (d) 0
3. (a) –8 (b) –1 (c) 6 (d) –12
4. (a) 6 (b) –7 (c) 1 (d) –19
5. (a) Income of ₹1000 (b) 50 km South
   (c) 5°C temperature rises (d) Lost by 2 seconds
6. (a) + ₹500 (b) –4 km (c) –15°C (d) +7 kg
7. Do it yourself.
8. (a) 2 (b) 9 (c) 12 (d) 9
9. (a) –4 (b) 1
10. (a) –3 (b) –8 (c) –3 (d) 3
11. (a) 3 (b) E (c) A (d) 7
12. (a) 0 (b) –2 (c) 3

Exercise 7.2

1. (a) T (b) T (c) F (d) T
2. (a) < (b) > (c) < (d) <
   (e) = (f) > (g) > (h) >
3. (a) –3, –2, –1, 0, 1, 2, 3, 4
   (b) – 10, –9, –8, –7, –6, –5, –4
   (c) –2, –1 (d) –5, –4, – 3, –2, –1, 0, 1
4. (a) –7, –5, –4, 0, 4, 7, 10
   (b) –6, –4, –2, 0, 7, 8
   (c) –15, –10, –8, –6, 9, 10
5. (a) 6, 1, 0, –2, –3, –4 (b) 16, 1, 0, –3, –12, –14
   (c) 17, 13, +11, –2, –6, –8
6. (a) 5 (b) 7 (c) 0 (d) 11

Exercise 7.3

1. (a) 8 (b) 5 (c) –10 (d) –8
   (e) 0
2. (a) 65 (b) 22 (c) –16 (d) –12
   (e) 240 (f) –70
3. (a) –8 (b) – 1 (c) 74 (d) 60
4. 0 5. 10
6. (a) = (b) < (c) < (d) <
   (e) >

Exercise 7.4

1. (a) 1 (b) –10 (c) 9 (d) +2
2. (a) –6 (b) +8 (c) – 4 (d) –11
3. (a) –5 (b) 5 (c) 5 (d) –5
4. (a) 27 (b) –60 (c) 32 (d) –540
5. –70 6. 1750 m
7. (a) < (b) > (c) < (d) =
8. (a) 5 (b) 10 (c) 5
9. –2500

Competency Based Exercise

1. (a) ii (b) ii (c) ii (d) ii
2. (a) –68 (b) –80 (c) 72 (d) 29
3. *m* = –19 4. –50 5. +5°C
6. (a) –1 (b) 4 (c) –8 (d) 2
7. 4038 8. 0

Challenge!

1. –9
2.

	–6	
2	–5	3
	4	
	7	

Let's Work in Mind

1. –14 2. 49 3. –20 4. – ₹430
5. 11 6. –99

Assertion-Reasoning Questions

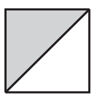
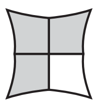
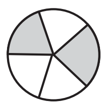
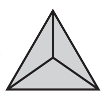
1. (a) 2. (b) 3. (a) 4. (a)
5. (a) 6. (d) 7. (a) 8. (c)

Self Assessment – 1

1. (a) ii (b) i (c) iv (d) iii
2. (a) 8 (b) 21 (c) twin (d) segment
3. (a) ii (b) iv (c) i (d) ii
4. (a) 150 (b) 450 5. 3
6. (a) CXXXII (b) DLXXIV (c) MMC 7. 102
8. (a) 7347 (b) 212787 (c) 937500 (d) 7360000
9. 56 10. 64 cm
11. (a) *b* and *m* (b) *c* and *n* (c) *l* and *m*, *m* and *n*, *l* and *n*
    (d) K, E, B, J (e) C
12. 5; ∠AOC, ∠BOD, ∠AOD, ∠BOE, ∠AOE
13. (a) Cuboid (b) Hexagonal prism
    (c) Cylinder (d) Triangular prism

Chapter 8: Fractions

Warm-up

1. (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{4}{9}$ (d) $\frac{1}{3}$
2. (a)  (b)  (c)  (d) 
3. (a) numerator = 1, denominator = 6
   (b) numerator = 5, denominator = 7

(c) numerator = 8, denominator = 11
(d) numerator = 6, denominator = 17

4. Yes, Harish

Exercise 8.1

1. $\frac{8}{12}$ or $\frac{2}{3}$ **2.** 56 **3.** $\frac{15}{19}$

4. (a) two-thirds (b) three-eighths (c) one-fifth (d) four-ninths

5. (a) 3 pens (b) 12 balloons (c) 20 hours

6. (a) proper (b) mixed (c) improper (d) Improper (e) proper (f) improper

7. (a) < (b) < (c) = (d) >

8. (a) $2\frac{3}{5}$ (b) $3\frac{2}{7}$ (c) $6\frac{1}{5}$ (d) $2\frac{4}{19}$

9. (a) $\frac{11}{3}$ (b) $\frac{16}{3}$ (c) $\frac{37}{5}$ (d) $\frac{125}{9}$

10. (a) $5\frac{2}{5}$ (b) $\frac{48}{5}$ (c) 9

11. 20 pencils **12.** $\frac{9}{12}$ or $\frac{3}{4}$

Exercise 8.2

1. (a) 7 and 8 (b) $3\frac{1}{2}$ (c) ii (d) $3\frac{1}{4}$

2.-3. Do it yourself.

4. $P = 2\frac{4}{7}$; $Q = 1\frac{4}{7}$; $R = 4\frac{2}{7}$; $S = \frac{3}{7}$; $T = 3\frac{6}{7}$

Exercise 8.3

1. (a) iii (b) ii (c) i

2. (c)

3. (a) $\frac{20}{35}$ (b) $\frac{36}{24}$ (c) $\frac{12}{15}$ (d) $\frac{21}{35}$

4. (a) 49 (b) 45 (c) 55 (d) 81 (e) 105

5. (a) 18 (b) 16 (c) 35 (d) 56 (e) 12

6. (a) $\frac{4}{5}$ (b) $\frac{8}{10}$

7. (a) iv (b) iii (c) v (d) i (e) ii

8. (a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{9}$ (e) $\frac{6}{7}$ (f) $\frac{1}{2}$

9. (a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

10. (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{8}{15}$ (d) $\frac{7}{10}$

11. No, Pankaj coloured $\frac{1}{12}$ of the book.

12. 19, $\frac{8}{19}$

13. (a) $\frac{3}{5}$ (b) $\frac{5}{9}$ **14.** $\frac{2}{7}$

15. Lali = $\frac{2}{3}$, Mili = $\frac{3}{7}$, Joly = $\frac{5}{9}$

Exercise 8.4

1. (a) like (b) unlike (c) like (d) unlike

2. (a) $\frac{8}{15}, \frac{9}{15}$ (b) $\frac{11}{20}, \frac{14}{20}$ (c) $\frac{56}{105}, \frac{20}{105}$ (d) $\frac{28}{60}, \frac{33}{60}$

3. (a) > (b) < (c) < (d) >

4. (a) $\frac{5}{6}$ (b) $\frac{5}{8}$ (c) $\frac{10}{11}$ (d) $\frac{4}{5}$

5. (a) T (b) T (c) T (d) T

6. $\frac{1}{2}$

7. (a) $\frac{11}{15} > \frac{7}{10} > \frac{2}{5}$ (b) $\frac{25}{32} > \frac{3}{4} > \frac{11}{16}$ (c) $\frac{5}{6} > \frac{19}{24} > \frac{5}{12}$

8. (a) $4\frac{1}{5} < 4\frac{3}{7} < 4\frac{2}{3}$ (b) $5\frac{1}{6} < 5\frac{2}{3} < 5\frac{3}{4}$

9. Shilpa **10.** Minu

Exercise 8.5

1. (a) $1\frac{3}{8}$ (b) $9\frac{1}{3}$ (c) 16

2. (a) $\frac{1}{2}$ (b) $1\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $1\frac{1}{2}$ (e) $1\frac{7}{8}$ (f) $1\frac{1}{6}$

3. (a) $7\frac{4}{5}$ (b) $5\frac{11}{14}$ (c) $15\frac{1}{2}$ (d) $12\frac{8}{15}$ (e) $12\frac{3}{8}$ (f) $8\frac{14}{15}$

4. $\frac{5}{8}$ **5.** $1\frac{5}{12}$ L **6.** $\frac{14}{15}$ **7.** $5\frac{1}{4}$ hours

8. ₹$24\frac{1}{4}$ **9.** $31\frac{1}{15}$ kg **10.** 14 km **11.** 13 m

Exercise 8.6

1. (a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{15}$

2. (a) $\frac{7}{18}$ (b) $2\frac{2}{3}$ (c) $2\frac{2}{7}$ (d) $4\frac{1}{15}$

3. (a) $3\frac{1}{5}$ (b) $3\frac{5}{12}$ (c) $2\frac{3}{8}$

4. $5\frac{1}{15}$

5. (a) $3\frac{3}{8}$ (b) $\frac{1}{4}$ (c) $2\frac{7}{12}$ (d) $5\frac{11}{14}$

6. 3 **7.** $\frac{3}{10}$ **8.** $2\frac{1}{4}$ m **9.** $2\frac{3}{4}$ L

10. $2\frac{1}{4}$ minutes **11.** $1\frac{3}{4}$ buckets

12. 4 m

Competency Based Exercise

1. (a) iii (b) i (c) ii (d) iv
(e) ii (f) ii (g) ii

2. (a) $\frac{31}{35}$ (b) $5\frac{2}{7}$ (c) $\frac{3}{4}$ (d) $8\frac{7}{8}$
(e) 6 (f) $3\frac{3}{20}$

3. $\frac{7}{10} < \frac{3}{4} < \frac{4}{5} < \frac{7}{8}$

4. $6\frac{1}{6}$ km **5.** $3\frac{3}{4}$ litres **6.** Maths **7.** $2\frac{11}{15}$

8. $\frac{3}{10}$ **9.** $47\frac{1}{8}$ kg

Challenge!

1. $\frac{1}{5}$ **2.** $1\frac{1}{8}$

Let's Work in Mind

1. $\frac{3}{4}$ **2.** 8 **3.** No

4. 15 **5.** $5\frac{2}{3}$ **6.** 0 and 1 **7.** $\frac{1}{6}$

Assertion-Reasoning Questions

1. (a) **2.** (d) **3.** (c) **4.** (c)
5. (a) **6.** (c) **7.** (b)

Chapter 9: Decimals

Warm-up

1. A = 0.5, B = 1.3

2. (a) $\frac{7}{10} = 0.7$ (b) $\frac{5}{10} = 0.5$

3. (a) True (b) $\frac{16}{32}, \frac{50}{100} = 0.50$

Exercise 9.1

1. (a) 0.7 (b) 0.23 (c) 0.18 (d) 0.092
(e) 2.765

2. (a) 0.007 (b) 8 (c) $\frac{5}{100}$ (d) 3.405
(e) $\frac{1}{10}$ (f) decimal fractions

3.

Number	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Number name
(a) 32.6		3	2	6			Thirty-two point six
(b) 102.37	1	0	2	3	7		One hundred two point three seven
(c) 479.002	4	7	9	0	0	2	Four hundred seventy-nine and two-thousandths
(d) 97.023		9	7	0	2	3	Ninety-seven point two three

4. (a) 0.09 (b) 0.4 (c) 0.006
(d) 0.004 (e) 0.05

5. (a) 306.8 (b) 204.68 (c) 620.9 (d) 9.04

6. (a) 3 and 4 (b) 5 and 6 (c) 2 and 3
(d) 15 and 16

7. (a) 570.904 (b) 700.106

8. 0.09

9. (a) 7.047 (b) 53.472 (c) 130.507
(d) 325.034 (e) 270.065

10. (a) $20 + 3 + \frac{1}{10} + \frac{7}{100} + \frac{2}{1000}$

(b) $\frac{1}{10} + \frac{9}{100} + \frac{3}{1000}$

(c) $10 + 5 + \frac{3}{100} + \frac{8}{1000}$ (d) $80 + \frac{2}{10} + \frac{3}{100}$

(e) $300 + 20 + 5 + \frac{9}{10} + \frac{8}{100} + \frac{6}{1000}$

Exercise 9.2

1. (a) iii (b) iv (c) ii (d) iv
   (e) i
2. (a) 5.700, 6.150, 7.012 (b) 0.5000, 0.0500, 0.0050
   (c) 4.20, 42.00, 9.16 (d) 1.100, 1.110, 1.111
3. (a) 0.57 (b) 3.75 (c) 41.5 (d) 12.34
   (e) 3.75 (f) 12.07
4. (a) $<$ (b) $<$ (c) $>$ (d) $=$
   (e) $>$ (f) $<$
5. (a) 47.03, 40.12, 35.37, 35.2
   (b) 144.2, 143.7, 15.002, 14.9, 13.9
   (c) 75.12, 75.012, 75.0012, 75.0
   (d) 4.12, 4.113, 4.1, 4.013
6. (a) 0.697, 5.72, 6.9, 55.02
   (b) 0.1576, 1.576, 15.76, 157.6
   (c) 35.8, 35.81, 36.01, 42.11
   (d) 0.0123, 0.12, 0.273, 10.13

Exercise 9.3

1. (a) iii (b) i (c) iii
2. (a) $\frac{9}{20}$ (b) $\frac{1}{50}$ (c) $\frac{3}{40}$ (d) $\frac{5}{8}$
   (e) $\frac{2}{5}$ (f) $\frac{13}{100}$ (g) $\frac{1}{125}$
3. (a) $2\frac{1}{200}$ (b) $5\frac{1}{40}$ (c) $15\frac{1}{25}$ (d) $25\frac{3}{50}$
   (e) $44\frac{1}{4}$ (f) $19\frac{3}{4}$
4. (a) 0.66 (b) 1.009 (c) 0.007 (d) 0.08
   (e) 0.375
5. (a) 0.32 (b) 3.25 (c) 4.5 (d) 3.8
   (e) 46.5
6. (a) ₹0.55 (b) 28.5 cm (c) 25.558 kg
   (d) 3.045 L (e) 87.450 km
   (f) 0.015 km (g) ₹350.15
   (h) 0.028 kg (i) 0.65 m (j) 5.04 m
   (k) 40.040 g (l) 15.050 kL
7. (a) 0.6 hour (b) 0.02 hour (c) 2.4 hours (d) 0.055 hour
8. $\frac{5}{8}$, 0.625
9. $\frac{3}{10}$ or 0.3 century

Exercise 9.4

1. (a) 12.48 (b) 21.75 (c) 107.8 (d) 107.482
   (e) 2.718
2. (a) 0.453 (b) 108.587 (c) 318.81 (d) 650.2
3. ₹63.75
4. 14.55 km
5. ₹257.20
6. 3.05 km
7. 12.55 m
8. 17.25 kg
9. 39.4°C

Exercise 9.5

1. (a) 0.311 (b) 72.4 (c) 90.33 (d) 0.0073
   (e) 215.32
2. (a) 3.695 kg (b) ₹464.75 (c) 23.455 km
   (d) 1.62 L
3. (a) 45.652 (b) 1.621 (c) 143.214
4. (a) 19.35 (b) 42.13 (c) 12.27
5. 15050 g
6. ₹52.25
7. ₹7.70
8. 4.6°F
9. 6.2 kg
10. 10.3 m

Competency Based Exercise

1. (a) iv (b) iv (c) iii (d) ii
   (e) iii
2. (a) T (b) F (c) T (d) F
   (e) T
3. (a) iii (b) i (c) ii
4. (a) $\frac{7}{100}$ (b) $\frac{2}{1000}$ (c) $\frac{2}{10}$
5. (a) 2 (b) 3
6. (a) 0.007 km (b) 0.032 L (c) 5023.007 (d) 300.005
7. (a) $\frac{7}{10}+\frac{9}{100}+\frac{8}{1000}$ (b) $50 + 7 + \frac{8}{100}+\frac{7}{1000}$
   (c) $30 + \frac{7}{100}+\frac{2}{1000}$ (d) $300 + 20 + 7 + \frac{8}{10}+\frac{2}{100}$
8. (a) 0.903 (b) 372.057 (c) 0.009 (d) 8.3
9. (a) 0.1, 0.112, 1.01, 1.1 (b) 3.14, 3.141, 3.157, 3.2
10. (a) 83.71, 83.7, 8.37, 0.8371
    (b) 12.011, 11.98, 11.907, 11.9
11. (a) 0.28 (b) 18.5 (c) 0.125 (d) 9.4 (e) 17.375
12. (a) $27\frac{12}{25}$ (b) $12\frac{16}{25}$ (c) $4\frac{1}{50}$ (d) $63\frac{31}{100}$
13. (a) 661.842 (b) 4.062 (c) 15.68 (d) 111.472
    (e) 45.06 (f) 8.85
14. 50.128
15. 15.515 kg
16. ₹5.67
17. 85.939
18. 4 cm
19. 239.145, 199.98

Challenge!

1. 9.105
2. 26.987
3. $\frac{9}{10}$

Let's Work in Mind

1. 0.9
2. ₹13.10
3. ₹21.91
4. 200
5. 7 and 8
6. 3.5 m

Assertion-Reasoning Questions

1. (a)
2. (c)
3. (d)
4. (b)
5. (a)

Chapter 10: Data Handling

Warm-up

Do it yourself

Exercise 10.1

1.

Outcomes	Tally marks	Number of times
1	𝍸 |	6
2	||||	4
3	𝍸	5
4	|||	3
5	𝍸 |||	8
6	||||	4

2. (a) 33, 37, 37, 39, 41, 43, 49, 50, 51, 52, 52, 55, 63, 63, 67, 68, 71, 71, 75, 76, 78, 80, 81, 87, 91
(b) 33 (c) 6 students (d) 7 students

3.

Shirt Size	Tally marks	Number of students
32	|||	3
34	𝍸 |	6
36	𝍸 ||	7
38	𝍸 ||	7
40	𝍸 ||||	9
42	𝍸 |||	8

4.

Number of children	Tally marks	Number of families
1	𝍸	5
2	𝍸 ||	7
3	|||	3
4	|||	3
5	||	2

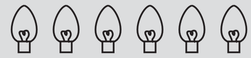
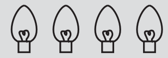
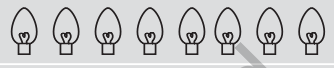
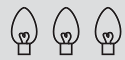
(a) 3 (b) 12

5. (a) Do it yourself. (b) 1 (c) 5
6. (a) raw data (b) numbers (c) 𝍸
(d) tabular (e) frequency

Exercise 10.2

1. (a) Saturday (b) Tuesday and Friday (c) 54,000
2. (a) 175 (b) Red, 350 (c) 150
3. (a) March and May (b) 50000
(c) July (d) 2,24,000

4.

Day	Number of bulbs sold
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

5. (a) 7 symbols (b) Village B

6.

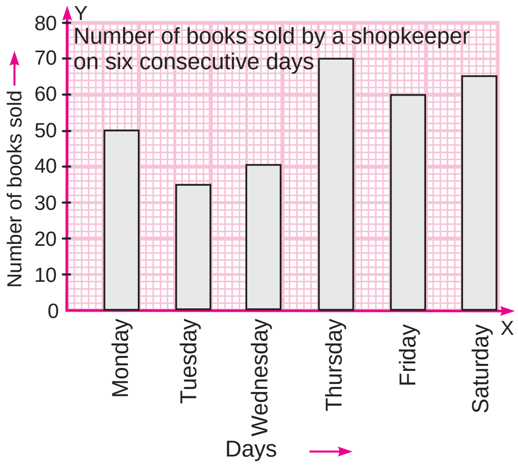
Class	Students
I	
II	
III	
IV	
V	

Exercise 10.3

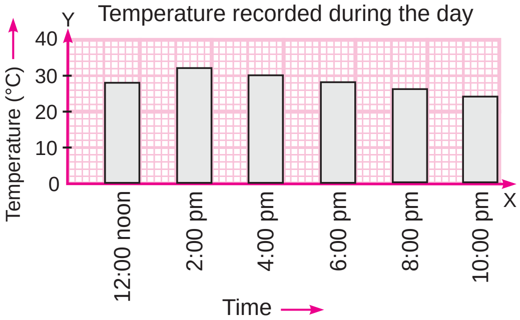
1. (a) May (b) 200 (c) 3 : 5
2. (a) 6:00 am – 7:00 am (b) 7:00 am – 8:00 am
(c) 2100
3. (a) 48 (b) Water park (c) 10
4. (a) 12 (b) Truck
5. (a) Tuesday (b) Sunday (c) 9°C
6. (a) ₹4000 (b) Wednesday
(c) Monday (d) ₹13,000

Exercise 10.4

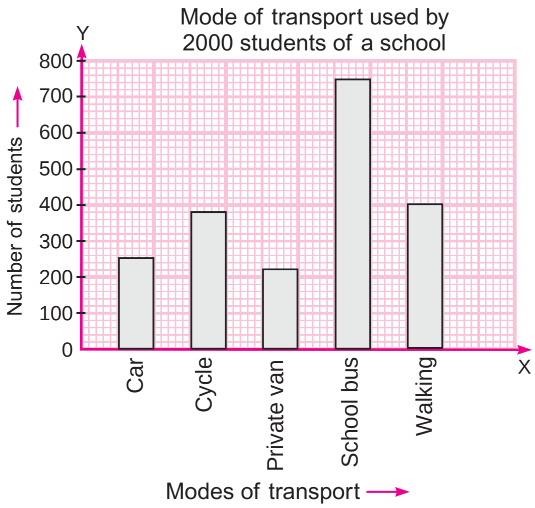
1. (a)



(b)

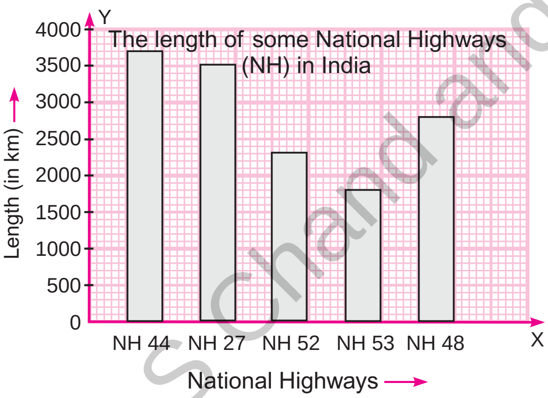


(c)



2. (a) 7 students (b) $\frac{19}{50}$

3.



Competency Based Exercise

1. (a) iv (b) i (c) iii (d) iv

2.

Blood group	Tally marks	Number of families
A	卌 卌 |	11
B	卌 ||	7
AB	卌 |	6
O	卌 |	6

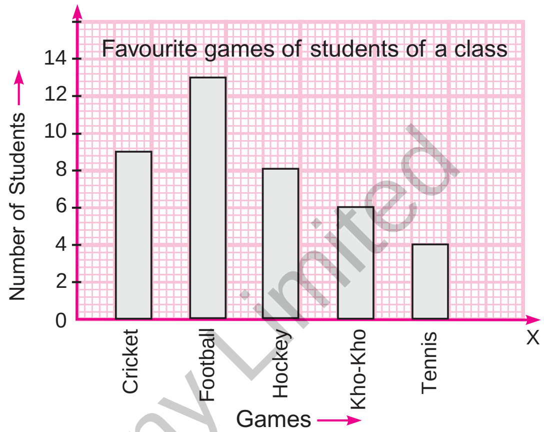
3. (a) 9 (b) 13 (c) 18

4. (a) Saturday (b) 100 (c) ₹11,000
(d) Do it yourself.

5. Do it yourself.

6. (a) 68 (b) Class X (c) Class IX

Challenge!



Let's Work in Mind

1. 13 **2.** 4 **3.** Tally marks

4. 25 persons **5.** 80 flowers

Assertion-Reasoning Questions

1. (a). **2.** (a) **3.** (a) **4.** (c)

Chapter 11: Mensuration

Warm-up

1. (a) perimeter (b) perimeter (c) area
(d) area (e) perimeter (f) area

2. (a) 24 cm (b) 24 cm

3. (a) ₹5 coin (b) ₹10 note

Exercise 11.1

1. (a) 22 cm (b) 24 cm (c) 39 cm (d) 30 cm
(e) 38 m (f) 38.4 m (g) 54 cm (h) 68 cm
(i) 36 cm (j) 32 cm (k) 22 cm

2. (a) 20 cm (b) 26 cm (c) 37.82 m (d) 55 cm
(e) 144 cm

3. (a) 4 cm (b) 22 cm (c) 2.1 cm (d) 100 cm

4. 10 cm **5.** 80 cm **6.** ₹13,800

7. 10.5 m **8.** 8 m **9.** 800 m **10.** 25 cm

11. 380 m, ₹3230 **12.** 18 cm **13.** 18 m

14. Radha, 60 m **15.** 120 m, 90 m

16. 6 cm **17.** 78.5 cm **18.** 6 rounds **19.** 6 cm

Exercise 11.2

1. (a) 16 sq cm (b) 19 sq cm (c) 16 sq cm (d) 8 sq cm
(e) 8.5 sq cm (f) 9 sq cm (g) 9 sq cm (h) 12 sq cm

2. (a) P = 22 cm, A = 12 sq cm
(b) P = 32 cm, A = 16 sq cm **3.** 26 sq cm

Exercise 11.3

1. (a) area = 120 sq cm
   (b) length = 160 cm, area = 14,400 sq cm
   (c) perimeter = 32 cm, area = 64 sq cm
   (d) side = 9 cm, perimeter = 36 cm
2. 40 cm
3. (a) 63 sq cm (b) 9 sq cm
4. kitchen = 91 sq m, living room = 200 sq m
5. 8 m **6.** (a) 31 sq m (b) 146 sq cm
7. 110 m **8.** 72 **9.** 160
10. ₹16,800 **11.** 16 cm **12.** 500 sq cm

Competency Based Exercise

1. (a) iii (b) ii (c) i (d) ii
2. 4 cm **3.** ₹24,700
4. (a) 50 sq m (b) 550 sq m **5.** 448
6. (a) 22 sq cm (b) 5 sq m **7.** 24 cm
8. 216 sq cm **9.** 56 cm

Challenge!

1. 666 cm **2.** 164 cm^2 **3.** 45 cm

Let's Work in Mind

1. area **2.** 450 sq m **3.** 10 cm
4. 8 m **5.** 4 units **6.** No

Assertion-Reasoning Questions

1. (a) **2.** (a) **3.** (b) **4.** (c)
5. (d) **6.** (c)

Chapter 12: Algebra

Exercise 12.1

1. (a) $2x + 1$ (b) $3x + 1$ (c) $5x + 1$
2. (a) $3n$ (b) $5n$ (c) $4n$ (d) $2n$
3. $7t$ km **4.** $5p$ cadets **5.** $(x - 7)$ laddus
6. $40x$ oranges

Exercise 12.2

1. (a) $7l$ (b) $4l$ (c) $5l$ (d) $6l$
2. $2(x + y)$ **3.** $d = 2r$
4. $2(l + b)$ **5.** $2x + y$ **6.** $4(x + y + z)$
7. xy **8.** $2(a + b + c)$
9. (a) T (b) T (c) T (d) T
   (e) T (f) F

Exercise 12.3

1. (a) $2a, 3b$ (b) $7x, -4x^2$ (c) $8, 2x, -y^2$
   (d) $4a, 3b, -c, \frac{1}{2}d$
2. (a) $x + 2y$ (b) $7a + 8b - 1$ (c) $x^2 + y^2 + 4z^3 + 1$
3. (a) $2, x, y, z$ (b) $7, a, a, b$ (c) $\frac{1}{5}, l, m, n, n$
   (d) $2, 3, \frac{1}{13}, a, a, b, c$ (e) $5, 5, 5, y, y, y$
   (f) $-3, 2, 2, a, a, a, z, z, z, z, x, x, x, x, x, x$
4. (a) 12 (b) $3y$ (c) $\frac{1}{2}yz^2$ (d) -1
5. (a) $3a^2b, \frac{1}{2}ba^2$ (b) $3yz, 4zy; 7xz, -5zx$
6. (a) Monomial (b) Monomial
   (c) Trinomial (d) Binomial
   (e) Monomial (f) Trinomial
   (g) Monomial (h) Binomial
   (i) Monomial
7. 19
8. (a) 4 (b) 1 (c) 29 (d) 19

Exercise 12.4

1. (a) $m - 3$ (b) $11y$ (c) $\frac{x}{4}$ (d) $\frac{4}{x}$
2. (a) $3x - 22$ (b) $2x + 5$ (c) $3y - 11$
3. (a) $3x + 4$ (b) $7x + 3$ (c) $13 - 5x$ (d) $12 - \frac{x}{7}$
4. (a) $\frac{4 + x}{3}$ (b) $4(x - 6)$ (c) $2n - 3$ (d) $3x + 5$
5. Set I : (a) iii (b) iv (c) i (d) ii
   Set II: (a) ii (b) i (c) iv (d) iii
6. (a) $(4b - 8)$ m (b) $(5b - 6)$ m
7. (a) $(4v + 10)$ km (b) $(3v + 15)$ km
8. (a) Bini = $s + 6$, Tini = $s - 3$ (b) Bini = $s + 5$, Tini = $s - 6$
9. $(4x - 3)$ years
10. Length = $7h$ cm and breadth = $(7h - 5)$ cm, where height = h cm
11. (a) Rohit scores 15 runs more than what Shikhar scores.
    (b) Rony has in his wallet six times the number of coins he has dropped in a saving bank.
    (c) Jenni's father is 3 times older than Jenni and her mother is 2 years younger than her father.
    (d) The number of girls in our class is 7 more than twice the number of boys in our class.
12. (a) iii (b) i (c) iv (d) ii
    (e) i

Exercise 12.5

1. (a) Equation (b) Equation (c) Expression
   (d) Expression (e) Equation
2. No **3.** Yes **4.** No
5. (a) $x = 4$ (b) $x = 8$
6. (a) $m = 10$ (b) $p = 4$ (c) $y = 8$

7. (a) $x + 3 = 5$; $x = 2$ (b) $9 - x = 3$; $x = 6$
(c) $3z = 15$; $z = 3$
(d) $\frac{1}{4}x = 6$; $x = 24$ (e) $y + 1 = 12$; $y = 11$
(f) $16 - 2x = 10$; $x = 3$

8. (a) $x = 3$ (b) $x = 11$ (c) $x = 12$ (d) $x = 4$
(e) $x = 7$ (f) $x = 2$ (g) $y = 6$ (h) $y = 4$

Exercise 12.6

1. (a) $x = 5$ (b) $y = 6$ (c) $z = 9$ (d) $x = 21$
(e) $p = 2$ (f) $t = 4$ (g) $u = 49$ (h) $z = \frac{1}{2}$
2. (a) $x = \frac{1}{5}$ (b) $x = 3$ (c) $x = 1$ (d) $z = 42$
(e) $p = 5$ (f) $p = 10$
3. (a) 15 (b) $p = 9$ (c) $x = 4$ (d) $g = 3$
4. 7 years 5. $y = 6$ 6. $\frac{7}{2}$ 7. 47
8. 27

Competency Based Exercise

1. (a) iv (b) iii (c) ii (d) iii
(e) ii (f) ii
2. (a) $x = 22$ (b) $x = 33$ (c) $n = 256$ (d) $x = 5$
(e) $a = 6.57$ (f) $y = 26$
3. 75
4. (a) $\frac{36}{x} = 6$; $x = 6$ (b) $p + 15 = 26$; $p = 11$
5. Uncle = $3z$ years, Aunt = $(3z + 4)$ years

Challenge!

1. $(2x + 10)$ oranges 2. ₹$(12x + 38)$

Let's Work in Mind

1. $50m$ matchsticks 2. p
3. ₹$(x - 1600)$ 4. 12
5. $5z$ 6. No

Assertion-Reasoning Questions

1. (a) 2. (c) 3. (b) 4. (b)
5. (a) 6. (d) 7. (c)

Chapter 13: Ratio and Proportion

Exercise 13.1

1. (a) 3 : 4 (b) 3 : 2 (c) 1 : 1
2. (a) 3 : 8 (b) 15 : 1 (c) 4 : 7 (d) 8 : 3
(e) 3 : 25 (f) 1 : 10 (g) 1 : 20 (h) 20 : 1
3. (a) 19 : 3 (b) 23 : 4 (c) 23 : 4 (d) 1 : 3
4. 5 : 74
5. (a)

Breadth	18	9	27
Length	30	15	45

(b)

Breadth	25	45	15
Length	35	63	21

6. (a) > (b) < (c) =
7. 1 : 2 8. 14 : 9
9. Manu: 12 pencils, Kanta: 18 pencils
10. (a) 4 : 11 (b) 1 : 3 11. 5 : 2 12. 5 : 4
13. 60 14. ₹192 15. 2 : 3

Exercise 13.2

1. (a) No (b) Yes (c) Yes (d) Yes
2. (a) Yes (b) No (c) No (d) No
(e) Yes (f) No
3. (a) 72 (b) 10 (c) 45 (d) 8
4. (a) 10 (b) 75 (c) 95 (d) 231
(e) 27
5. (a) 9 (b) 8 (c) 6 (d) 12
6. (a) 18 (b) 40 (c) 150
7. 27 8. 63 9. 15 oranges
10. ₹400 11. 8 kg 12. ₹8000

Exercise 13.3

1. ₹100 2. ₹198 3. ₹24,000 4. 365 L
5. 13 kg 6. 6 bananas 7. ₹12,800 8. 3°C
9. 18 kg 10. ₹252 11. 3 envelopes
12. 98 km 13. ₹198 14. 8 hours 15. ₹57,600

Competency Based Exercise

1. (a) i (b) ii (c) i (d) iii
(e) ii (f) ii (g) i (h) ii
2. 40 : 25 3. 1 : 12 4. 200 5. 149 : 160
6. ₹426
7. (a) 8, 40, 5, 25 (b) 2, 3, 8, 12 8. 20,880
9. 19 sweets 10. 980 : 1000
11. Manav = 200 stamps, Shourya = 150 stamps, and Gopesh = 100 stamps
12. ₹1890

Challenge!

1. 16 2. 50 toffees 3. 44 kg, 52 kg

Let's Work in Mind

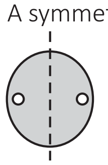
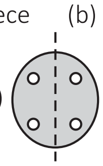
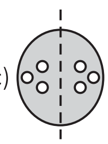
1. 35 cm 2. ₹2400 3. 60° 4. 2 : 3
5. ₹130

Assertion-Reasoning Questions

1. (a) 2. (a) 3. (c) 4. (d)
5. (c) 6. (b)

Chapter 14: Symmetry

Warm-up

1. (a) A symmetrical piece (b) iii
2. (a)  (b)  (c) 

Exercise 14.1

1. (a) *l* (b) *l* and *m* (c) *l* and *m* (d) none (e) *l* (f) *l* and *m* (g) *m*
2. (a) DE (b) AB (c) BE (d) AC, BD
3. No
4. Four; divider, protractor, set square and ruler
5. Yes

Exercise 14.2

1. (a) Yes (b) No (c) No (d) Yes (e) Yes
2. (a) horizontal (b) horizontal (c) vertical (d) none (e) vertical (f) none (g) horizontal (h) none (i) vertical (j) none (k) both horizontal and vertical (l) both horizontal and vertical (m) both horizontal and vertical (Every line through the centre) (n) none (o) vertical
3. (a) 2 (b) 0 (c) 1 (d) 3 (e) 6 (f) 1 (g) 8
4. Do it yourself. **5.** LM
6. (a) 0 (b) rectangle, rhombus (c) H, I, O X (d) infinite (e) 1

Exercise 14.3

1. (c), (f) and (h) **2.-6.** Do it yourself.

Competency Based Exercise

1. (a) ii (b) iii (c) ii (d) i (e) ii
2. (a) line symmetry (b) 5 (c) 2 (d) 100 (e) perpendicular bisector (f) diagonals (g) no (h) 1
3. (a) T (b) F (c) T (d) F (e) T (f) F
4. (a) and (c) **5.** (b) **6.** 2; S and R
7. 4; H, I, O, X **8.** 33 and 88
9. (a) 6 (b) 1 (c) 4 (d) 2 (e) infinite (f) 0
10. E, C and S

Challenge!

1. 12, 12 **2.** 6

Let's Work in Mind

1. AC **2.** Yes
3. 1 **4.** 30°–60°–90° set square, compasses

Assertion-Reasoning Questions

1. (d) **2.** (a) **3.** (d) **4.** (c)
5. (a) **6.** (c) **7.** (d) **8.** (a)

Chapter 15: Practical Geometry

Warm-up

1. Scale or ruler **2.** Protractor **3.** Compasses and divider
4. Set squares

Exercise 15.1

9. At centre of the circle **10.** AB = 5 cm, rectangle

Exercise 15.2

6. ∠AEB = 30°

Competency Based Exercise

1. (a) 90° (b) triangle (c) $27\frac{1}{2}°$ (d) protractor (e) line segment
10. Square

Challenge!

1. AB is perpendicular bisector of XY.
2. Yes, PM = PN

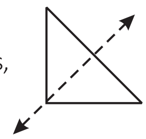
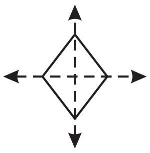
Let's Work in Mind

1. AQ = 4.4 cm **2.** ∠ROP = 30°
3. In the exterior of the circle

Assertion-Reasoning Questions

1. (a) **2.** (b)

Self Assessment – 2

1. (a) F (b) F (c) F (d) T
2. (a) 35.5 (b) equal (c) 10(*x* – 3) (d) right
3. (a) i (b) ii (c) iii (d) ii
4. Parallelogram
5. Line segments — Rays
   (a) $\overline{OA}, \overline{OB}, \overline{AC}, \overline{BD}$ — $\overrightarrow{AC}, \overrightarrow{BD}$
   (b) $\overline{BD}, \overline{DF}, \overline{BA}, \overline{DC}, \overline{FE}$ — $\overrightarrow{BA}, \overrightarrow{DC}, \overrightarrow{FE}$
6. ₹(*x* – 1600) **7.** –2500 **8.** (a) *x* = 5 (b) *x* = 7
9. (a) No (b) Yes,  (c) Yes, 
10. (a) 3 : 4 (b) 1 : 1

11.-12. Do it yourself

13. (a) 180 (b) Sujata's , $\frac{1}{35}$